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DIFFERENTIAL EVOLUTION METHOD APPLIED IN PARAMETER IDENTIFICATION AND MODEL UPDATING OF FLEXIBLE PLATE FEM MODEL

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Abstract. *The present work aims to study the dynamic behaviour of a thin steel square plate by means of theoretical and experimental analysis using the Finite Element Method (FEM) and Differential Evolution algorithm. The FEM by itself is capable of modelling with great accuracy many physical structures, even those more complicated. Although it provides good predictions, when the matter is experimental measures there are some uncertainties associated with component geometry, materials constants and boundary conditions that lead to significant differences between experimental results and theoretical ones. In order to deal with this problem, model updating methods are widely used to minimize these discrepancies and, therefore, obtain an accurate model that truly represents the physical structure in analysis. In this context, this paper identifies unknown parameters that needs to be updated such as elasticity modulus, suspension stiffness and damping constants of modal test so that the Differential Evolution algorithm using multiple experimental and theoretical frequency response functions (FRFs) can find the best solution within a specified parameter range. The updated model is reliable enough to be used in further analysis like structural modification and substructuring. In the end, experimental and theoretical FRFs are compared to verify the quality of the fit. Furthermore, the Artificial Bee Colony updating method is compared to Differencial Evolution algorithm in order to discuss characteristics of both formulations involved.*

Keywords: *Optimization method, modal analysis, Artificial Bee Colony*

1. INTRODUCTION

Finite element model updating is, by definition, the improvement of theoretical model using experimental data. In most cases, the compatibility between theoretical and experimental model is verified comparing frequency response functions (FRFs) in terms of amplitude and natural frequencies. In this context, model updating came to improve the correlation of these models from modifications in stiffness, mass and damping matrices or even parameters that describe the system itself, so the numerical model becomes more accurate compared to the experimental one. The results of model updating are of great interest, since the updated model is reliable enough for further applications.

The procedure of updating can be perfomed by two types of methods: direct or iterative. The direct ones are those who don't iterate, performing pontual changes in the system. Usually, they return less accurate results when compared to iterative ones. Iterative methods, in turn, seek to adjust the difference between theoretical and experimental results based on algorithms that evaluate the model many times in each iteration and verify the influence of parameter changes in the whole system. This study aims on Differential Evolution algorithm (DEA), an iterative and FRF based method, since it directly uses FRFs information while updating. As most of the updating methods, DEA requires the user to define a cost function to be minimized, a set of parameters to be updated and the range of each parameter so that the algorithm can seek for the best solution in that range (Friswell and Mottershead, 1995).

The DEA is one of the most used population based methods. Its popularity is mainly due to its simplicity and effectiveness in solving a class of problems in different areas, including multi-objetive, multi-modal, dynamic and constrained optimization problems. This method, and other evolutionary algorithms are based on the process of natural selection observed by Darwin. In this context, in a population, living entities are selected according to their fitness and the algorithm selects the best individual of the generation to minimize the cost function (Zaharie, 2012).

Over the years, DEA has been successfully applied in different engineering fields for various optimization problems. Tang *et al.* (2008) performed a structural identification problem in an 20-DOF system to identify mass, stiffness and damping properties using DEA and particle swarm optimization. Ho-Huu *et al.* (2016) presented an improved differential evolution (IDE) and its application for solving shape and size optimization problems of truss structures with frequency constraints and the improved algorithm turned out to be much better than standard DEA in terms of computational costs. Cavalini *et al.* (2015) used in their work a self-adaptive differential evolution (SADE), a variation that dynamically updates required parameters such as population size, crossover rate and mutation scale factor. Also, the strategy was applied to update the FE model of a rotating machine composed by a horizontal flexible shaft, two rigid discs and two unsymmetrical bearings. Seyedpoor *et al.* (2015) used DEA in order to identify the multiple damages cases of structural systems by verifying natural frequency changes of a structure and then solving an optimization problem to find the site and extent of structural damage.

The approach used in this study explains the use of Differential Evolution algorithm in engineering problems. This methodology seeks to be realible enough for situations encountered everyday by design engineers that must have an accurate model to replicate the physical behaviour of the system. By changing parameters like elasticity modulus, suspension stiffness and damping constants related to experimental modal testing, the algorithm seeks for the global optimum within a parameter range.

