COBEM 2021 26th International Congress of Mechanical Engineering
ABCM Associação Brasileira de Engenharia e Ciências Mecânicas
November 22-26, 2021, Virtual Congress, Brazil

QUANTIFYING SPATIAL UNCERTAINTY AND INFERRING THE STOCHASTIC WAVE ATTENUATION

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Abstract. *Variability is an inherent aspect of manufacturing processes such as machining and mechanical assembly. Deformations during the useful life of a structure are also a cause of variability. Statistical methods can be used to quantify variability in geometry or material properties. The variability can be in the form of random variables or stochastic processes, and the results obtained with the stochastic models will also be stochastic. The objective of the present study is to present how to use some statistical tools to simulate structural spatial variability and to infer on the stochastic vibration attenuation results. The stochastic simulations with homogeneous structures, and one-, two-, and three-dimensional spatially varying properties are presented. The Bayesian inference is applied to the stochastic attenuation response for homogeneously varying and one-dimensional height depending spatial variability cases.*

Keywords: *Uncertainty quantification, spatially correlated variability, Bayes' factor, statistical inference, manufacturing variability.*

1. INTRODUCTION

Manufacturing processes does not yield perfectly built designed systems. The system functionality can be compromised depending on the type and intensity of variability. There are some cases, where the variability os the mechanical properties is considerably smaller within batches than between batches, such as, in some concrete manufacturing (Aît-Mokhtar *et al.* (2013)) or for natural composite materials, such as bamboos, where the mechanical property variability can be almost homogeneous for a given section, but can vary along the plant height, and it can vary considerably from plant to plant (Ribeiro *et al.* (2019)). However, according to the way a system or part is manufactured, for example in a 3D printer where each manufactured slice presents small variability, the model can consider homogeneous properties for a given "slice" but varying for different slices along its length (Fabro *et al.* (2020); Beli *et al.* (2019)). If the manufacturing process is not very precise, the variability can be modeled along the structure in a three-dimensional form (Papazetis and Vosniakos (2019)).

When dealing with the influence of variability on vibration attenuation, statistical methods have been used to infer on the mechanical property and geometry statistical distribution, to simulate spatially correlated variability, and to infer on the stochastic results (Ribeiro *et al.* (2020a); Fabro *et al.* (2010); Souza *et al.* (2020); Liu *et al.* (2019)).

The objective of the present study is to present some methods for simulating different types of one- , two-, and three-dimensional spatial variability inherent to the structural system or part (Ribeiro *et al.* (2020b,a)). Statistical inference is also performed for some results obtained as stochastic vibration attenuation for some of the proposed cases (Ribeiro *et al.*

(2019)).

2. MODEL PARAMETER ESTIMATION

First, the mechanical property distributions can be estimated using frequentist or Bayesian estimators. Some common frequentist estimators are maximum likelihood (Fonseca *et al.* (2005)) and accept-reject estimators (Ribeiro *et al.* (2020d)) that use information related to the observed mechanical properties.

The Bayesian statistics is based on the Bayes' theorem, but using probability densities instead of probabilities (Robert (2007)). This method uses a prior distribution that is defined according to information obtained previous the observation is actually measured (Congdon (2014)). The posterior distribution carries all the information the researcher has about the variable to be inferred: the prior distribution and the likelihood function that carries information about the observations (Robert (2007)). The posterior distribution formula is shown in Eq. (1)

$$\pi(\boldsymbol{\theta} \mid \boldsymbol{x}) = \frac{\pi(\boldsymbol{x} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\pi(\boldsymbol{x})}, \tag{1}$$

where $\pi(\boldsymbol{\theta} \mid \boldsymbol{x})$ is the posterior distribution, $\pi(\boldsymbol{x} \mid \boldsymbol{\theta})$ is the likelihood function, $\pi(\boldsymbol{\theta})$ is the prior distribution (Congdon (2014)).

