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SPECTRAL MODEL AND VIBRATION ANALYSIS OF BEAMS CONNECTED BY BOLTs-JOINT

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Abstract. *Structures are commonly jointed using fasteners such as rivets or bolts arranged in various configurations depending on the required performance. Bolted structures are widely utilised in industrial structures and equipment due to the numerous advantages they possess. They are an important part of the system and can influence its dynamic response. In this paper, two beams are connected with a bolt-joint and modelled using the spectral element method (SEM). The equivalent spectral model represents the bolt-joint and the beams. The identification of dynamic characteristics of the system is validated by comparing the results to the commercial FEA software ANSYS. It is found that the vibration responses obtained with SEM are in good agreement with the dynamic responses estimated via FEM. The advantages of using SEM are the reduced number of elements in the mesh and high accuracy in the vibration response prediction.*

Keywords: *Vibration characterisation, Bolt-joint, Spectral element method, Finite element method*

1. INTRODUCTION

Most industrial structures and equipment consist of screws for joining their components together because they can be repeatedly assembled and disassembled during an operation. The bolts main function is to create an adequate connection force across the joint–tension load, which can withstand external vibrations, dynamic loads or thermal variations without slackening. There are at least two elements associated with each screw, and this set is called a bolt-joint. Properly tightened screws make use of their elastic properties, behaving like springs. When a load is applied, the bolt stretches and tries to return to its original length. This creates compressive force across the joint parts.

The study of bolted joint modelling has been carried out by several researchers, with the finite element method (FEM) being one of the main numerical methods used (Oldfield *et al.*, 2003; Knight *et al.*, 2008; Zeng and Zhao, 2011), due to its accurate and efficient representation. Kim *et al.* (2007b), performed numerical simulations using the implicit FEM software package ANSYS for the pre-stress effect and contact behaviour in the bolt-joint of a marine diesel engine. Alfattani (2020), describes in this article the structural analysis performed in ANSYS software on a preloaded bolted joint. Their parametric studies were conducted using elastic, large strain, and nonlinear finite element analysis to determine the influence of various factors on the response. Such factors considered included bolt preload, contact surfaces, edge boundary conditions, and joint segment length. Liu *et al.* (2015), used a three-dimensional finite element model in Abaqus FEA software to simulate the bolted joint under harmonic shear displacement. The stress concentration factors at the roots of the screw with preload and the contact conditions between the surfaces were studied. Soo Kim and Kuwamura (2007), also used the same software to investigate the structural behaviour of shear connections bolted with thin-walled stainless steel plate.

The spectral element method (SEM) is a mesh method similar to FEM, where the functions of approximate forms of the element are replaced by functions of the exact solution of differential equations of government. Therefore, a single element is enough to model any continuous and uniform part of the structure. This feature significantly reduces the number of elements required in the structure model and improves the accuracy of the dynamic system solution. An extensive study of the fundamentals and vairous new applications of SEM, such as composite laminates, periodic structure, damage detection was presented in (Lee, 2009; Doyle, 1997), and the behaviour of the waves in a conductor cable of transmission line in (Dutkiewicz and Machado, 2019a,b; Machado *et al.*, 2020).

Many studies have focused primarily on the estimation of contact stress and screw preload. It is necessary to evaluate the dynamic behaviour of the bolted joint in-service condition to avoid failure due to resonance. In this paper, two beams are connected with a bolt-joint and modelled using the spectral element method (SEM). The equivalent spectral model represents the bolt-joint and beams. The identification of dynamic characteristics of the system is validated by comparing the results to the commercial finite element ANSYS-WORKBENCH and the published by Kim *et al.* (2007a).

2. FINITE ELEMENT BACKGROUND

Structures with bolt-joints are often simplified as combined beam or spring elements and then both end nodes of the elements are coupled with piece nodes fixed along the bolt holes. Modelling approaches represent in joint interfaces such as spring-damper have been investigated in the works of Gant *et al.* (2011) and Ahmadian and Jalali (2007).

FEM is a numerical method based on the principle that an approximate solution to any complex problem in different areas of knowledge can be obtained by subdividing a larger complex structure into smaller components called elements, which are connected by nodes. Nodes will have nodal displacements or degrees of freedom that can include translations, rotations and derivatives of higher-order displacements. The description of the finite element method initializes by rewriting the differential equation as a variational equation, the variational principle for dynamic of systems or deformable solid is called Hamilton's principle Petyt (2010). The form of Hamilton's principle to be used is therefore

$$\int_{t_1}^{t_2} \left(\delta\left(T-U\right)+\delta W\right)+dt=0, \tag{1}$$

where δ is virtual operator, T is Kinetic energy, U is elastic potential (strain) energy and W is work done by applied forces.

