COB-2021-1096

OPTIMIZING SWING FOOT TRAJECTORY OF HUMANOID ROBOT WALKING FOR ENERGY EFFICIENCY

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Abstract. *In this paper, we develop a method using an evolutionary strategy (ES) algorithm to find the parameters that make the humanoid robot walk more efficiently. The robotic bipedal walk is limited for several reasons, one of which being the design. The kinematic chain is restricted by geometry or components operating limits, such as servos. We intend to find parameters that help the walk without changing the design of the robot. Our contribution is to implement the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) to find parameters that define the oscillating foot trajectory. The equation of the dynamics of the movement can be derived using Lagrangian Mechanics. We validate our approach by implementing the parameters generated by the proposed method in to our custom-made robot Chape. In this robot we are able to vary the speed to suit different situations using the parameters obtained, knowing which is the best step size to be defined to the walk has the lowest energy cost.*

Keywords: *Humanoid Robot, Metaheuristic Optimization, Energetic Efficient Walking, Swing Leg Movement.*

1. INTRODUCTION

In the context of mobile robotics, there are several research problems to be studied. One of them is the bipedal walk in humanoid robots. These robots are fascinating because they have a human-like appearance, but in reality, their behavior differs from the human behaviour. Several reasons limit the bipedal walk, one of which is the design. Geometry or components (such as servos) restrict the kinematic chain of a humanoid. As a consequence, there movement comes with high energy expenditure.

For this reason, the battery cannot last for a long time, limiting the use of the robot. For example, a state-of-the-art Asimo robot uses more than ten times the energy of a person during their walk, according to Takenaka *et al.* (2009). This difference is due to the complexity of robotic movements from the point of view of control caused by non-linearity, subactuation, and high dimensionality, which represents a significant challenge for the latest generation control techniques, accordingly to Collins *et al.* (2005).

The complexity of controlling these robots justifies the difficulty of humanoids in adapting to different terrain types, such as sand, ice, or uneven ground. In Yi *et al.* (2015), Hong and Lee discuss how uneven terrain is harmful, especially for small humanoids. As a solution, Hong and Lee proposed a controller that adjusts the ankle projection position for desirable elevation angles of the toe-off and heel strike depending on the slope of the terrain. To prove this, they carried out a simulation and experiment on a THOR-RD robot.

The same authors show in Yi and Lee (2016) the controller for toe-off and heel strike for THOR-RD full-sized humanoid robot and DARwIn-OP miniature humanoid robot. With this, the robots gained a more human-like walk, in addition to overcoming their kinematic restrictions and exploring other terrains.

Robotic movements more similar to humans decrease energy expenditure in gait. In Silva *et al.* (2019), we showed that by varying the trajectory of the height of the center of mass and adopting a more human behavior during the gait, it is possible to reduce energy expenditure. This was proved through a realistic Gazebo simulator (Koenig and Howard, 2004).

In robotics, the trajectory defines the path that the actuators follow, and can be used to conserve energy. In classical robotics literature (Khatib, 2008), path planning by polynomial interpolation is a common technique, which can also be used to plan the joint's positions of humanoid robots (Cuevas *et al.*, 2010).

From an energetic perspective, we cannot use only one possible trajectory; we must use an optimized one. For example, in Fattah and Fakhari (2010), the trajectories of the hip and foot employ polynomial interpolation to optimize the position of the ankle joint of the oscillating foot for less energy expenditure.

In this paper, we intend to show a way to save energy during the robotic bipedal walk. Our contribution is to implement the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) to find parameters that define the swing foot trajectory, including the toe-off and heel strike movements. For this purpose, we simulate a phase of a walk of a small humanoid robot made in our laboratory, Chape (Tonaco *et al.*, 2019). We are using only the CMA-ES and not any other algorithm because CMA-ES is used to study humanoid movement, especially in 3D Humanoid Robot Soccer (Urieli *et al.*, 2011).

The remainder of this work is organized as follows. Section 2 presents a problem overview. Section 3 presents a mathematical model of swing foot. Section 4 determines the methodology for two situations. In Section 5 presents the results obtained for our approach. Finally, Section 6 presents a conclusion of the paper and futures works.