The posterior distribution can be estimated using an algorithm Markov chain Monte Carlo (MCMC) (Gamerman and Lopes (2006)). Any location (mean, median, mode) and dispersion estimator (variance, confidence interval) can be applied on the posterior distribution (Congdon (2014)). The Bayes' factor can also be used to infer on the statistical distribution of stochastic attenuation result (Ribeiro *et al.* (2020c,b)). Equation (2) presents the Bayes' factor formula that used the ratio of chances of a random variable being in some regions on the parameter space related to the prior ($o(1,0)$) and posterior distributions ($o(1,0 \mid \boldsymbol{x})$) (Robert (2007); Ribeiro *et al.* (2019)).

$$FB_{1,0} = \frac{o(1,0 \mid \boldsymbol{x})}{o(1,0)}, \qquad \text{where} \qquad o(1,0 \mid \boldsymbol{x}) = \frac{\pi(\theta_1 \mid \boldsymbol{x})}{\pi(\theta_0 \mid \boldsymbol{x})}, \qquad o(1,0) = \frac{\pi(\theta_1)}{\pi(\theta_0)}. \tag{2}$$

3. MODELING SPATIAL VARIABILITY

There are in the literature some procedures to simulate spatially correlated fields, such as the discrete Karhunen–Loève (KL) expansion (De Cursi and Sampaio (2015); Ribeiro *et al.* (2020b)), the expansion optimal linear estimation (EOLE) (Li and Der Kiureghian (1993); Ribeiro *et al.* (2020a)), and the Kernel smoother (Scott (2015)). All these methods require either a known correlation function that relates the spatial varying property or the correlation matrix estimated from the observations. A common assumption used in modeling such spatially correlated field is a exponential function related to the distance between the positions. The exponential usually has the correlation length parameter (λ) and the larger its value, more a position depends on near neighbors, and less it depends on far neighbors. As it is graphically presented in Fig. 1, the field becomes "smoother" - and it converges to be the field mean value - as λ increases.

If a vector $\boldsymbol{E}_x$ of length n is to be smoothed, it can be made premultiplying $\boldsymbol{E}_x$ by the Kernel matrix $\boldsymbol{K}$, whose i-th row and j-th column element is given by Eq. 3

$$k_{ij} = \frac{K\left(\frac{y_i - y_j}{\lambda}\right)}{\sum_{j=0}^{n} K\left(\frac{y_i - y_j}{\lambda}\right)}, \tag{3}$$

where the function $K(r)$ can be an exponential function of the spatial distance (r) with λ (Aydin (2007)). Then, the discrete and smooth vector ($\hat{\boldsymbol{E}}_x$) is obtained according to Eq. 4

$$\hat{\boldsymbol{E}}_x = \boldsymbol{K}\boldsymbol{E}_x. \tag{4}$$

4. HOMOGENEOUS ELEMENTS

First, the Young's modulus (E) is considered homogeneous for the entire structure (within), but different between structures. In the current research, the frame unit-cell (Fig. 3b) and frame structure presented (Fig. 3b) are proposed. It is also assumed that the between mechanical property variability was correctly estimated as the distribution in Fig. 2. Figure 4 graphically represents the first three structures with E given by the three first samples of this distribution.

5. SPATIALLY VARYING PROPERTIES

Now, cases where a structure with one- two- and three-dimensional spatially varying mechanical property are shown.

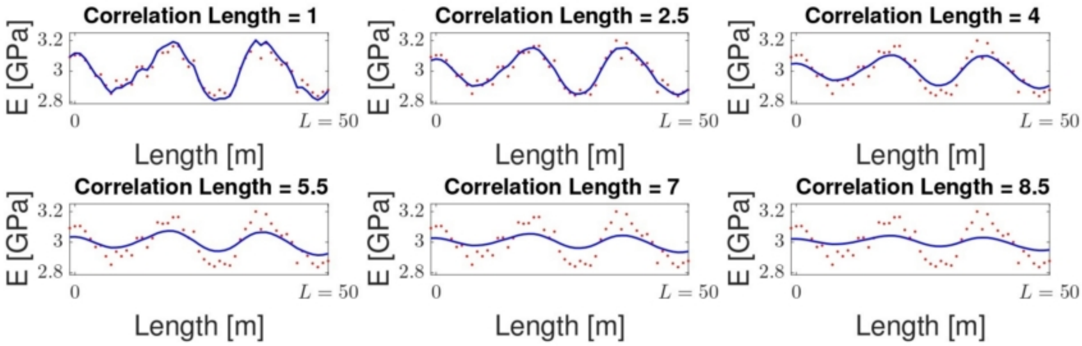


Figure 1: Kernel smoother applied to the same data using exponential function with different correlation length values.

