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MULTIBODY MODELING AND OPTIMIZATION OF A DYNAMIC VEHICLE SUSPENSION SYSTEM

Jefferson Gomes de Carvalho Souza
Elias Dias Rossi Lopes
Gustavo Simão Rodrigues
Mechanical Engineering Section, Instituto Militar de Engenharia, Praça General Tibúrcio, 80, 22290-270, Rio de Janeiro, RJ, Brasil
E-mail: jeffersongomes@ime.eb.br, eliasrossi@ime.eb.br, simao@ime.eb.br

Taís Gomes de Carvalho Souza
Production Engineering Laboratory, Universidade Estadual do Norte Fluminense Darcy Ribeiro, Av. Alberto Lamego 2000, 28013-602, Campos dos Goytacazes, RJ
E-mail: taisgdcsouza@pq.uenf.br

Abstract. *The vehicle suspension system performs a fundamental role for the driveability, comfort and proper application of different vehicles, being responsible for reducing the vibrations induced by ground irregularities, increasing the vehicle components life cycle and reducing possible damages to the road. In this context, the purpose of this paper is to study and analyze the dynamic response in the time domain of a truck front active suspension system model, in a state space formulation based on Euler-Lagrange equations. Also, closed-loops control systems are designed and compared in order to optimize the suspension properties and response, based on two different strategies: Poles-placement/Full State Feedback (FSFB) and Linear Quadratic Regulator (LQR). The results for the numerical analysis show that for similar specifications of overshoot and settling time for the dynamic system, the LQR model can reduce Quadratic Performance Index/Cost Function (J) still obtaining better dynamic response.*

Keywords: *Active Suspension, LQR, Structural Optimization, Controllers, Dynamics.*

1. INTRODUCTION

Vehicle suspension systems are essential for the management of the driveability and vertical dynamics of a vehicle. In terms of vertical dynamics, the suspension systems must be able to support the vehicle's chassis, guarantee the contact between the tires and the ground, and mainly, attenuate the vibrations induced by road irregularities.

In the Dynamics field, mechanical vibrations are described as oscillations capable to deviate a body from its static state, which can be characterized as free or forced. Forced vibrations, in turn, are classified as physical phenomena caused by disturbing external forces, leading the mechanical system to forced oscillations (MCCALLION, 1973).

In this context, the design of a suspension system must consider parameters such as stiffness and damping coefficients, in order to obtain the behavior of the vehicle's suspended mass within acceptable and desirable levels.

Generally speaking, suspension systems can be considered passive, semi-active or active. In passive systems, the components have specific and constant characteristics, without the presence of an external energy source (SILVEIRA; PONTES JUNIOR; BALTHAZAR, 2014). In semi-active systems, the control action consists of adjusting the damping factor. This control can be achieved by changing the fluid flow of a shock absorber, or using magneto-rheological fluids, for example. In case of active systems, they may have sensors and actuators to apply an external control force to the system. For instance, the electromechanical and electropneumatic suspension models are recognized as active suspension systems. These kinds of engineering resources can neutralize with much higher efficiency the effects of forced vibrations.

Active suspension systems are an important topic in the actual vehicular dynamics tematic, since its main goal is to provide better dynamic response to the road irregularities, combining at the same time proper driveability and stability. In this context, some authors are implementing control techniques and metaheuristic algorithms in order to optimize active and semi-active suspension models. SOUBHIA (2011) presents the use of genetic algorithm and simultaneous strategy for optimization of plant and controller parameters, while PASCHOAL (2011) evaluates an approach with Fuzzy Logic.

This paper proposes a truck front active suspension system mechanism modeled by the action of a closed-loop electro-hidraulic system for a typical ¼ vehicle model, using the Euler-Lagrange formulation and the Optimal Quadratic Linear Regulator (LQR). In parallel, a Poles-placement controller is developed based on the design specifications (maximum overshoot and settling time). The controllers' performance are compared using the quadratic performance index, specified by the cost function (J). After simulating the 3 models in the state space form on Simulink, the dynamic response for the 2 degrees of freedom model are analyzed, considering a bump-type input. Then, comparisons are made between the systems subject to a unit step-type excitation in order to verify the controllers' efficiency.

2. DYNAMIC SYSTEM MODEL

The truck suspension system can be represented by a ¼ vehicle model as showed in figure 1. For the preliminary study of the vehicular dynamics, applied to a passenger vehicle, this model can be expressed by a multibody mechanism consisting of a coupled double mass-spring-damper (Elmadany and Abduljabbar (1999) e Fathy *et al.* (2003)).

Using the Lagrange-Euler's formulation for the conservative forces and adding the non-conservative damper forces and the control force $\boldsymbol{fa}(t)$ to the model, the two equations of motion are described as (1) below:

$$
\begin{aligned}
&M1\ddot{\boldsymbol{Z}}\boldsymbol{1}(t) = C2\left[\dot{\boldsymbol{Z}}\boldsymbol{2}(t) - \dot{\boldsymbol{Z}}\boldsymbol{1}(t)\right] - C1\left[\dot{\boldsymbol{Z}}\boldsymbol{1}(t) - \dot{\boldsymbol{Z}}\boldsymbol{r}(t)\right] + K2\left[\boldsymbol{Z2}(t) - \boldsymbol{Z1}(t)\right] - K1\left[\boldsymbol{Z1}(t) - \boldsymbol{Zr}(t)\right] - \boldsymbol{fa}(t);\\
&M2\ddot{\boldsymbol{Z}}\boldsymbol{2}(t) = -C2\left[\dot{\boldsymbol{Z}}\boldsymbol{2}(t) - \dot{\boldsymbol{Z}}\boldsymbol{1}(t)\right] - K2\left[\boldsymbol{Z2}(t) - \boldsymbol{Z1}(t)\right] + \boldsymbol{fa}(t)
\end{aligned}
\tag{1}
$$