2. THEORETICAL BASIS

2.1 Introduction

The optimization problems, in general, can be stated as:

$$Minimize\ or\ maximize\ f(x)\ subjected\ to\ x\ = \{x_1, x_2, ..., x_n\}^T \tag{1}$$

where $f(x)$ is the objective function and x is a set of design variables, while x_{min} and x_{max} are the lower and upper boundaries, respectively. Finite element optimization is a subject that has recieved acceptance and has applications in many areas of engineering, including aerospace, civil, mechanical and electrical. Due to some uncertainties associated with the model, there is a need update it in order to better reflect measured data. In structural dynamics, model updating techniques are frequently used in conjunction with vibrations measurements to find out unknown system parameters, such as material's properties, boundary conditions and constraints. Sometimes, these parameters have theoretical values that represent a very good first approximation but, in majority, as manufacturing process can exert great influence on them, it is essential to update the model and verify if the optimization problem converged to an expected solution (Friswell and Mottershead, 1995; Marwala, 2010).

It's also necessary to mention that optimization problems are totally dependent of parameters selection. The engineer must be able to choose correctly the set of design variables, once this stage is probably the most important task in model updating. The selected parameters must truly describe the region of the system which is supposed to be inadequately modelled and, of course, the output (eigenvalues, FRFs) must be sensitive to the chosen parameters so the algorithm can reconcile the model output with physical measurements (Friswell and Mottershead, 1995; Marwala, 2010).

2.2 Differential Evolution Algorithm

The Differential Evolution algorithm was proposed by Storn and Price (1995) and first appeared as a technical report. Since 1995, DEA has been widely used to solve problems with great accuracy in many different areas. In DEA community, the individual trial solutions are referred as parameter vectors or genomes. DEA is similar to other standard evolutionary algorithms (EAs) and operates through the same computational steps. However, DEA employs difference of the parameter vectors to explore the objetive function landscape, unlike standard algorithms. As DEA is a population-based method, it generates new points (trial solutions) that are perturbations of existing points. The algorithm perturbs current generation vectors with the scaled difference of two randomly selected population vectors. In order to produce a trial vector, in its simplest form, DEA adds the scaled, random vector difference to a third randomly selected population vector. In selection stage, the trial vector competes against the population vector of same index. Once the last trial vector has been tested, the survivors of all pair wise competitions become parents of the next generation in the evolutionary cycle (Das *et al.*, 2009).

Many variations of DEA have been proposed since its first appearance, so the version implemented throughout this text is DEA/rand/1/bin (DEA stands for Differential Evolution algorithm, rand stands for the random choice of perturbed vector in mutation scheme, 1 stands for the number of pair vectors considered in perturbation and bin means binomial crossover). As stated before, in the first step the user needs to define upper and lower limits of the vector variables and the cost function. In this work, the cost function to be minimized is a sum of the difference between theoretical and experimental FRFs, whose discrepancy must be reduced to obtain trustable parameters within the system. As this information has been specified, and if the $j-th$ parameter of the given problem has its lower and upper bound as $x_{min,j}$ and $x_{max,j}$ respectively and $rand_{i,j}(0,1)$ denotes $j-th$ instantiation of a uniformly distributed random number lying between 0 and 1 for the $i-th$ vector, then one may initialize the $j-th$ component of the $i-th$ population according to Eq. (2) (Das *et al.*, 2009).

$$x_{i,j} = x_{min,j} + rand_{i,j}(0,1)(x_{max,j} - x_{min,j}) \tag{2}$$

Once the population is initialized, the algorithm starts mutation scheme according to the variation chosen. To create a donor vector for each $i-th$ member of the current population (also called target vector), three other distinct parameter vectors, say the vectors $x_{1,j}$, $x_{2,j}$ and $x_{3,j}$ are randomly selected from the current population. The difference of any two of these three vectors is scaled by a scalar number F and this difference is added to the third one to obtain the donor vector $v_{i,j}$ according to Eq. (3).

$$v_{i,j} = x_{3,j} + F(x_{1,j} - x_{2,j}) \tag{3}$$

where the scalar F is a positive real number between 0 and 1 named mutation scale factor. Although some authors discuss the upper limit of F, it rarely exceeds 1.

To increase the potential diversity of the population, DEA uses a crossover operator after generating the donor vector through mutation scheme. The crossover scheme used in this, as stated before, was binominal. Basically, the donor vector exchanges components with the target vector in order to form a trial vector. Binomial crossover is performed on vector's variables whenever a randomly selected number between 0 and 1 is less than or equal to the crossover rate (CR) value. This scheme can be outlined in Eq. (4).

$$u_{i,j} = \begin{cases} v_{i,j}, & if\ (rand_{i,j}(0,1) \leq CR\ or\ j = j_{rand}) \\ x_{i,j}, & otherwise \end{cases} \tag{4}$$

where j_{rand} is randomly chosen vector index to ensure that the trial vector gets at least one component from the donor vector.