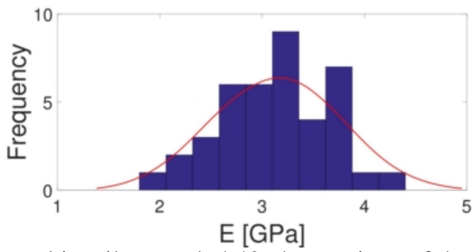
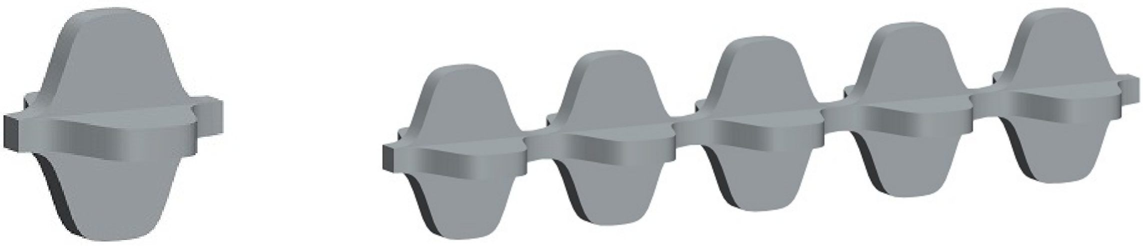


Figure 2: Estimated distribution from arbitrarily sampled 40 observations of the Young's modulus (E) from N(0.2,0.5).



(a) (b)

Figure 3: (a) - proposed unit-cell. (b) - proposed frame structure.

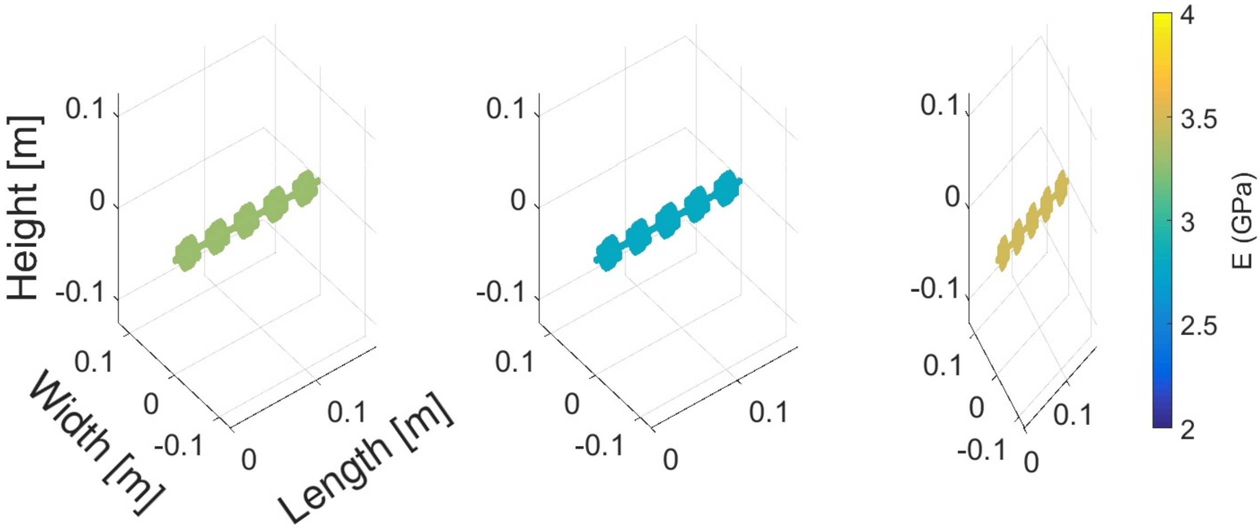


Figure 4: Three first observations sampled from Fig. 2 considered as different values for each entire structure.