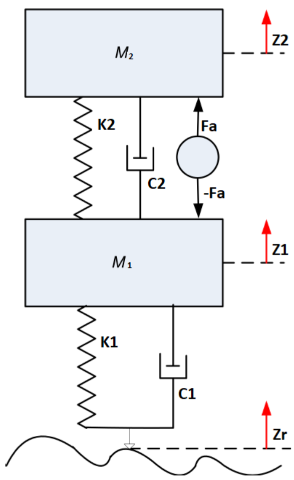

Figure 1: Quarter-car active suspension model. Available from: QUANSER Manual (2010).

Where the spring stiffness and suspension damping coefficient are $\boldsymbol{K2}$ and $\boldsymbol{C2}$**,** respectively. The tire is modeled as a spring-damp, also characterized by $\boldsymbol{K1}$ and $\boldsymbol{C1}$ coefficients**.** The displacement of mass $\boldsymbol{M1}$ is represented by $\boldsymbol{Z1}$ while the displacement of mass $\boldsymbol{M2}$ is represented by $\boldsymbol{Z2}$, and $\boldsymbol{fa}(t)$ represents an active control force.

The system above can be described in MATLAB/Simulink blocks representation as figure 2:

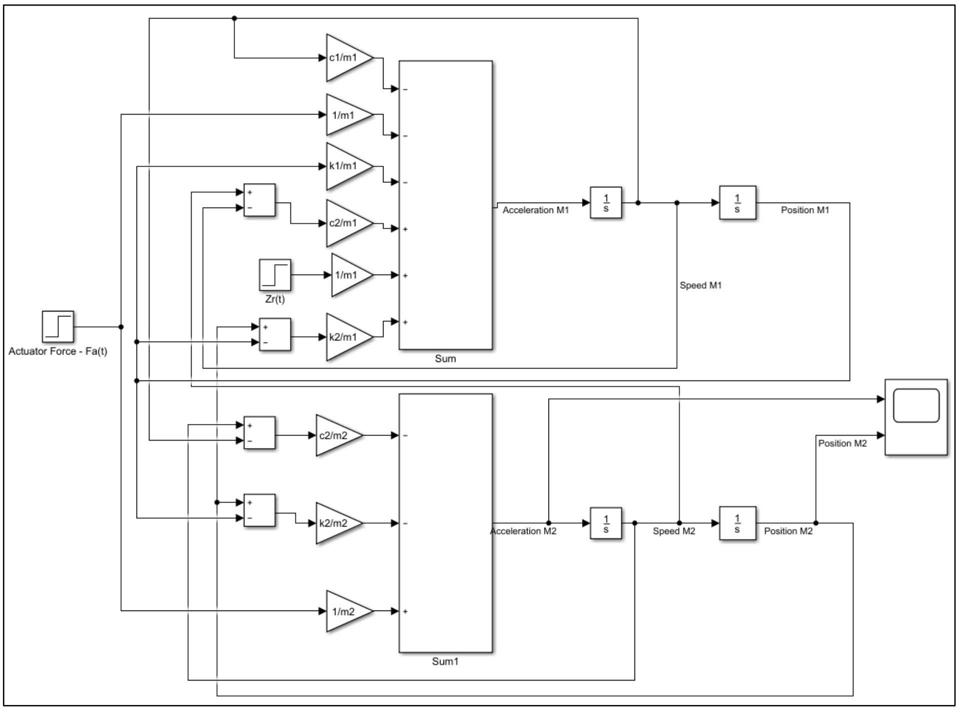

Figure 2: System Block Representation (Simulink)

When subjected to an unit step $Zr(t)$, $K1$ and $C1$ are immediately compressed establishing a new condition for the whole system. Besides, in order to perform proper simulations, the equations (2) and (3) are used to mathematically model the displacement of suspension in contact with the road, along the path of the quarter-car model (RUTHES, 2016):

$$Zr(t) = \frac{h}{2}\{1 - \cos[w.(t - \frac{d}{v})]\} \tag{2}$$

$$w = \frac{2\pi v}{L} \tag{3}$$

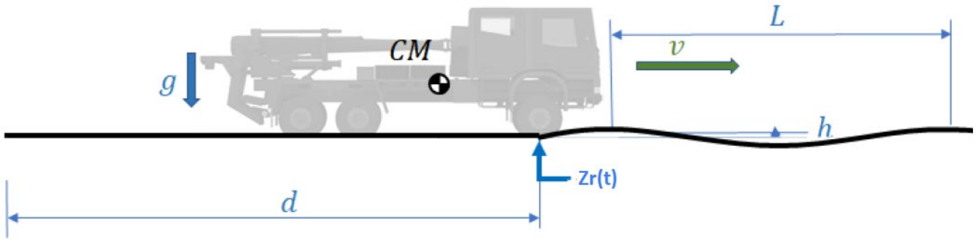

Figure 3: Road Bump Excitation (Author)

A sinusoidal function can perfectly describe the movement of the vehicle as it passes through the speed bump. However, in a real situation, the vehicle travels some distance before crossing the obstacle. Thus, equation (2) can model this scenario. In parallel, equation (3) considers the value of the angular velocity (w) of the vehicle as a function of the length of the speed bump (L) and the quarter-car speed. The unknown value (d) represents the distance traveled before the bump, while (v) corresponds to the constant speed of the vehicle's path on the trajectory $d + L$. For reference, $Zr(t)$ parameters also matches to Brazilian National Traffic Council (CONTRAN) resolution 39/98 construction parameters.