2. PROBLEM FORMULATION

2.1 CMA-ES

In engineering, we always seek to optimize processes, thus reducing material and energy costs. The search for optimal parameters has generated a lot of research on algorithms that can optimally choose design variables. For example, there is a class of algorithm based on Evolutionary Strategies (ES). First, the population is selected in this algorithm, later evaluates each individual with a fitness function. After that, the population is recombined for the next generation.

An ES is a stochastic search method inspired by principles of biological evolution. For example, the CMA-ES is an ES based on Gaussian mutation and deterministic selection. CMA-ES is considered one of the best choices against ill-conditioned, non-convex black-box optimization problems in the continuous domain, according to Szynkiewicz (2018). Hence, being the state-of-the-art of ES algorithms. Nikolaus Hansen first proposed the CMA-ES algorithm in 2001. In this algorithm, the population is chosen by a multivariate normal distribution.

In each generation the parent for the next generation is calculated with a weighted average of λ selected candidates from μ off-springs generated using a (λ, μ) selection. The next generation population is generated by sampling a multivariate normal distribution given by $\mathcal{N}(\mathbf{M}^{(g)}, (\boldsymbol{\sigma}^{(g)})^2 \mathbf{C}^{(g)})$ the step size σ influences the overall variance of the mutation at generation g, according to Gagganapalli (2015).

In every generation, k new individuals $\mathbf{x}_k^g \in \mathbb{R}^n$, follow Gagganapalli (2015), the equation can be written as:

$$\mathbf{x}_k^{g+1} \sim \mathcal{N}(\mathbf{M}^g, (\boldsymbol{\sigma}^g)^2 \mathbf{C}^g).k = 1, ...\lambda \tag{1}$$

where $\mathbf{x}_k^{g+1}$ is the kth sample generated at generation $(g+1)$. In order to pass to the next generation $(g+2)$, the $\mathbf{M}^{(g+1)}$, $\mathbf{C}^{(g+1)}$, $\boldsymbol{\sigma}^{(g+1)}$ needs to be computed.

2.2 Robotic platform

Chape is a humanoid robot built by ITA robotics research group ITAndroids. There are currently four Chape robots at LAB-SCA. Chape has 20 degrees of freedom (DoF), and the DoF are distributed as such: 2 in the neck, 3 in each arm, and 6 in each leg. A servomotor implements each DoF of this humanoid robot. Figure 1 shows Chape and schematic drawing of Chape's degrees of freedom (DoF), the names of the right-side joints are similar to the left-side.
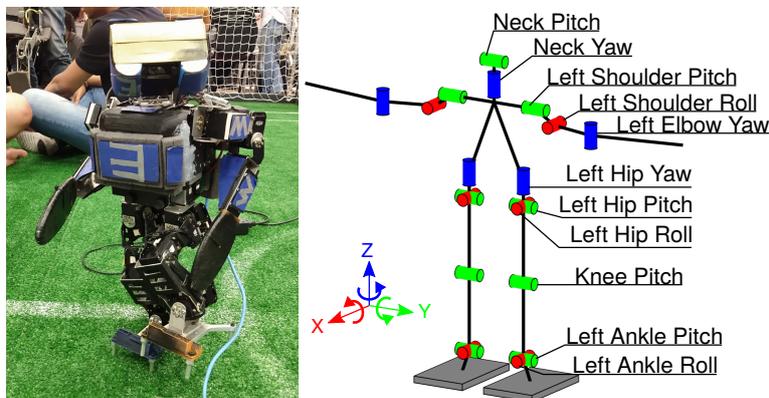

Figure 1. Real Chape and schematic drawing of Chape, respectively.

In Fig. 1, the servomotor in blue performs only rotation around Z, the green only rotation around Y, the red only

rotation around X.

For a complete gait cycle, the robot uses 12 degrees of freedom (DoF), i.e., the legs joints. Chape's walk consists of two phases. The first phase occurs when the two feet are in contact with the ground, known as the support phase. The other phase occurs when only one foot is in contact with the ground, known as the simple support or swing phase. The foot that is not in contact with the ground, it's also called the swing foot.