The last stage of DEA is called selection and is quite simple. In this stage, the algorithm decides who between the target vector $x_{i,j}$ and the recent formed trial vector $u_{i,j}$ will survive to the next generation. If the trial vector yields a better fitness value it will replace the target vector in the next generation. In case of minimization problem, the selection scheme can be outlined as in Equation (5).

$$x_{i,j+1} = \begin{cases} u_{i,j}, & if\ f(u_{i,j}) \leq f(x_{i,j}) \\ x_{i,j}, & otherwise \end{cases} \tag{5}$$

2.3 Summary of the algorithm

Using the terms cited before, one can sketch the following pseudo code for DEA:

Algorithm 1 The general structure of DEA

Population initialization $X(0) \rightarrow \{x_1(0), x_2(0), ..., x_n(0)\}$
$j = 0$
Compute $\{f(x_1(j)), f(x_2(j)), ..., f(x_n(j))\}$
while the stopping condition is false **do**
  for $i = 1, n$ **do**
    $v_{i,j} \rightarrow generateMutant(X(j))$
    $u_{i,j} \rightarrow crossover(x_{i,j}, v_{i,j})$
    Compute $f(u_{i,j})$
    if $f(u_{i,j}) \leq f(x_{i,j})$ **then**
      $x_{i,j+1} = u_{i,j}$
    else
      $x_{i,j+1} = x_{i,j}$
    end if
  end for
  $j = j + 1$
end while

It's important to emphasize there are three main control parameters of DEA algorithm: mutation scale factor F, crossover rate CR and population size NP. According to Ronkkonen *et al.* (2005), a plausible choice of population size NP is between $3D$ and $8D$, where D is the amount of variables, the scaling factor $0.4 < F < 0.95$ with $F = 0.9$ being a good first choice. For CR, values between (0.8,1) are good when function's parameters are dependent.

2.4 Kirchhoff-Love plate model

The mathematical model of the plate structure was based on thin plate theory. It is known as *Kirchhoff-Love plate model*, being an extension of *Euler-Bernoulli's* beam theory for two-dimensional problems. In addition, the model assumes a plane stress state in the plane of the plate structre (Reddy, 2006).

The schematic finite element plate model is presented in Fig. 1. Each node has three degrees of freedom: one translation along z axis and two rotations along x and y axes. According to Fig. 1, the degrees of freedom for node i are: w_i, θ_{xi} and θ_{yi}. For nodes j, m and n the reasoning is analogous. Therefore, each finite element has 12 degrees of freedom (Logan, 2007).

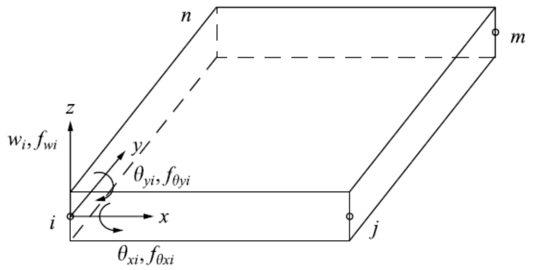

Figure 1: Rectangular plate with three degrees of freedom per node (Logan, 2007).

Equation (6) brings a 12-term polynomial in x and y to express the displacement function of the element

$$w(x,y) = \gamma_1 + \gamma_2 x + \gamma_3 y + \gamma_4 x^2 + \gamma_5 xy + \gamma_6 y^2 + \gamma_7 x^3 + \gamma_8 x^2 y + \gamma_9 xy^2 + \gamma_{10} y^3 + \gamma_{11} x^3 y + \gamma_{12} xy^3 \quad (6)$$

Expressing the nodal coordinates w, θ_x and θ_y as a function of the polynomial coefficients presented in Eq. (6), and knowing that $\theta_x = \frac{\partial w}{\partial y}$ and $\theta_y = -\frac{\partial w}{\partial x}$ one can obtain Eq. (7) (Logan, 2007).

$$\begin{Bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix} = \begin{bmatrix} 1 & x_i & y_i & x_i^2 & x_i y_i & y_i^2 & x_i^3 & x_i^2 y_i & x_i y_i^2 & y_i^3 & x_i^3 y_i & x_i y_i^3 \\ 0 & 0 & 1 & 0 & x_i & 2y_i & 0 & x_i^2 & 2x_i y_i & 3y_i^2 & x_i^3 & 3x_i y_i^2 \\ 0 & -1 & 0 & -2x_i & -y_i & 0 & -3x_i^2 & -2x_i y_i & -y_i^2 & 0 & -3x_i^2 y_i & -y_i^3 \end{bmatrix} \begin{Bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \\ \gamma_7 \\ \gamma_8 \\ \gamma_9 \\ \gamma_{10} \\ \gamma_{11} \\ \gamma_{12} \end{Bmatrix} \quad (7)$$