Therefore, it is very convenient to define the following state variables:

$$x1(t) = Z2(t), \quad x2(t) = \dot{Z}2(t), \quad x3(t) = Z2(t) - Z1(t), \quad x4(t) = \dot{Z}2(t) - \dot{Z}1(t). \tag{4}$$

The system above can be written in the state space form as follows, where $\frac{fa(t)}{Zr(t)} = u(t)$:

$$\dot{x} = Ax(t) + Bu(t), \tag{5}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{C2}{M2}\frac{C1}{M1} & 0 & \frac{C2}{M2}\left(\frac{C2}{M2} + \frac{C2}{M1} + \frac{C1}{M1}\right) - \frac{K2}{M2} & \frac{C2}{M2} \\ \frac{C1}{M1} & 0 & -\left(\frac{C2}{M2} + \frac{C2}{M1} + \frac{C1}{M1}\right) & 1 \\ \frac{K1}{M1} & 0 & -\left(\frac{K2}{M2} + \frac{K2}{M1} + \frac{K1}{M1}\right) & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{M2} & \frac{C2}{M2}\frac{C1}{M1} \\ 0 & -\frac{C1}{M1} \\ \frac{1}{M1} + \frac{1}{M2} & -\frac{K1}{M1} \end{bmatrix} \tag{6}$$

Before the action of any road input or active control force, the system is in stationary state, so zero initial conditions can be assumed:

$$x1 = 0, \; x2 = 0, \; x3 = 0, \; x4 = 0. \tag{7}$$

In vehicle active suspension optimization projects, the following indices are generally considered: the passenger comfort and suspension deformation (driveability). The use of this criteria imposes some restrictions to $x2 = \dot{Z}2(t)$ and $x1 = Z2(t)$, respectively, and for sure, to $fa(t)$ which is the control force to be developed by the controller.

3. QUADRATIC PERFORMANCE INDEX

At the optimization process, a performance index widely used in the literature is the quadratic performance index (Thompson (1976) and Elmadany and Abduljabbar (1999)), which is presented in (8).

$$J = \int_0^{\infty} q1.fa^2(t) + q2.Z2^2(t) + q3.\dot{Z}2^2(t) \tag{8}$$

Where 1 , $q2$ and $q3$ are the penalty constants. With the criteria described above, which considers $x2 = \dot{Z}2(t)$ and $x1 = Z2(t)$ to be optimized, we can obtain the Cost Function J in terms of the state variables $x(t)$ and the control force $fa(t) = u(t)$ on a matrix form.

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t) \tag{9}$$

Where the penalty matrices Q and R are taken to be positive semidefinite as follows:

$$Q = \begin{bmatrix} q2 & 0 & 0 & 0 \\ 0 & q3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; R = [q1] \tag{10}$$

4. CONTROLLER DESIGN

The controller projects are developed applying two different strategies, in order to evaluate the dynamic response and to compare both techniques. The first one is the Linear Quadratic Regulator (LQR), which consists of minimizing the cost function $\boldsymbol{J}$. The second method is the Full State Feedback Controller, which consists of discovering the gain matrix K that provides the desired behavior to the system, considering the specifications defined for the closed-loop system.

4.1 Linear Quadratic Regulator (LQR)

Optimal controller designs for linear systems using LQR are easily found in the literature (Kwakernaak and Sivan (1972) e Kailath (1979)). For a continuous system described as (5), the LQR problem is intended to determine a full state feedback (FSFB) controller that minimizes the Cost Function J. In order to find the minimum, both matrices Q and R are considered the weight matrices, such that $Q = Q' \geq 0$ and $R = R' \geq 0$.

The optimal controller LQR that minimize the Cost Function (9) is given by:

$$u(t) = Kx(t), \text{ when} \tag{11}$$

$K = -R^{-1}B'P$, where $P = P' \geq 0$, is the solution for the algebraic Ricatti's equation:

$$A'P + PA - PBR^{-1}\, B'P + Q = 0 \tag{12}$$

4.2 Full State Feedback Controller (Poles-placement)

The application of the poles-placement starts with choosing the desired poles, based on certain natural response or frequency specifications. These specifications can be, for example, the maximum overshoot Mp, damping ratio ζ and settling time Ts. Assuming that exist a closed-loop desired poles to match the project specifications, there is a gain matrix K capable, trough the feedback of states, to force the closed-loop poles of the system to allocate themselves in the desired positions, as long as the system is fully controllable and observable. In usual controllers project, it is possible to design a system to meet the specification of maximum overshoot and settling time, based on the natural frequency and the damping ratio of the dominant pair of complex poles, as follows:

$$Mp_{\%} = \exp(-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}) \tag{13}$$

$$Ts = \frac{4}{\zeta\, w_n} \tag{14}$$

5. NUMERICAL ANALYSIS AND CONTROLLERS PERFORMANCE

The numerical simulations and optimization of the control force $fa(t)$ were made through MATLAB/Simulink, where the whole truck parameters are applicable to a SCANIA P 410 CB 6 x 6 model. The most relevant parameters for the truck vertical dynamics study are described at table 1 (DING *et al,* 2012).