5.1 One-dimensional

Now, the Kernel smoother was used to simulate two types of spatially correlated variability. First, it is considered that E follows the same probability density function (PDF) (Fig. 2) along the structure length. Figure 5a and Fig. 5b presents respectively some simulations of such distribution and the stochastic field for 40 samples of such spatial variability. Figure 5c and Fig. 5d presents the same results when E is height depending. In other words, for a given section, E follows the distribution N(0.2h,0.2+30h), where h is the web ear height.

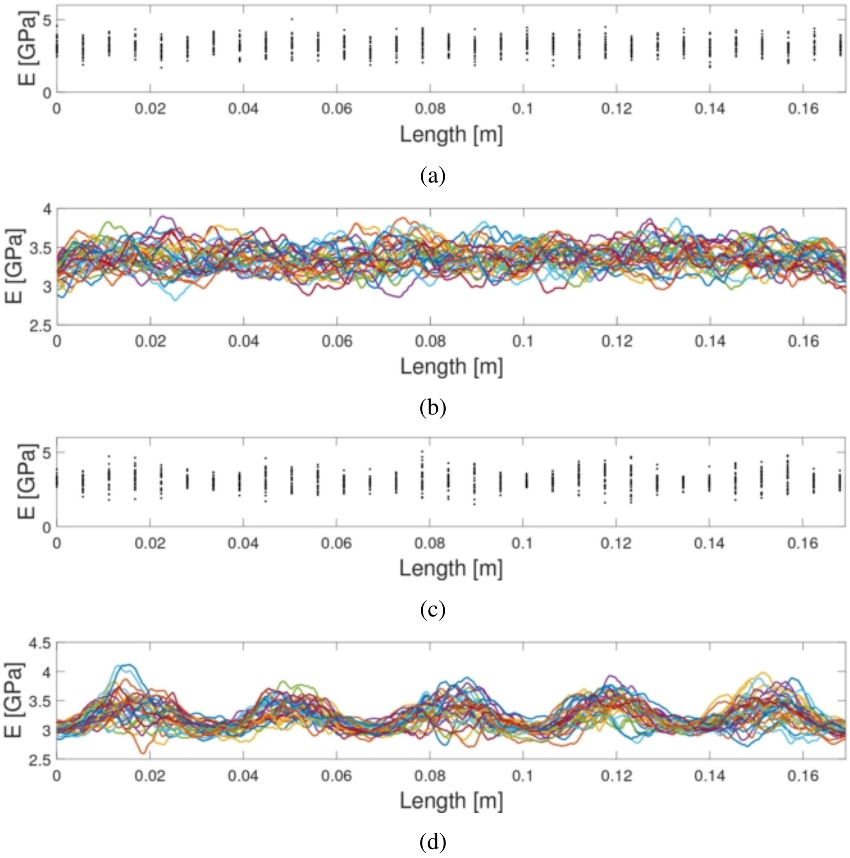


Figure 5: Samples along the structure length and simulated field using Kernel smoother from homogeneous ((a) and (b)) and height depending spatially varying Young's modulus ((c) and (d)).

Figure 6 the presents three first samples of structures with spatially varying E along their length homogeneously, i.e., following the simulated field in Fig. 5b. Figure 7 the presents three first samples of structures with the height depending E along their length homogeneously, i.e., following the simulated field in Fig. 5d. One can notice that there are high values in the web of the frames in Fig. 6 the probability of having larger values in the web is the same probability as it is for the we ear. Figure 7 presents no larger values of E in the web because its probability is much smaller than it is for the web ear, which presents some large values of E.

5.2 Two-dimensional

The two-dimensional example were obtained as the product of position depending PDFs in x and y: N(sin(x),0.3) and N(cos(y),0.3). Figure 8 presents samples and the smoother surface simulating a plate with spatially varying property.

5.3 Three-dimensional

Assuming again E varying spatially according to the distance a given position is far from the element center line (r) along its length: N(0.2r,0.2+30r). Figure 9 presents the three first frames, in slices, with E spatially varying according to r. After the values of E are samples for all the discrete positions, the three-dimensional Kernel, related to the distance r, is applied. Figure 10 presents the three first samples when the Three-dimensional Kernel is applied. On can notice that there are very different values in near positions in Figure 9 while it values of near positions are much more homogeneous in Figure 10.