For a better understanding of the phases of walking, we sketched Fig. 2. The walking starts with the support phase, then changes to the swing phase, and finally back to the double support phase. All this exchange between phases occurs during a total time, also called step duration, represented for T in Fig. 2. Still, in Fig. 2, T_{d1} is the duration of the first double support phase, T_{d2} is the duration of the second double support phase, t_b is the time of the beginning of the single support phase and t_e is the time of the end of the single support phase.

In our robot, 80% of the gait corresponds to the swing phase (a phase in which only one foot is in contact with the ground and the other is in balance). This value is typical for robots (Maximo *et al.*, 2016). In humans this value corresponds to 60% (Bonnefoy-Mazure and Armand, 2015).
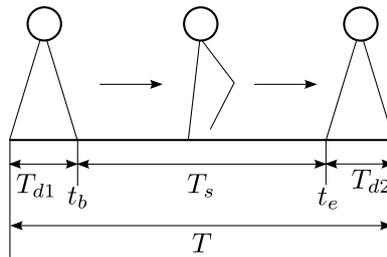

Figure 2. Walking cycle.

2.3 Proposed Optimization

As the swing phase corresponds to the predominant phase in the walk, we propose to find parameters that define the swing foot trajectory. We use energy as metrics to find three parameters, which correspond to the frequency of the steps, the toe support angle (at the beginning of the swing phase, i.e., toe-off), and the heel support angle (at the end of the swing phase, i.e., heel strike).

To find these parameters of the swing foot, the CMA-ES simulates the dynamics λ times by iteration. Simulating a complete walk would be very computationally costly. Therefore, a simplification needs be made due to the complexity of the gait. So, in our model, only a single support phase is simulated, considering the torso as fixed. Then we can assume that the swing foot movement is on a plane, using the servos of the lower limbs in the sagittal plane, i.e., the plane composed of the X direction and Z direction in Fig. 1.

Therefore, the measured energy corresponds to the efforts to obtain the pitch angles (angles performs by servos only rotation around Y in Fig. 1) in the swing leg joints. From the trajectory defined for the swing foot with the optimal parameters obtained by the CMA-ES, the inverse kinematics (IK) finds the swing leg joints' trajectory. After that, it uses them as a reference for the position control of each servo, corresponding to the hip, knee, and ankle joint.

3. MATHEMATICAL MODEL

3.1 Dynamics

For the dynamics of the swing leg, the kinematic chain considers three links. The equation of the dynamics of the movement can be derived using Lagrangian Mechanics, and the expression is as follows:

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{\theta} \in \mathbb{R}^{3 \times 1}$ is the joint variable vector, $\mathbf{M} \in \mathbb{R}^{3 \times 3}$ is the inertia matrix, $\mathbf{C} \in \mathbb{R}^{3 \times 1}$ is the vector of Coriolis and centripetal torques, $\mathbf{G} \in \mathbb{R}^{3 \times 1}$ is the vector representing gravitational torques, $\boldsymbol{\tau} \in \mathbb{R}^{3 \times 1}$ is the vector of control input torques of the three joints.

For the position control of each servo, a closed-loop model is considered. Then the calculation of the gain of the MX-28AT servos, follows Maximo *et al.* (2017b) for the modeling of this servo. To model the effect of the servomotor reduction, add the term $(J_m N^2 \eta)$ to the main diagonal of the inertia matrix $\mathbf{M}$.
where J_m, N, and η are the gearbox reduction, and gearbox efficiency of the servomotor, respectively.

Table 1 contains the mechanical parameters used to obtain the dynamic model of the three-link manipulator. In Tab. 1, l_i is the length of the link; m_i is the mass of the link; I_i is the moment of inertia of the link, where $i \in 1, 2, 3$.

Table 1. Parameters for the kinematic chain links.

Parameters	Value
l_1, m	0.093
m_1, kg	0.10653
I_1, kgm^2	0.00058
l_2, m	0.093
m_2, kg	0.04007
I_2, kgm^2	0.00058
m_3, kg	0.2825
I_3, kgm^2	0.00064

3.2 Trajectory

Currently, the trajectory of Chape's swing foot maintains a smooth movement to avoid accelerating the actuators due to foot impact on sharp edges. For this reason, the foot leaves and touches the ground with zero speed to reduce the impact. By Maximo (2015), the vertical trajectory is given by Eq. (3) and interpolating the Eq. (4), we have the forward trajectory given by Eq. (5).