Table 1. SCANIA P 410 CB 6 x 6 parameters.

SCANIA P 410 General Parameters	**Value**
Truck Mass	22.560 kg
Chassis Mass	18.470 kg
Front Unsprung Mass	490 kg
Intermediate Unsprung Mass	1.700 kg
Rear Unsprung Mass	1.700 kg
Tire (Michelin 16.00 R20 XZL)	
Cornering stiffness	5.600 N/º
Damping coefficient (C1)	2.660 N s/m
Tire Rigidity (K1)	770.700 N/m
Wheel Inertia momentum	14 kg m²
Indeformable radius	0,4515m
Static radius	0,542m

Figure 4: SCANIA P 410 CB 6x6. Available from: ACUÑA (2017)

The paper study consists of the ¼ active front suspension model for the SCANIA truck, with the parameters resumed at table 2:

Table 2. ¼ truck active front suspension model parameters.

Suspension Parameters	**Value**
Quarter Truck Mass (M2)	4500 kg
Suspension Mass (M1)	490 kg
Quarter Suspension Spring Coefficient (K2)	59.600 N/m
Wheel Spring Coefficient (K1)	770.700 N/m
Suspension Damping Coefficient (C2)	20.000 N s/m
Wheel Damping Coefficient (C1)	2.660 N s/m
Control Force $fa(t)$	To be optimized
Gain matrix K	To be optimized

Finally, for a realistic simulation of the road bump ($Zr(t)$ function), the following parameters are established in table 3 contents:

Table 3. Parameters for the speed-bump function.

Bump parameters	**Value**
Bump height (h)	0,08 m
Bump length (L)	1,5 m
Distance traveled before the bump (d)	20 m
Quarter-car speed (v)	30 km/h

The open-loop system is exactly the passive suspension model. In this case, the values observed are just a result of the simulation process for the content of table 2, while the gain matrix $K = 0$, since there is no controller or control force designed.

The controller project was made based on an input from the control action $fa(t)$ since the system presents itselfs as a regulator, keeping its current state and, once subject to disturbances, it performs a control action in order to return to your previous position. Through MATLAB functions “crtb” and “obsv”, it is possible to assure that the system is controllable and observable.

In order to choose the poles-placement for the FSFB, a project specification for the closed loop performance is needed. With the state space model designed, a maximum overshoot $Mp = 25\%$ and the settling time $Ts = 1.8\ sec$ for $x1 = Z2(t)$ response will be assumed. With MATLAB by setting the quarter-truck mass ($M2$) position ($x2$) as the output, the open-loop Laplace transform transfer function described below is found:

$$\frac{24.13\,S^2 + 7062\,S + 20830}{S^4 + 50.69\,S^3 + 1732\,S^2 + 7062\,S + 20830} \tag{15}$$

Also using MATLAB, it is possible to find out the open-loop poles of the passive suspension system. In order to meet the specifications and considering equation (13), let’s consider there is a gain matrix K which places the closed-loop dominant poles as follows: $[-2.31 \pm 2.938]$. The second pair of poles were fixed in order to keep the distance of the dominant poles as follows: $[-23.233 \pm j31.343]$.

After finding the closed-loop poles for the project specifications defined, we can obtain the gain K matrix using MATLAB in two steps:

1) Set the location of the closed-loop poles;
2) Find gain matrix K.

Finally, the LQR model is an optimized closed-loop system, which consists of the plant parameters, described on table 1, R, and Q. Gain K, and control force $fa(t) = u(t)$ are to be found. To calculate Ricatti’s equation and consequently, the LQR gain matrix K, let’s assume that $q1 = 10^2$, $q2 = 2$, $q3 = 1$.

With Simulink State Space models as showed on figure 3, the initial conditions described (7), $Zr(t)$ described (2) and the unit-step response, both controllers for the active and the passive suspension can be analyzed.