There are several techniques for determining approximate solutions to Hamilton's principle. One of the most used methods is the Rayleigh-Ritz, known as the finite element displacement method. The element model is shown in Fig. 1, the generalized coordinates at each node are v, the total deflection and θ, the total slope. This results in an element with four degrees of freedom, a two-node element with two-DoF at each node, i and j are local node numbers.

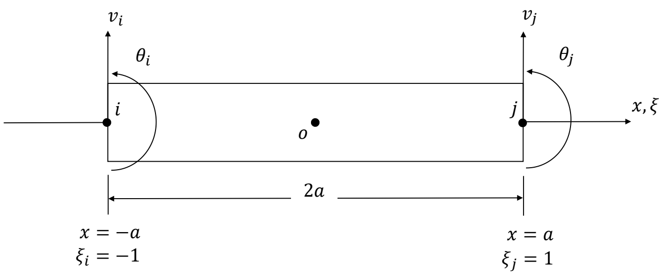


Figure 1: Beam Element.

Using the local coordinate (ξ) and beam length $2a$ defined in Fig. 1, the displacement v can be written in matrix form as follows:

$$v\left(\xi\right)=\left[\mathbf{N}\left(\xi\right)\right]\left\{\mathbf{v_e}\right\}, \tag{2}$$

where $\mathbf{v_e}$ is displacement vector of the element and $\mathbf{N}$ is a matrix of shape functions given by

$$\left[\mathbf{N}\left(\xi\right)\right]=\left[\mathbf{N_1}\left(\xi\right)\mathbf{N_2}\left(\xi\right)\mathbf{N_3}\left(\xi\right)\mathbf{N_4}\left(\xi\right)\right], \tag{3}$$

The energy functions for beam are given by:

$$T=\frac{1}{2}\int_0^l \rho A\dot{v}^2 dx, \tag{4}$$

$$U=\frac{1}{2}\int_{-1}^{+l} EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx, \tag{5}$$

where is A Area Mass, ρ is the mass per unit volume of the material, E is Young's modulus for the material and I is the second moment of area (or moment of inertia) of the cross-section.

Substituting the displacement Eq. (2) into the kinetic energy Eq. (4). The parameter a transforms from the x coordinate to the local coordinate ξ. Thus, $dx = ad\xi$.

$$T = \frac{1}{2}\int_{-1}^{+1} \rho A \dot{v}^2(\xi)\, ad\xi = \frac{1}{2}\{\dot{\mathbf{v}}_{\mathbf{e}}\}^T \left\{\rho A a \int_{-1}^{+1} [\mathbf{N}(\xi)]^T [\mathbf{N}(\xi)]\, d\xi\right\}\{\dot{\mathbf{v}}_{\mathbf{e}}\}, \quad (6)$$

such that

$$T = \frac{1}{2}\{\dot{\mathbf{v}}_{\mathbf{e}}\}^T [\mathbf{M}_{\mathbf{e}}]\{\dot{\mathbf{v}}_{\mathbf{e}}\}, \quad (7)$$

where $\dot{\mathbf{v}}_{\mathbf{e}}$ is the velocity vector of the beam element, $\mathbf{M}_{\mathbf{e}}$ is the mass matrix of the beam element.

Substituting the displacement Eq. (2) into the strain energy Eq. (5) gives

$$U = \frac{1}{2}\int_{-1}^{+1} EI\frac{1}{a^4}\left(\frac{\partial^2 v(\xi)}{\partial \xi}\right)^2 ad\xi = \frac{1}{2}\{\mathbf{v}_e\}^T \left\{\frac{EI}{a^3}\int_{-1}^{+1} [\mathbf{N}''(\xi)]^T [\mathbf{N}''(\xi)]\, d\xi\right\}\{\mathbf{v}_{\mathbf{e}}\}, \quad (8)$$

such that

$$U = \frac{1}{2}\{\mathbf{v}_{\mathbf{e}}\}^T [\mathbf{K}_{\mathbf{e}}]\{\mathbf{v}_{\mathbf{e}}\}, \quad (9)$$

where $\mathbf{K}_{\mathbf{e}}$ is element stiffness matrix.