$$Z(\phi) = A \left(0.5 - 0.5 \cos \left(2\pi \frac{\phi_- \phi_b}{\phi_e - \phi_b} \right) \right), \tag{3}$$

$$H(\phi) = \left(0.5 - 0.5 \cos \left(\pi \frac{\phi_- \phi_b}{\phi_e - \phi_b} \right) \right), \tag{4}$$

$$X(\phi) = H(\phi)x_f + (1 - H(\phi))x_o, \tag{5}$$

where A is amplitude; ϕ is the normalized time within a time step; ϕ_b is the normalized time of the beginning of the single support phase; ϕ_e is the normalized time of the end of the single support phase; x_o is the center of the foot's starting point to the forward direction; x_f is the center of the foot's endpoint to the forward direction.

However, we wanted to add the toe and heel lift corresponding to the toe-off and heel strike. So, we needed to add the angles, specifically the angle between the toe and the ground (θ_{toe}) and the angle between the heel and the ground (θ_{heel}). Nonetheless, we intend to keep the movement smooth until the foot reaches θ_{toe} and after θ_{heel} until the foot is flat on the ground. Figure 3 illustrates these angles, and the angles between the links are θ_1, θ_2, and θ_3.

Figure 3.a illustrates the beginning of the swing phase, i.e., toe-off, and Fig. 3.b demonstrates the end of the swing phase, i.e., heel strike. Therefore, we proposed that the same forward trajectory is kept, changing only the starting and ending points of the center of the foot, since they are in a function of θ_{toe} and θ_{heel}, respectively. So the new path forward is given by:

$$X_{new}(\phi) = H(\phi)x_f(\theta_{heel}) + (1 - H(\phi))x_o(\theta_{toe}), \tag{6}$$

And the new equation for the vertical trajectory is given by a fifth-degree polynomial:

$$Z_{new}(\phi) = a_1(\phi)^5 + a_2(\phi)^4 + a_3(\phi)^3 + a_4(\phi)^2 + a_5(\phi) + a_6, \tag{7}$$

where a_1, a_2, a_3, a_4, a_5, a_6 are the polynomial coefficients;ϕ is the normalized time within a time step.

To find the polynomial coefficients in Eq. (7), we use the position and velocity of three situations as a condition, which are: the toe-off, the maximum foot height, and heel strike. Furthermore, in our proposal, the swing foot has rotation, i.e., it has an attitude angle, as the foot leaves the toe-off and reaches the heel strike. The attitude angle (θ_a) varies in function of ϕ. θ_a is given by interpolation function of θ_{toe} and θ_{heel}.

$$\theta_a(\phi) = \theta_{toe} + (\phi - \phi_b) \left(\frac{\theta_{heel} - \theta_{toe}}{\phi_{heel} - \phi_{toe}} \right), \tag{8}$$
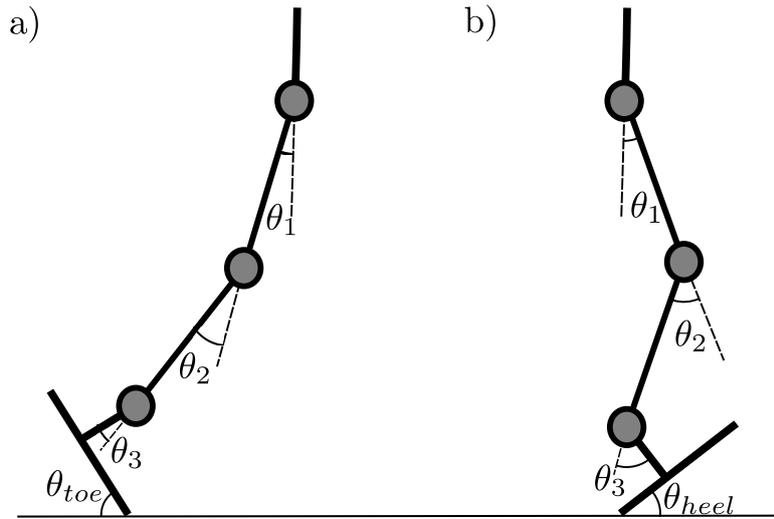
Figure 3. a) The beginning of the swing phase, i.e., toe-off. b)The end of the swing phase, i.e., heel strike.