6. INFERRING THE STOCHASTIC ATTENUATION

As examples, median, Bayes' factor with values of 3, 10, 30, and 100 (BF3, BF10, BF30, and BF100) were used to infer on the stochastic imaginary part of the flexural (of a beam), longitudinal (of a rod), and torsional (of a shaft)

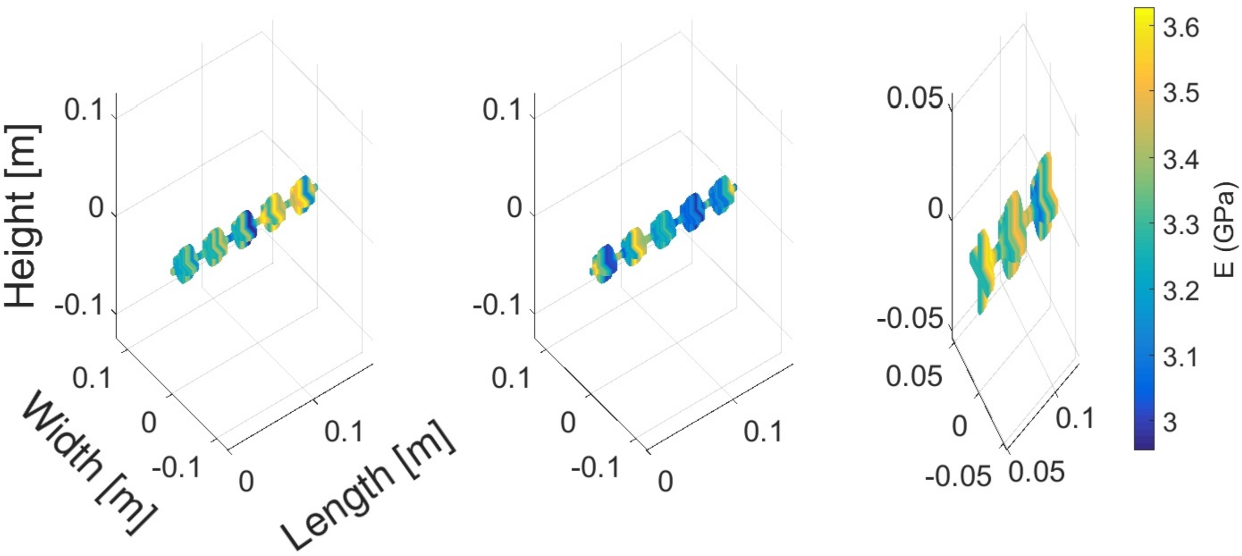


Figure 6: Three simulated beams with spatially varying E following the simulated field 5b. The third one is in zoom.