Equation (7) relates the nodal coordinates for one node with polynomial coefficients and one can expand it to the four nodes, obtaining Eq. (8).

$$\{a\}_{12x1} = [A]_{12x12} \{\gamma\}_{12x1} \quad (8)$$

where $\{a\}$ is a vector of generalized coordinates, $\{\gamma\}$ is a vector of all coefficients and $[A]$ is a matrix that relates both. In this context, one can express elementary deformations according to Eq. (9), relating them to the vector $\{a\}$.

$$\{\varepsilon\} = \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6x & -2y & 0 & 0 & -6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2x & -6y & 0 & -6xy \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & -4x & -4y & 0 & -6x^2 & -6y^2 \end{bmatrix} \begin{Bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \\ \gamma_7 \\ \gamma_8 \\ \gamma_9 \\ \gamma_{10} \\ \gamma_{11} \\ \gamma_{12} \end{Bmatrix} \quad (9)$$

Therefore, by definition, it is possible to obtain from Eq. (9) the matrix $[H]$ that relates deformations and polynomial coefficients, according to Eq. (10).

$$\{\varepsilon\}_{3x1} = [H]_{3x12} \{\gamma\}_{12x1} \quad (10)$$

Using matrix $[H]$, the gradient matrix $[B]$ can be obtained in Eq. (11) (Shames and Dym, 1995).

$$[B]_{3x12} = [H]_{3x12} [A]_{12x12}^{-1} \tag{11}$$

To properly obtains the element stiffness matrix, one still needs the constitutive matrix $[D]$, presented in Eq. (12).

$$[D] = \frac{Eh^3}{12(1-v^2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \tag{12}$$

where E is the material elasticity modulus, v is Poisson's ratio and h is the plate thickness. Therefore, the element stiffness matrix $[K]$ can be obtained using Eq. (13).

$$[K] = \int_V [B]^T [D] [B] \, dV \tag{13}$$

Similarly, the element mass matrix can be deduced from Eq. (14).

$$[M] = \int_V \rho [N]^T [N] \, dV \tag{14}$$

where ρ represents material density and $[N]$ is the shape function matrix, according to Eq. (15).

$$[N]_{1x12} = [R]_{1x12} [A]_{12x12} \tag{15}$$

where vector $[R]$ can be obtained by the relation between the polynomial from Eq. (6) and the coefficients, presented in Eq. (16).

$$[R] = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2y & xy^2 & y^3 & x^3y & xy^3 \end{bmatrix} \tag{16}$$

From stiffness and mass matrices, considering also hysteretic proportional damping, the equation of dynamical system can be written as Eq. (17) (Ewins, 2000; Fu and He, 2001).

$$[M] \{\ddot{q}\} + ([K] + i [C]) \{q\} = \{f(t)\} \tag{17}$$

where $\{\ddot{q}\}$, $\{q\}$, $\{f(t)\}$ are the acceleration, displacement and input force vectors associated to generalized coordinates, respectively. The hysteretic damping matrix $[C]$ is "proportional", typically as follows in Eq. (18) (Ewins, 2000).

$$[C] = \beta [K] + \eta [M] \tag{18}$$

The system also has viscous damping to represent nylon's wire damping properties and this parameter is contemplated in the general damping matrix. The addition of this term is an attempt to model a flexible nylon wire used in experimental modal test to represent free-free condition. The purpose here is to include wire's stiffness and damping in order to obtain better results regarding the updating process.

From matrices $[K]$, $[M]$ and $[C]$ or, in other words, from the spatial model, the system can be dynamically represented in an equivalent way by the modal model. This model can be obtained by performing an eigenvalue problem that leads to the system's spectral $[\lambda]$ and modal $[\psi]$ matrices, where $[\lambda]$ contains eingenvalues and $[\psi]$ the corresponded mode shapes under analysis, which are the eigenvectors. Therefore, the response of a system q_i to harmonic excitation Q_k, considering a system with N degrees of freedom and hysteretic damping, in terms of receptance, is stated by Eq. (19) (Maia and Silva, 1999; Fu and He, 2001).