3.3 Cost Function

The cost function used for the optimization of the CMA-ES is based on the energy formulation follow Maximo *et al.* (2017a). Therefore, only energy in the swing phase of one leg is considered.

The total energy (E_{total}) is composed of two terms: the total mechanical energy (E_{mech}) and the electric energy dissipated by the servos (E_{diss}).

$$E_{total} = E_{mech} + E_{diss}. \tag{9}$$

$$E_{mech} = \int_{t_0}^{t_f} \boldsymbol{\tau}(t)^T \boldsymbol{\omega}(t) dt. \tag{10}$$

$$E_{diss} = \int_{t_0}^{t_f} R \left\| \mathbf{i}(t) \right\|^2 dt = \frac{R}{K_t^2 N^2 \eta^2} \int_{t_0}^{t_f} \left\| \boldsymbol{\tau}(t) \right\|^2 dt. \tag{11}$$

where R, K_t, N, and η are the electrical resistance, torque constant, gearbox reduction, and gearbox efficiency of the servomotor, respectively; $\boldsymbol{\omega}(t)$ are vectors of the angular speed of the servo; $\mathbf{i}(t)$, and $\boldsymbol{\tau}(t)$ are vectors containing the currents and torques of all joints (at the output axes) at time instant t; and t_0 and t_f are the initial and final times considered in the measurement. It is important to note that the energy measurement in an actual robot could be performed by measuring the battery voltage. Follow Maximo *et al.* (2017b), Tab. 2 contains the values of the MX-28AT servo constant.

Table 2. Parameters for MX-28AT.

Parameters	Value
R, Ω	8.3
K_t, Nm/A	0.0107
N	193
η	0.836
J_m, kgm^2	0.0000000868

4. METHODOLOGY

In this section, we propose two approaches. In the first approach, we define the frequency of the step to compare the energy expenditure between the situation in which the toe-off and heel strike occurs with the situation in which the foot remains parallel to the ground, which is the case for most humanoid robots. In the second approach, we include the frequency of step with a parameter to be optimized.

4.1 Fixed Step Frequency

The first case analyzes only two parameters for a robot with a fixed frequency of step: θ_{heel} and θ_{toe}. The entire simulation was done using the MATLAB software; the CMA-ES input hyperparameters are in Tab. 3.

Table 3. CMA-ES hyperparameters for fixed step frequency.

Hyperparameters	Value
Number of Variables(NV)	2
Maximum Number of Iterations(MNI)	100
Lower Bound of Decision Variables(LBDV)	-60°
Upper Bound of Decision Variables(UBDV)	60°

From the hyperparameters contained in Tab. 3, the size of the population (λ) is obtained; the parent population for the next generation (μ) is obtained, and step size control (σ) is obtained.

According to Tab. 4, the total time of a trajectory is 0.32 seconds, i.e., 80% of step duration. For numerical integration of the simulation, it used the MATLAB ode45 function. It penalized the cost function if the angle of inclination of the foot exceeded $60°$ in any rotation sense.

Table 4. Parameters for the Simulation.

Parameters	Value
Maximum foot height, m	0.0854
Step duration, s	0.4
Step length, m	0.08
Sample time of robot, s	0.008
Double support ratio	0.2
Sample time of servo, s	0.001

4.2 Optimized Step Frequency

As already mentioned in this paper, the fact that we can change the step frequency is helpful in bipedal walking. Therefore, we include the frequency of the step as a third optimization parameter of the CMA-ES. Currently, our robot moves with the same frequency at different speeds.

However, due to the difference in parameters' scale, we normalized the minimum and maximum values of the optimized parameters from 0 to 1, respectively. Our humanoid has no viable speed above $0.4\ m/s$, so Tab. 5 contains the parameters of the CMA-ES for the study of three cases.

Collins *et al.* (2005)compare energy expenditure while walking between robots and humans. To this end, they divide trajectory energy by the product of the weight by the distance covered. In our case, the comparison is between the robot itself. Therefore we divide trajectory energy only by distance covered. Therefore, CMA-ES will not be biased to find a frequency that reduces the trajectory.

Table 5. CMA-ES hyperparameters for optimized step frequency.