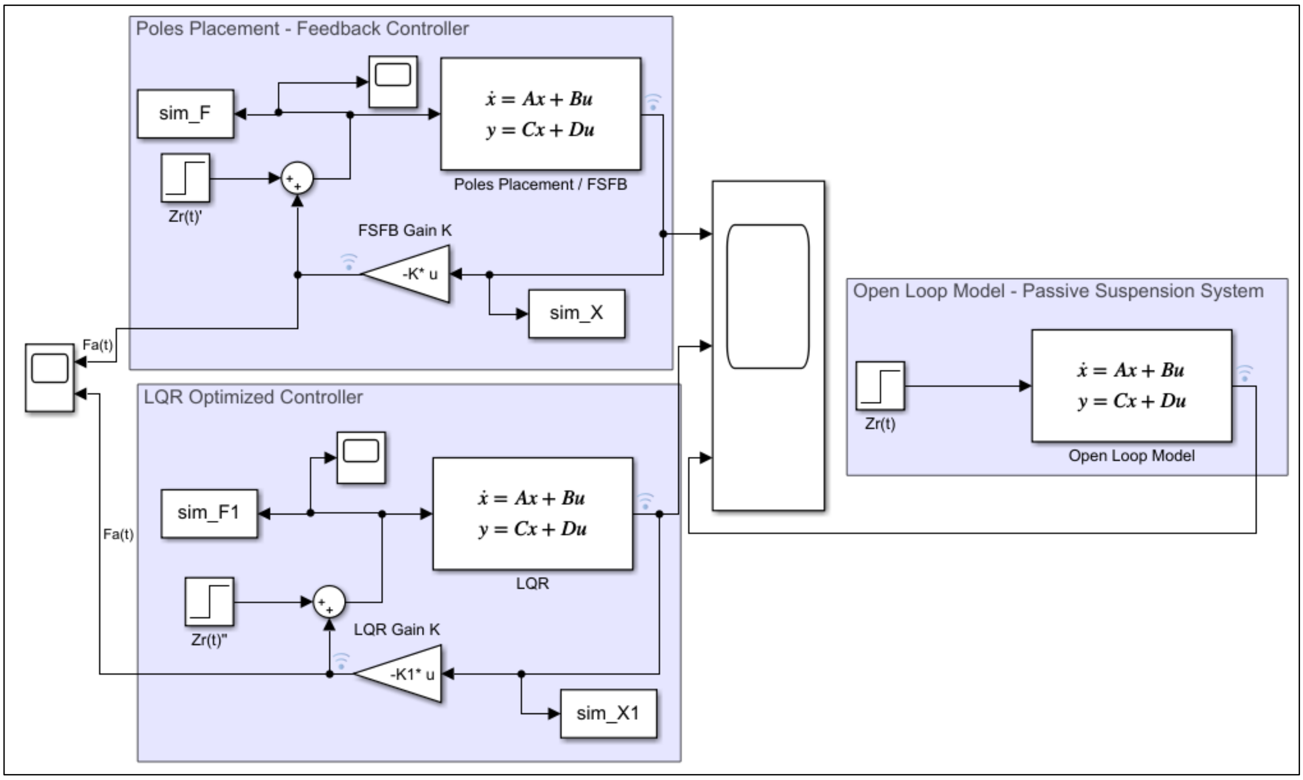

Figure 5: State Space Model for LQR, FSFB and Open loop

The figure 6 and 7 show the states $x1$ and $x2$ behavior during 3.5 seconds, respectively, for each LQR, Pole-placement and open-loop model, once subject to a unit step function and the control force $fa(t)$, modeled by the closed-loop A matrix $A_{cl} = A + BK$, for each designed K matrix.

At figure 4, it is possible to note that the highest $Z2(t)$ overshoot occurs for the open-loop model, followed by the Pole-placement model, while the same behavior is seen for $\dot{Z2}(t)$ curve. As the control project requirement, both pole-placement and LQR models presents values for settling time below 1.8 sec.