$$\alpha_{j,k} = \frac{q_i}{Q_k} = \sum_{r=1}^{N} \frac{({}_r\phi_j)({}_r\phi_k)}{({}_r\lambda^2 - \omega^2)} \tag{19}$$

where ${}_r\phi_j$ and ${}_r\phi_k$ are the elements j and k of a mode r from modal matrix. In addition, it's convenient to express the complex variable ${}_r\lambda^2$ in terms of real and imaginary contributions, according to Eq. (20).

$${}_r\lambda^2 = \omega_r^2(1 + i\zeta_r) \tag{20}$$

with ω_r being the natural frequency of mode r and ζ_r the damping of the same mode. Therefore, the accelerance is presented in Eq. (21)

$$I_{j,k} = \frac{\ddot{q}_j}{Q_k} = -\omega^2 \alpha_{j,k} = \sum_{r=1}^{N} \frac{({}_r\phi_j)({}_r\phi_k)}{1 - (\frac{\omega_r}{\omega})^2(1 + i\zeta_r)} \tag{21}$$

From this last equation, it is possible to analyze the dynamic behaviour of the structure in frequency domain in a theoretical viewpoint.

3. TEST BENCH

A test bench was built to measure experimental FRFs of a steel square plate, whose geometrical and inertia properties are in Tab. 1.

Table 1: Geometrical and inertia properties of typical theoretical steel square plate (Callister and Rethwisch, 2012).

Parameter	**Value**
E, GPa	210
L, m	0.48
h, m	0.0027
ρ, kg/m^3	7860
v	0.3

The plate was discretized in 100 elements, 10 in each direction (x and y), and therefore it was a mesh with 121 nodes along the structure. Figure 2(a) presents the plate used in this study, as well as the mesh and the nodes numbered. Also, there's a coordinate system in the lower left corner.

(a)

(b)

Figure 2: (a) Square plate (b) Plate suspended by nylon wire in a modal testing

The procedure performed was a hammer test and the experimental modal analysis was conducted in a free-free condition. The plate has been suspended in nodes 2, 10, 112, 120 using nylon wire of 0.5mm, as can be seen in Fig. 2(b). After this, the modal test started and 121 experimental FRFs were obtained in a range from 0 to 400Hz by exciting all 121 nodes with the hammer and measuring response at node 3 from uniaxial high sensitivity accelerometer. The hammer test setup can be seen in Fig. 3.

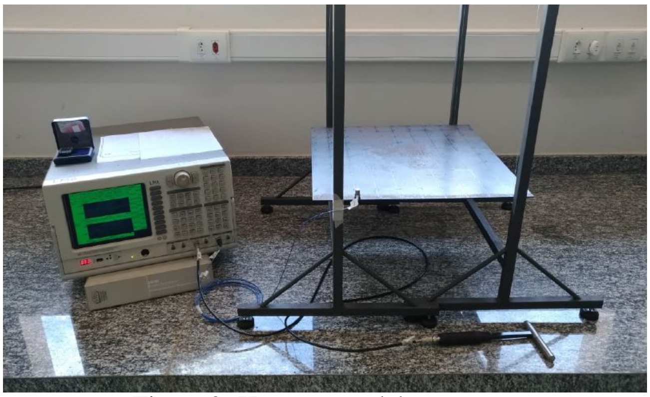

Figure 3: Hammer modal test setup.

In order to process the obtained experimental data and results, a dynamic signal analyzer SRS (Stanford Research Systems) model SR785 was used. The BNC connector from impact hammer was connected to channel 1, input "A" of the analyzer and the BNC connector from PCB uniaxial accelerometer model 352C33, serial LW252667, was connected to channel 2, input "A". The experimental procedure was performed in Applied Mechanics Laboratory of Mechanical Engineering Department of the University of Lavras. Table 2 presents main settings adjusted in the analyzer during data acquisition process.

Table 2: Settings of dynamic signal analyzer to perform modal test using impact hammer

Parameter	**Adjust**	**Parameter**	**Adjust**
Span, Hz	400	Window	Force/Exp
Line Width, mHz	500	Channel 1 Window	Force
Acquisition Time, s	2	Channel 2 Window	Exp
FFT Lines	800	Force Length, ms	8.78906
Start Freq., Hz	0	Expo TC, s	1
Center Freq., Hz	200	Trigger Mode	Auto Arm
End Freq., Hz	400	Trigger Source	Ch 1
Ch1 Coupling	ICP	Trigger Level	5%
Ch2 Coupling	ICP	Compute Average	Yes
Ch1 Eng Units	On	Average Type	Exp/Cont.
Ch1 EU Label	N	Avgs	5000
Ch1 EU/Volt, V/EU	0.002402	Average Preview	Manual
Ch2 Eng Units	On	Overload Reject	On
Ch2 EU Label	m/s^2	Ch2 EU/Volt, V/EU	0.01029

4. NUMERICAL SIMULATION AND EXPERIMENTAL VERIFICATION

After modal test, the experimental FRFs were directly compared to theoretical ones as in Fig. 4 and the need to update the model was evident. $I_{j,k}$ means measurement in degree of freedom j and excitation in degree of freedom k.