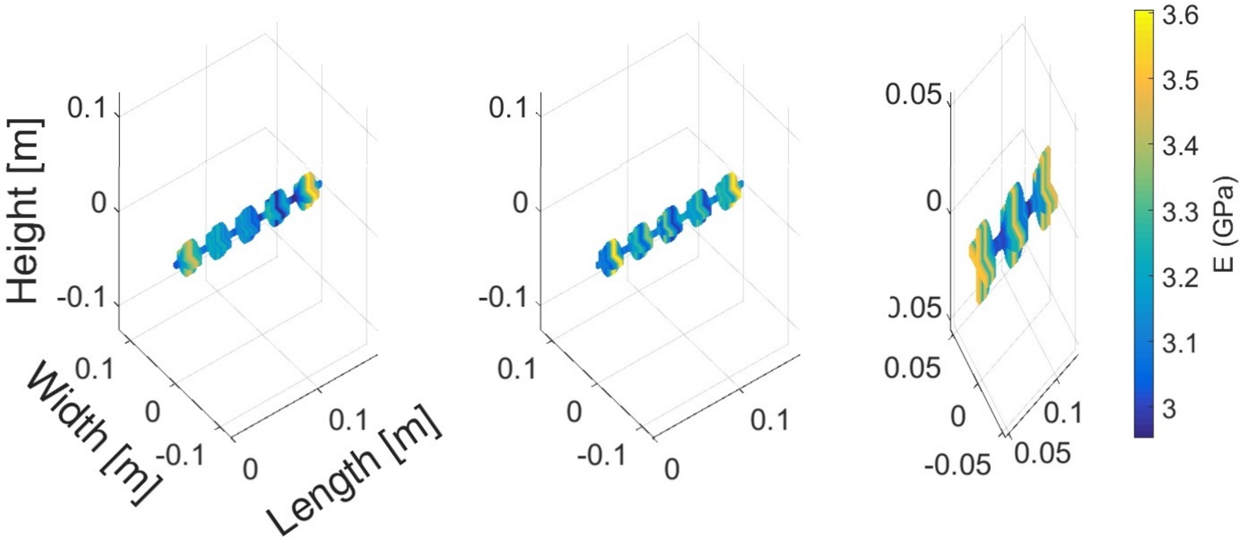


Figure 7: Three simulated beams with spatially varying E following the simulated field 5d. The third one is in zoom.

wavenumbers similarly as performed in (Ribeiro *et al.* (2020c)).

6.1 Homogeneous structures

First, inferences using Bayes' factor on the case of homogeneous frames, simulated according to Fig. 4, are presented in Fig. 11. The elements present the following attenuation bands in kHz: beam (1.4-1.6, 5.9-14.3), rod (8.5-25), and shaft (0.35-9, 10-24.2). Considering the FB100 (cyan line), for example, there is at least 100 of chances of occurring a complete band gap (intersection of all type waves attenuation bands) from 8.5 to 9 kHz and from 10 to 14.3 kHz.

6.2 Spatially varying properties

The same inference as previously presented when E is height depending (Fig. 5d). The results are presented in Fig. 12. The elements present the following attenuation bands in kHz: beam (1.1-2, 5-18, 20.6-25), rod (7.3-25), and shaft (0.3-9.5, 10.3-18, 18.3-25). Considering the FB100 (cyan line), for example, there is at least 100 of chances of occurring a complete band gap (intersection of all type waves attenuation bands) from 7.3 to 9.5 kHz and from 10.3 to 18 kHz.

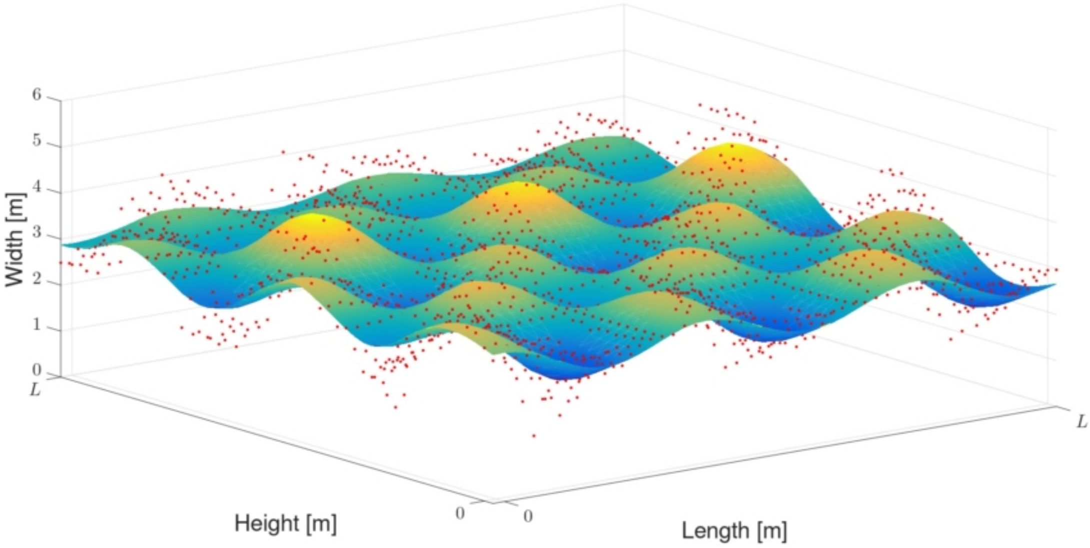


Figure 8: Sample from the product of the x and y depending PDFs: N(sin(x),0.3) and N(cos(y),0.3). The surface is the Kernel smoother using the distance in the two-dimensional space. It was used $\lambda = 3$.