Hyperparameters	Value
Number of Variables (NV)	3
Maximum Number of Iterations (MNI)	100
Lower Bound of Decision Variables (LBDV)	0
Upper Bound of Decision Variables (UBDV)	1
Maximum time of Step Duration, s	1
Minimum time of Step Duration, s	0.2

5. RESULTS

5.1 Results for Fixed Step Frequency

It is important to emphasize that the cost function includes the energy cost of the swing foot trajectory, the cost to reach the toe-off angle, and the cost of the heel-strike until the foot is flat on the ground. Thus, we ensure that the choice

of angles takes into account the expense of reaching them.

We observed that with a few iterations the algorithm has already converged, for this reason we use only 100 iterations. Figure 4 shows the best value of the cost function at each interaction of CMA-ES. The function converges to 9328.90 J/s, with parameters with the following value: $\theta_{toe} = 25.87°$ and $\theta_{heel} = 24.03°$. In comparison, the swing foot without toe-off and heel strike have a cost of 1719.24 J/s. Therefore, we can infer a cost reduction of 53.53% due to toe and heel lift.
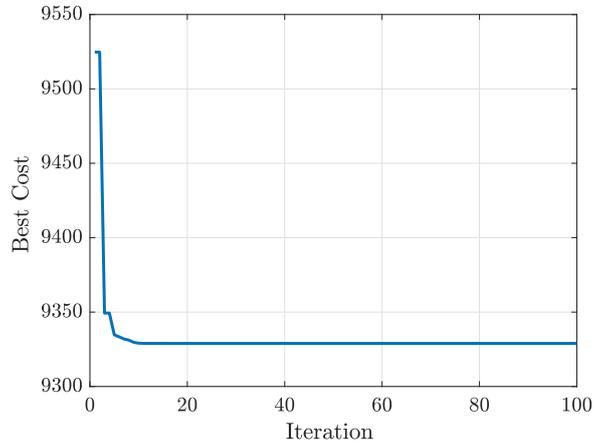
Figure 4. Best cost function per iteraction with toe-off and heel strike.

5.2 Results for Optimized Step Frequency.

Then, we simulate three cases in which the speed of the robot change. We find three parameters optimized for trajectory: θ_{toe}, θ_{heel}, and step frequency. Again, the cost function includes the energy cost of the swing foot trajectory, the foot's cost to reach the toe-off angle, and the heel strike until the foot was flat on the ground.

In Fig. 5, we see the convergence of the cost function parameterized by step length. We consider that the step length is equal to the product of the velocity and the step duration (i.e., the inverse of step frequency).

We observed that as the speed increases, the parameterized cost of the function decreases. In fact, a higher speed generates a longer step and reduces energy expenditure compared to shorter step. Thus, from this point of view, the speed of 0.3 m/s is the most advantageous.
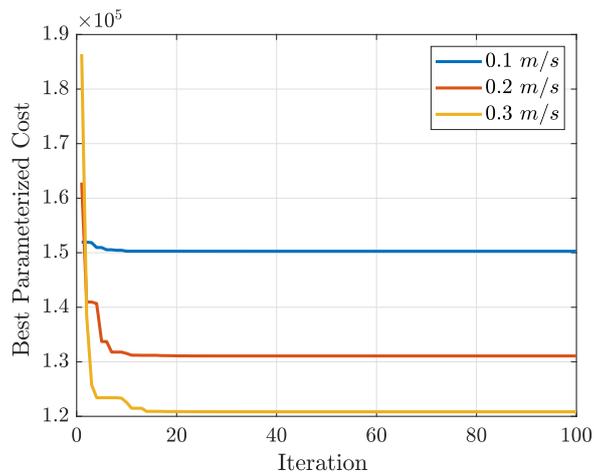

Figure 5. Best parameterized cost function per iteraction.

Table 6 contains the results of the parameters optimized for different speeds and the absolute cost of the path of the swing foot alone. It is evident that the higher the speed, the lower the step duration, i.e., increasing the speed, the robot will take more steps.

For a broader understanding of what movement looks like after optimization. Figure 6 illustrates some frames of the simulation of the speed of 0.1 m/s, in which we can see the beginning and end of the swing foot, as well as the maximum

Table 6. Parameter results optimized for different speeds.