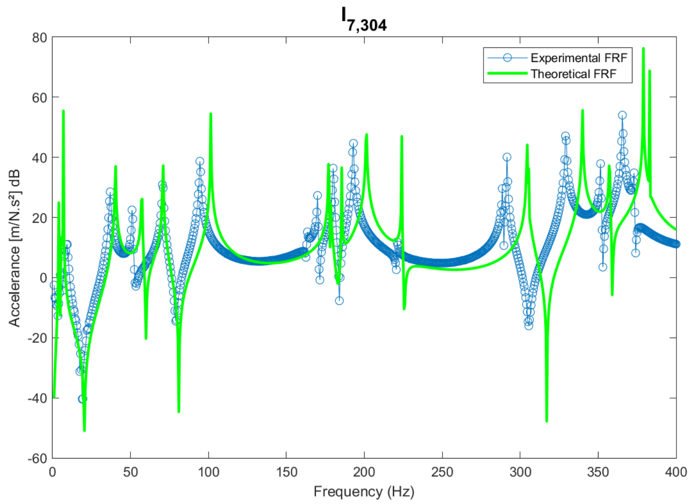

Figure 4: Comparison between experimental and theoretical FRF 7, 304.

As stated by Friswell and Mottershead (1995), the disagreement was a result of the invalid assumptions about the model properties used in the finite element model of the structure. First, to use Differential Evolution algorithm to parameter identification and FEM model updating accordingly, the parameters and their range needed to be established. In this context, the selected parameters for updating were: elasticity modulus E, nylon stiffness k (used as suspension in modal test in free-free condition), nylon viscous damping c and hysteretic "proportional" damping β. The parameters range are in Tab. 3.

Table 3: Limits of parameters

Parameter	Max. Value	Min. Value
E, GPa	260	150
k, N/m	2000	700
c, N.s/m	1.5	0.2
β	9.10^{-3}	9.10^{-8}

Then, the objective function was defined involving theoretical and experimental FRFs at each iteration of the method, according to Eq. (22).

$$OF = \sum_{r=1}^{n} \frac{\|FRF_{exp,r} - FRF_{theo,r}\|}{\|FRF_{exp,r}\|} \tag{22}$$

where $n = 6$ is the number of FRFs used, FRF_{exp} are experimental FRFs and FRF_{theo} are FRFs from mathematical finite element model. Regarding the OF, the theoretical FRFs amplitudes are subtracted from the experimental ones in each frequency of the discretization and the norm of this operation is compared to the norm of the experimental FRF in the same frequency for the whole frequency range, considering multiple FRFs. For the optimization process, an initial population of 32 inidividuals was considered, with $F = 0.8$ and $CR = 0.8$. After running the algorithm, the lowest value of the objetive function and the adjusted parameter values obtained are in Tab. 4.

Table 4: Updated parameter values

Parameter	Value
E, GPa	194.331
k, N/m	1171.571
c, N.s/m	1.5
β	9.10^{-3}
OF	3.3406

Also, a comparison between experimental and updated natural frequencies is presented in Tab. 5, as well as relative error. The first three modes are rigid body modes (RBM) and the others are flexible body modes (FBM). In this context, it is possible to observe that the value of k (used as suspension in modal test in free-free condition) was able to provide a good adjustment of RBM. Furthermore, the FBM were quite adjusted in frequency as well, according to the relative error in fourth column.