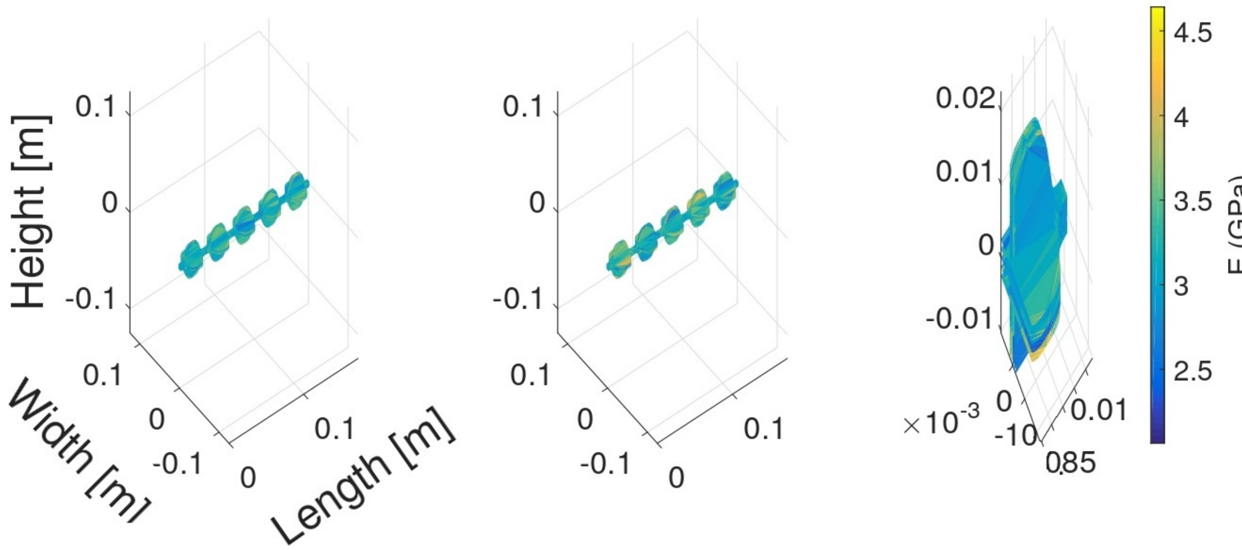


Figure 9: Three simulated beams with spatially varying E following a three-dimensional PDF. The third one is in zoom.

7. Concluding remarks

In the present study, some techniques for the modeling of one-, two-, and three-dimensional stochastic variability throughout structures are presented. These methodologies are applied to frame and plate structures to model their mechanical property spatial variability for different assumptions and using different spatial correlations. It is shown that these representations may be used to predict robust attenuation bands in quasi periodic structures.

8. ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the São Paulo Research Foundation (FAPESP) through processes number 2019/00315-8, 2018/15894-0, and 2014/19054-6, and the Brazilian National Council of Research CNPq (Grant Agreement ID: 420304/2018-5).