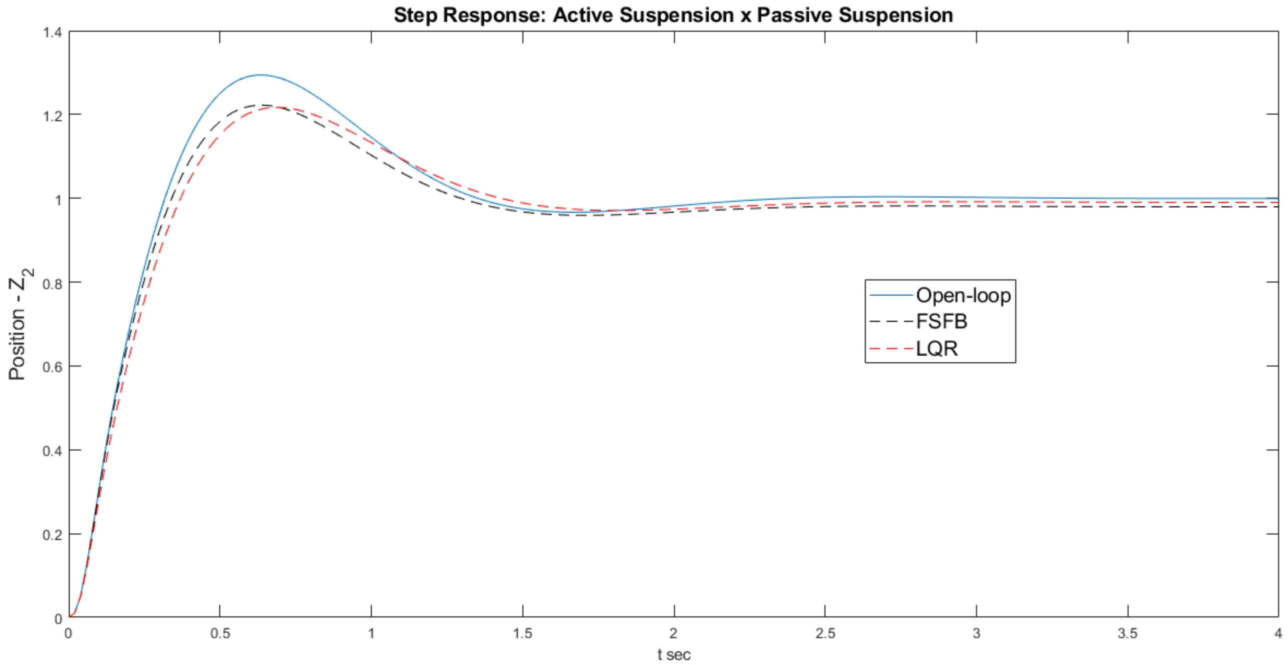

Figure 6: Step response curve for $Z2(t)$

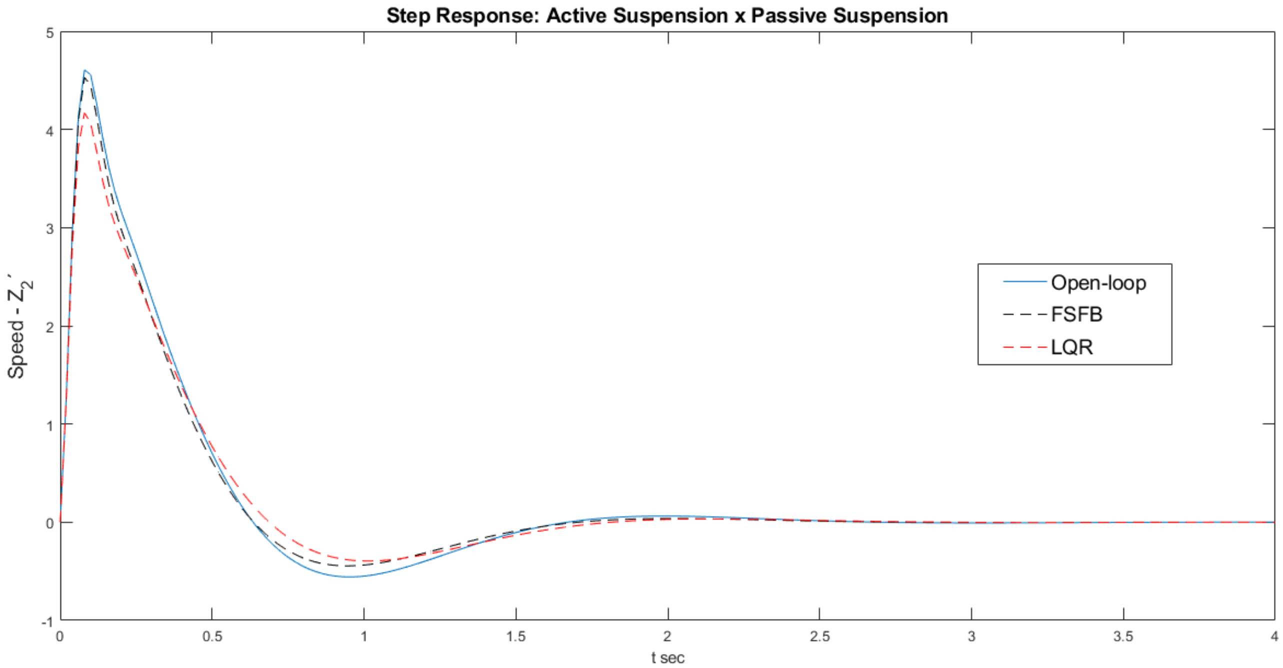

Figure 7: Step response curve for $\dot{Z}2(t)$

The figure 8 and 9 show the states $x1$ and $x2$ behavior for a $Zr(t)$ function, which was modelled by a trajectory condition described on table 2. Considering that the car touched the road bump after 2.5 seconds, it is observed that the LQR model was able to stabilize both $\mathrm{Z}2(t)$ and $\dot{\mathrm{Z}}2(t)$, which are, respectively the driveability and the passenger comfort as mentioned.

It is also clear that for the position and speed of the quarter-truck mass graphs, the lowest peak level was obtained with the LQR model, followed by pole-placement FSFB model. Checking both graphics parameters, it is possible to see that, compared with passive suspension system, both controllers bring the system to the stationary regime at least 0.5 seconds before, which is a representative improvement considering a single road bump as described.