Table 5: Theoretical and experimental natural frequencies

Mode	Updated, Hz	Experimental, Hz	|Relative Error|, %
1	4.8	4.5	6.67
2	6.8	7	2.86
3	8.5	9	5.55
6	68.4	70.5	2.97
10	171.4	170.0	0.82
11	178.4	180.0	0.88
12	193.6	193.0	0.31
14	292.8	288.5	1.49
15	293.9	291.5	0.82
16	327.0	329.0	0.61
17	343.7	351.5	2.22
18	364.7	365.5	0.22
19	368.4	373.0	1.23

The results in terms of FRFs are in Fig. 5. As all the experimental measurements were in node 3, the degree of freedom j is always 7. Some regions were not in good match between experimental and updated FRFs, especially in the range of 150-225Hz. This fact can be explained due to the geometry of the plate that isn't perfectly square because of manufacturing process and, therefore, there are some experimental close modes that don't appear in finite element model once the plate was considered to be square. Although this happened, the updated model was able to represent in a reliable way the modal test. It is also good to emphasize that the parameters converged to values consistent with the literature for

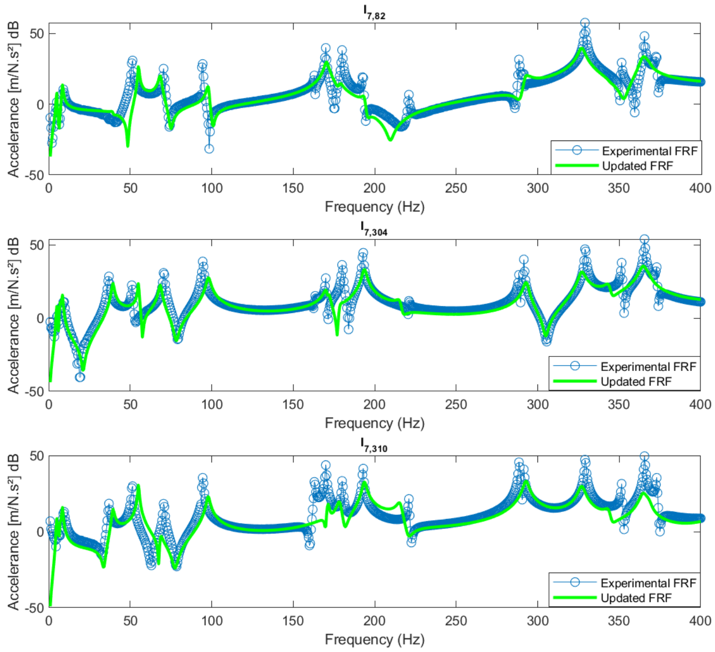

Figure 5: FRFs 7,82; 7,304; 7,310.

metallic structures. The elasticity modulus E is close to values for steel, the amplitude of the FRFs are reasonably adjusted and regarding the nylon wire parameters, those presented were the ones who best fit the curves. In addition, several modes and 6 FRFs were contemplated, all modes were adjusted, specially in frequency, including rigid body modes.

In order to verify the performance of DEA in model updating, the results obtained by this algorithm were compared with those of Artificial Bee Colony (ABC) or Bee Algorithm (BA), a popular metaheuristic algorithm inspired by foraging behaviour of honey bee swarm and proposed by Karaboga and Basturk (2007). There are scout bees in each swarm whose main task is to find food sources for their hives. As they find new food sources, these bees provide the information of the direction, distance and amount of nectar in the gardens for worker bees. And then, worker bees fly to the detected location proportionally to the amount of nectar available, i.e. more worker bees are sent to gardens which have more nectar and shorter distance to the hive. Figure 6 represents the convergence of DEA and BA methods for finite element model updating of the square plate using same parameters. BA showed a faster convergence but after 27 iterations both algorithms reached same OF value. Although BA performs better, the choice to use DEA was because of its known robustness and wide amount of variants: five mutation schemes and two crossover strategies. Regarding the mutation schemes in other variants, the main difference is that the donor vector, used to perturb each population member, is the best vector in the population with respect to fitness and more vectors can be used in the process, while in the scheme used in Eq. (3) the vectors are ramdonly selected. Each mutation scheme combined with either the exponential or binomial crossover yields a total of 10 DEA variants that allows the user to choose the best combination for the current analysis. This is a huge advantage of DEA over BA, since the latter in its standard version does not have variants to adapt the algorithm.