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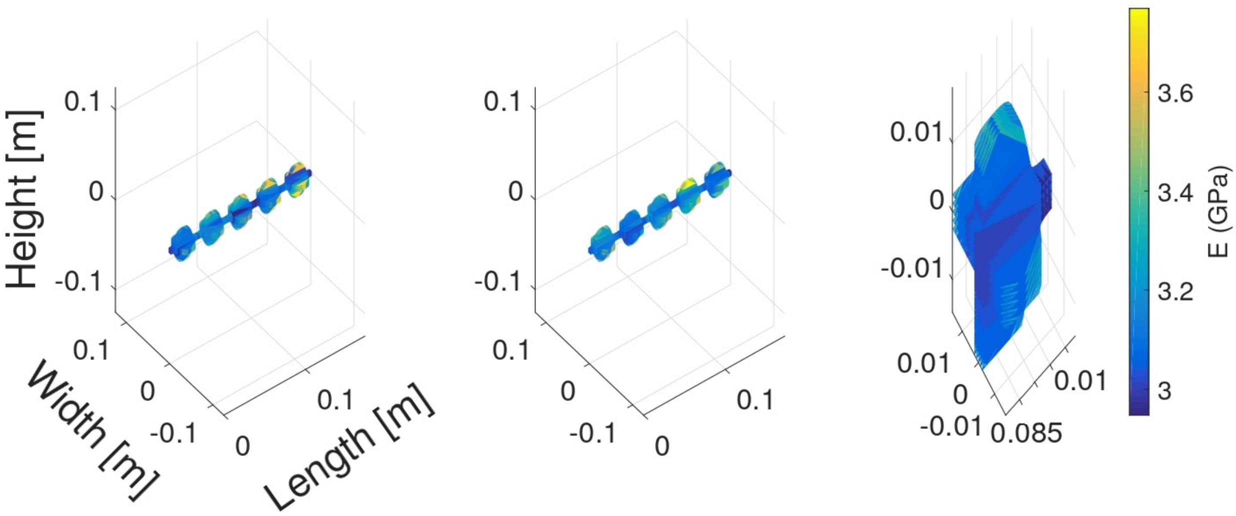

Figure 10: Three simulated beams with spatially varying E following a three-dimensional PDF after applied the Three-dimensional Kernel. The third one is in zoom.

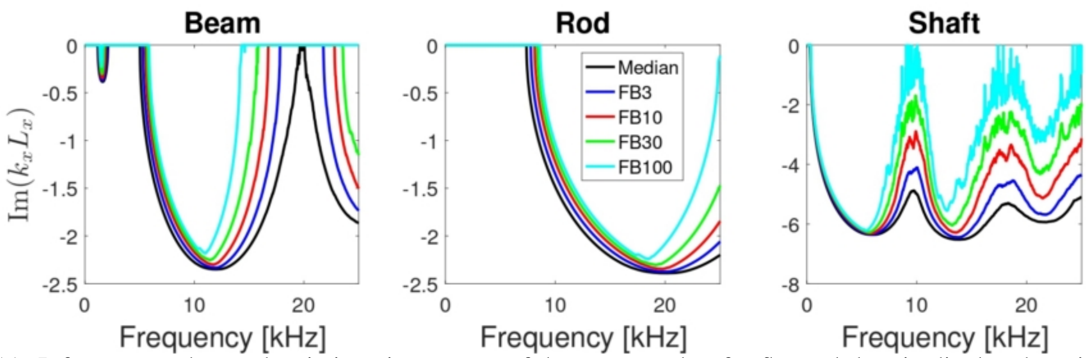

Figure 11: Inference on the stochastic imaginary parts of the wavenumber for flexural, longitudinal and torsional waves for variability presented in Fig. 4.

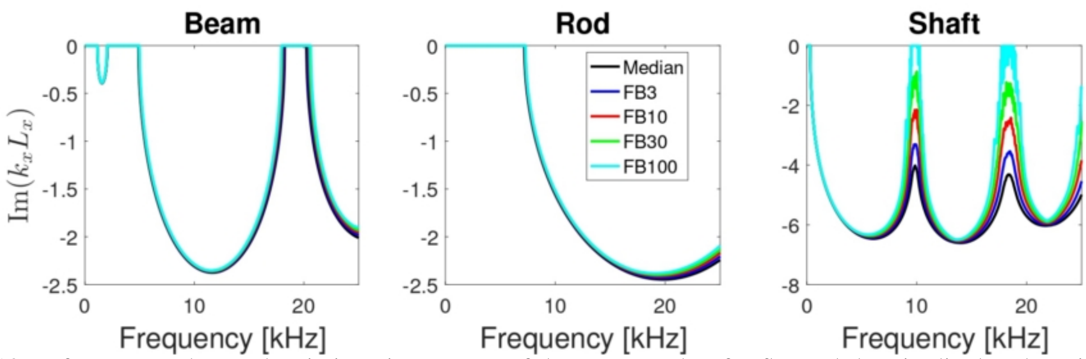

Figure 12: Inference on the stochastic imaginary parts of the wavenumber for flexural, longitudinal and torsional waves for variability presented in Fig. 5d.

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