Velocity (m/s)	$\theta_{toe}(°)$	$\theta_{heel}(°)$	Step duration (s)	Cost (J/s)
0.1	15.90	-15.30	1	8025.51
0.2	19.49	-22.76	0.64	8550.10
0.3	16.71	-29.40	0.61	9885.95

height of the foot. Moreover, we made a video[1] .

6. CONCLUSIONS

We conclude that the toe-off and heel strike movement is economically advantageous compared to the same trajectory in which it does not happen. The CMA-ES proved to be very useful for our problem, returning good results. Also, it possibles established walking speed for the entire gait, based only on the energy expenditure of the swing phase alone. We emphasize that all of this is done without physically altering our robot.

Based on these results, we can change the speed in different situations, knowing the best step size to be defined. Biologically, humans spend more energy when they walk faster. Our paper proves that, for our robot, this influence of speed on energy expenditure has similar behavior to humans (Gast *et al.*, 2019).

Finally, we may apply the same methodology in other robots, needing to change the parameters used to model the kinematic chain and actuators. In the future, we intend to measure the energy used for the full gait, using all the robot dynamics, in a realistic simulator, especially Gazebo. After in our real robot, since Yi and Lee (2016) showed in experiments with physical robots that the foot tilt does not generate instability or falls during gait.

7. ACKNOWLEDGEMENTS

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[1]https://www.youtube.com/watch?v=t2EW5fxBT3w

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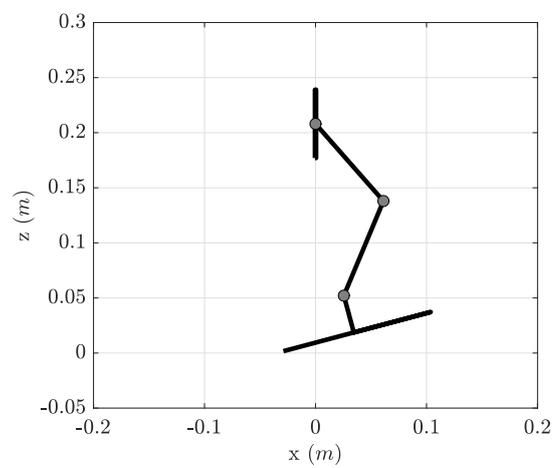## 9. RESPONSIBILITY NOTICE

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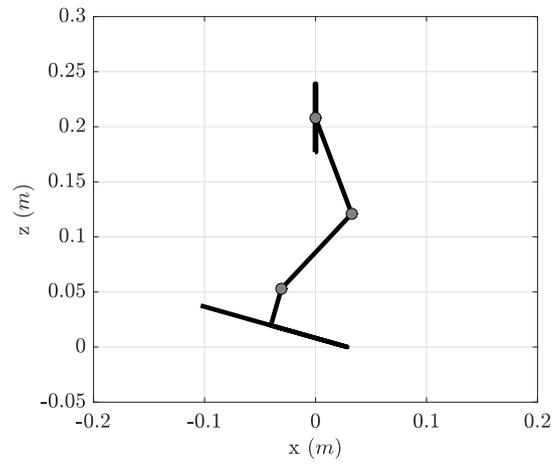
(a) Situation of toe-off.
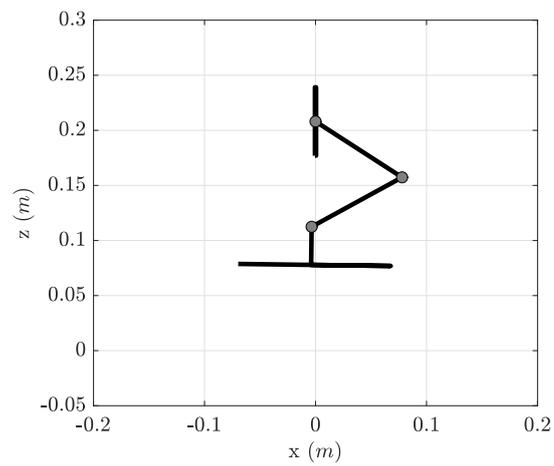
(b) Situation of the maximum foot height

(c) Situation of heel strike.

Figure 6. Optimized path of swing foot of the speed of $0.1m/s$.