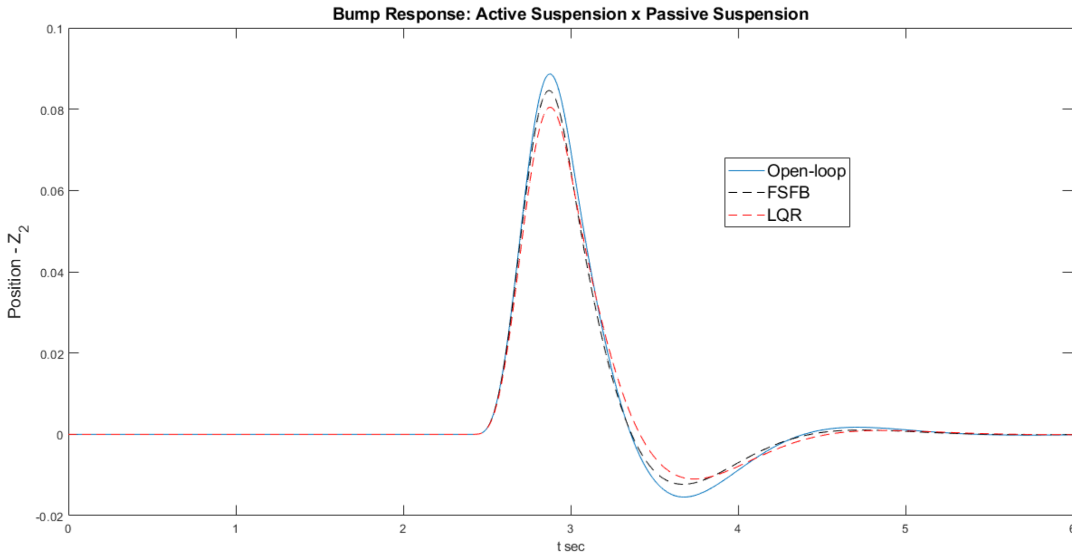

Figure 8: Road bump response curve for $Z2(t)$

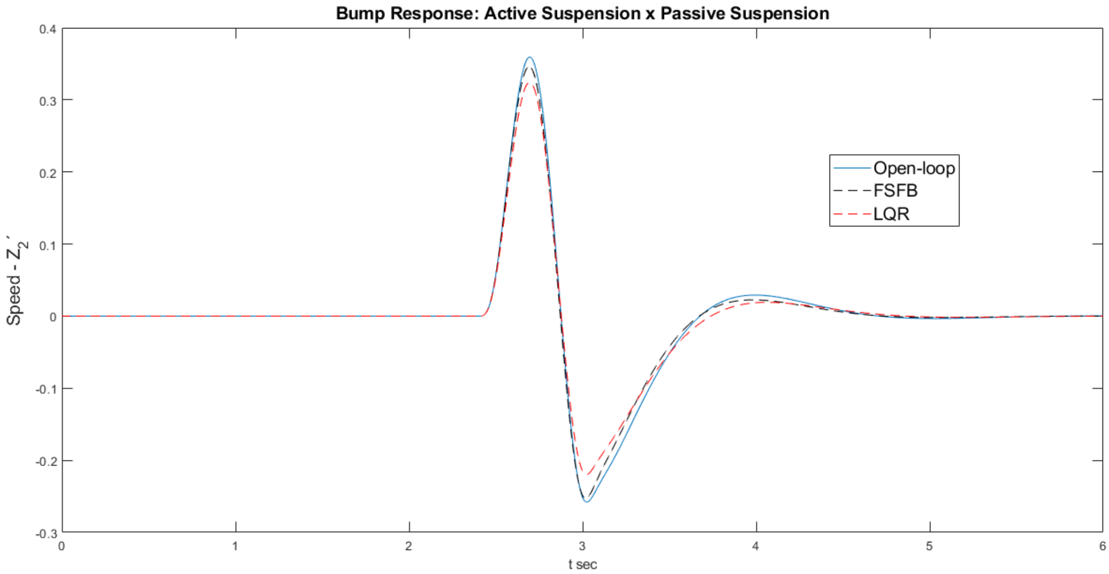

Figure 9: Road bump response curve for $\dot{Z}2(t)$

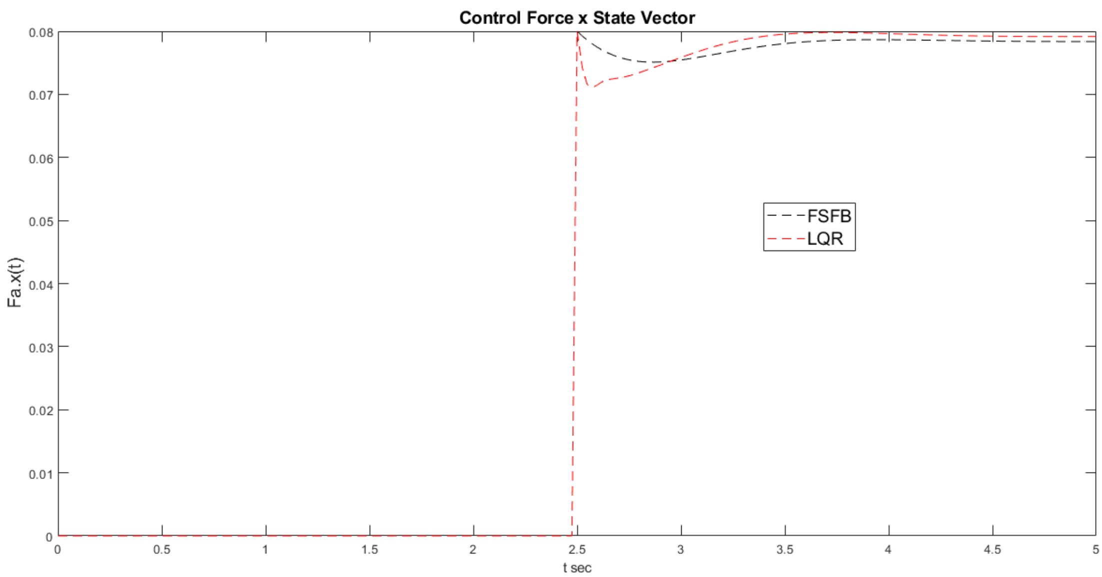

Figure 10: Control force $fa(t) = K.x(t)$ actuation curve

The figure 10 shows the characteristic of the control force $u(t) = K.x(t)$, in order to bring the system to stationarity. It is possible to verify how LQR can control faster and with less energy than the pole-placement controller. The controller is able to get better results since it can actuate in an independent manner at the different states $x1$ and $x2$, while the pole-placement is focused on managing the output based on the damping ratio and the overshoot, acting trough all the states, what can actually turns the technique unfeaseble since some dynamic systems, like active suspension, are not designed to have all the states measured.

Finally, using matlab comands and functions to reach the performance index J, the gain K matrix, the overshoot Mp and settling time Ts for each active and passive suspension systems, where applicable, when the system is submitted to the unit-step, it is possible to compare the accomplishment and resources needed for the time dynamic response found.