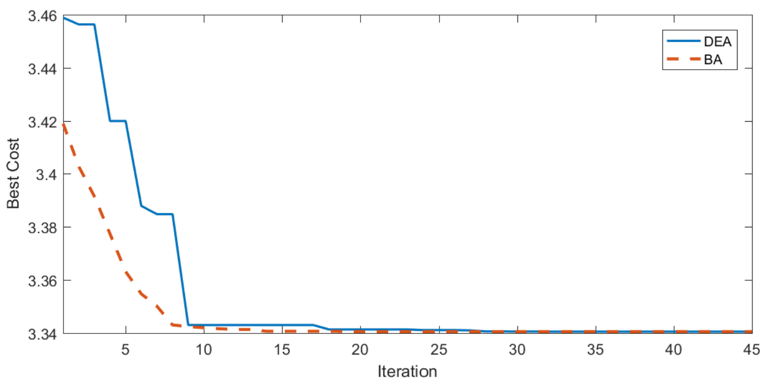

Figure 6: Evolution of the best function values

5. FINAL REMARKS

This study presented a brief explanation of Differential Evolution algorithm as well as computational implementation in one of its variations applied in model updating of a square plate structure using FRF data to adjust the theoretical model. A finite element plate model was also evidenced to obtain the system's spatial model in order to solve the eigenproblem and thus reach the modal model to generate FRFs to be compared. Regarding the model updating, DEA once again proved to be very robust, accurate and suitable for such optimization problems. The updated parameters were consistent with the literature and the experimental and theoretical curves were in good match in almost entire frequency range, despite some regions with great modal density that couldn't be reproduced by the theoretical model because of geometry issues. Also, to verify DEA's performance, another metaheuristc method called Artificial Bee Colony was used to minimize the same objective function and update the same parameters. Both methods reached exactly the same best value for OF and therefore provided identical updated parameter values. The results in this paper demonstrate that these methods were able to solve the inverse problem and identify the unknown parameters that best fit the curves and, in general, both algorithms were satisfactory once they achieved their purpose.

6. REFERENCES

Callister, W. and Rethwisch, D., 2012. *Fundamentals of Materials Science and Engineering: An Integrated Approach*. Wiley. ISBN 9781118061602.

Cavalini, A., Lobato, F., Koroishi, E. and Steffen, Jr, V., 2015. "Model updating of a rotating machine using the self-adaptive differential evolution algorithm". *Inverse Problems in Science and Engineering*, Vol. 24.

Das, S., Abraham, A. and Konar, A., 2009. *Metaheuristic Clustering*. Studies in Computational Intelligence. Springer Berlin Heidelberg. ISBN 9783540921721.

Ewins, D., 2000. *Modal Testing: Theory, Practice and Application*. Engineering dynamics series]. Wiley. ISBN 9780863802188.

Friswell, M. and Mottershead, J., 1995. *Finite Element Model Updating in Structural Dynamics*. Solid Mechanics and Its Applications. Springer Netherlands.

Fu, Z. and He, J., 2001. *Modal Analysis*. Elsevier Science. ISBN 9780080511788.

Ho-Huu, V., Vo-Duy, T., Luu-Van, T. and Le, L., 2016. "Optimal design of truss structures with frequency constraints using improved differential evolution algorithm based on an adaptive mutation scheme". *Automation in Construction*, Vol. 68, pp. 81–94.

Karaboga, D. and Basturk, B., 2007. "Artificial bee colony (abc) optimization algorithm for solving constrained optimization problems". Vol. 4529, pp. 789–798. ISBN 978-3-540-72917-4.

Logan, D., 2007. *First Course in the Finite Element Method*. Thomson. ISBN 9780534552985.

Maia, N. and Silva, J., 1999. *Modal Analysis and Testing*. NATO ASI series: E. Kluwer Academic Publishers. ISBN 9780792358930.

Marwala, T., 2010. *Finite Element Model Updating Using Computational Intelligence Techniques: Applications to Structural Dynamics*. Springer-Verlag Londons. ISBN 978-1-84996-322-0.

Reddy, J., 2006. *Theory and Analysis of Elastic Plates and Shells, Second Edition*. Series in Systems and Control. Taylor & Francis. ISBN 9780849384158.

Ronkkonen, J., Kukkonen, S. and Price, K., 2005. "Real-parameter optimization with differential evolution". Vol. 1, pp. 506 – 513 Vol.1. ISBN 0-7803-9363-5.

Seyedpoor, S., Shahbandeh, S. and Yazdanpanah, O., 2015. "An efficient method for structural damage detection using a differential evolution algorithm based optimization approach". *Civil Engineering and Environmental Systems*.

Shames, I. and Dym, C., 1995. *Energy and Finite Element Methods in Structural Mechanics*. New Age International Publishers Limited. ISBN 9788122407495.

Storn, R. and Price, K., 1995. "Differential evolution: A simple and efficient adaptive scheme for global optimization over continuous spaces". *Journal of Global Optimization*, Vol. 23.

Tang, H.S., Xue, S. and Fan, C., 2008. "Differential evolution strategy for structural system identification". *Computers and Structures*, Vol. 86, pp. 2004–2012.

Zaharie, D., 2012. "Differential evolution: From theoretical analysis to practical insights". *Mendel*, pp. 126–131.

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