Table 4. Results for models simulations

Suspension System Model	**Cost Function (J)**	**K matrix**	**% Mp**	**Ts**
Open-loop model (Passive)	-	-	29.429	1.9762 sec
Pole-placement FSFB Control (Active)	204.67	$0.020 \quad 0.022 \quad 0.101 \quad -0.0002$	24.74	1.7915 sec
LQR Control (Active)	185.59	$0.009 \quad 0.039 \quad 0.159 \quad -0.0020$	22.89	1.3772 sec

In order to reinforce that the application of a control system can guarantee a proper behavior for the dynamic system, a sensitive analysis is provided in figure 11, evaluating the effects of changes on the plant parameters and on the control system properties. Establishing 10% of reductions of plant parameters such as suspension damping coefficient and suspension spring coefficient and 10% of reduction of control system properties (LQR matrix index), we are able to understand these effects.

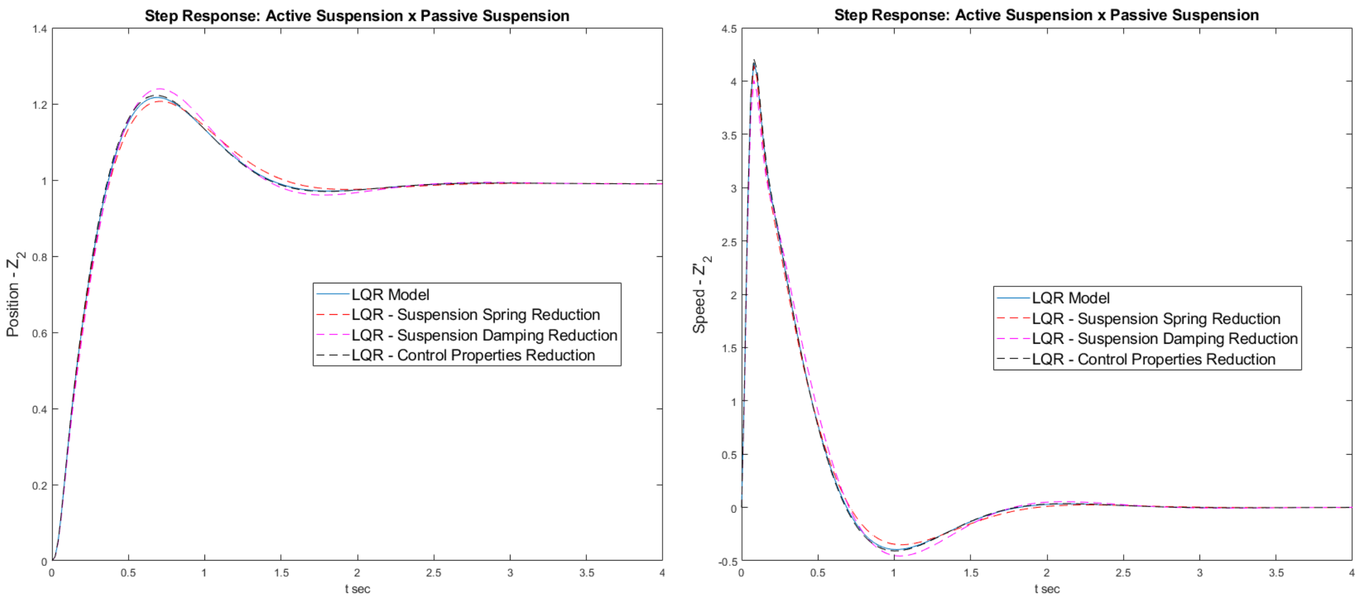

Figure 11: Sensitivy Analysis for properties reductions

The figure 11 shows how the states $x1$ and $x2$ behave during a unit step type excitation for four different changes in the system parameters. The table 5 presents the impacts to the systems time response occasioned by the properties reduction.

Table 5. Results for sensitive analysis

Suspension System Model	**% Mp**	**% Increment**	**Ts**	**% Increment**
LQR Model	22.89	-	1.3772 sec	-
LQR (10% $K2$ reduction)	21.85	-5,4 %	1.4547 sec	5,6 %
LQR (10% $C2$ reduction)	25.17	9,9 %	2.0618 sec	49,7 %
LQR (10% Control Properties Reduction)	23.43	2,3 %	1.8603 sec	35 %

6. CONCLUSION

This paper was intended to apply two different techniques of an active suspension model, in order to compare which strategy should present the best performance for the suspension system, optimizing the cost function.

First, the passive suspension was described in the state space model and the initial conditions were established. Then, considering an speed-bump input, the controllers were designed to represent a control force applied to the active systems, in order to recognize how the system should behave for the same plant parameters. Finally, for the unit-step response, both active suspension systems designed, the closed-loops LQR and FSFB models were simulated and compared, in order to find the gain matrix K needed to match the project specifications.

The numerical results show that the management of the gain matrix K together with the penalty matrices R and Q for the LQR model, the dynamic system project specifications can be assured even obtaining considerable reductions of the overshoot (Mp) and the settling time (Ts) for the unit step response while also reducing the Cost Function (J). The simulations also show better behavior of the LQR model for the road bump profile considering driveability and quarter-truck mass speed variables.

Trough MATLAB/Simulink tools, it was verified that the LQR was able to optimize the cost function J, while stabilizes the system much faster than the FSFB does. The reductions observed were around 10% for the cost function, 8% for the overshoot and 30% for the system settling time. From the passive system, these values achieves 28% and 43% of reduction for the overshoot and settling time, respectively.

Finally, a sensitive analysis shows that the impact produced by the control system to the system response is almost equivalent that the influence of the suspension damping coefficient, which is reasonable and justify its application.

Considering that the LQR is an intuitive optimization process, it is still possible to achieve better results using a metaheuristic algorithm, specially if some of the plant parameters are able to be adjusted.

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