2.1 ANSYS Mechanical simulation

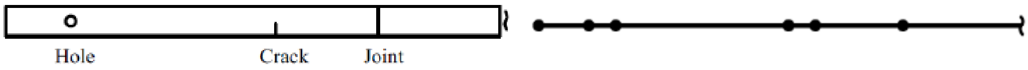
For the analysis of a structure, ANSYS may be broken into the following three stages (Stolarski *et al.*, 2006): Preprocessing, processing and Postprocessing. In particular, model generation (element type, mesh and material/geometric properties) or defining the problem is done in the Preprocessor and application of loads, constraints and the solution is performed in the Solution Processor, this is used for obtaining the solution for the finite element model that is generated within the Preprocessor. Finally, the results are viewed in the Postprocessor. This includes plotting contours, deformed shapes, vector displays, and listings of the results.

Modal analysis is used to calculate the natural frequencies and vibration modes of a structure, which are important design parameters for loading conditions. It can also be a starting point for another, more detailed, dynamic analysis, such as transient dynamic analysis, harmonic response analysis, or spectrum analysis (Wael A. Altabey *et al.*, 2018). There are different mode extraction methods, such as Block Lanczos, PCG Lanczos, Subspace, Supernode, Reduction (Householder) damped and QR damped. In this work, the Block Lanczos extraction model was used, which is an acceptable eigenvalue extraction method for problems with a high number of nodes and for presenting a good convergence rate when applied to problems with symmetric matrices.

Harmonic analysis is used to determine the response of a structure to time-varying cyclic loads. Therefore, gives you the ability to predict the sustained dynamic behaviour of your structures, thus enabling us to verify whether your designs will successfully overcome resonance, fatigue, and other harmful effects of forced vibrations. Analyses can generate plots of displacement amplitudes at given points in the structure as a function of forcing frequency.

3. SPECTRAL ELEMENT METHOD

In dynamic system analysis and SHM, it is crucial to have an efficient and economic numerical technique. The Finite Element Method (FEM) is one of the most common computational methods employed in several science areas. However, in medium and high-frequency wave propagation problems, this method requires too high a computational cost. The SEM was first proposed by Narayanan and Beskos (1978), further improved and named SEM by Doyle (1997) and Lee (2009). The SEM consists of the exact displacement of the wave equation of the analytical solution in the frequency domain. It is equivalent to an infinite number of finite elements. This characteristic and the spectral domain make SEM more suitable to solve the crack problem. The advantage of SEM is the reduced number of elements required to model the system compared to other computational methods, as demonstrated in Figure 2.



(a) (b)

Figure 2: Representation: (a) Physical structure; (b) Spectral element model.

SEM is similar in style to the FEM, written in the frequency domain, and the element interpolation function is the exact analytical solution of the differential equation. These futures allow a no mesh element requirement associated with high accuracy in solving structural dynamic problems. The number of elements required for a spectral model will coincide with the number of discontinuities of the structure. Another significant advantage of using SEM is the throw-off element that consists of conducting to propagate energy out of the system, it works as an anechoic termination dissipating the remaining energy in the system.

3.1 Beam spectral element

The beam is assumed as slender with transversal and rotational nodal displacement, shear and momentum nodal forces. By neglecting shear deformations, the differential equation of movement in its spectral form can be written as

$$\frac{d^4\hat{v}}{dx^4} - k^4\hat{v} = F, \tag{10}$$

with the homogeneous solution given by

$$\hat{v}(x,\omega) = a_1 e^{-ikx} + a_2 e^{-kx} + a_3 e^{-ik(L-x)} + a_4 e^{-k(L-x)}, \tag{11}$$

where

$$\mathbf{e}(x,\omega) = \begin{bmatrix} e^{-ikx} & e^{-kx} & e^{-ik(L-x)} & e^{-k(L-x)} \end{bmatrix},$$

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T,$$

for L being the beam length. The wavenumbers, k, k_1 and k_2 are given by

$$k^2 \equiv \sqrt{\frac{\omega^2\rho A}{EI}}, \quad k_1 = \pm k, \quad k_2 = \pm ik, \tag{12}$$

where ω is the circular frequency, E is Young's modulus, A is the cross-section area, ρ is the density, I is the inertia moment, and $i = \sqrt{-1}$. By using a complex Young's modulus, $E_c = E(1 + i\eta)$, the internal structural damping is introduced where η is the hysteretic structural loss factor. Figure 3 illustrates two nodes healthy beam spectral element model with two degrees of freedom (dof) per node. The nodal displacements are $\hat{v}$ and $\hat{\phi}$ and the nodal forces $\hat{V}$ and $\hat{M}$.

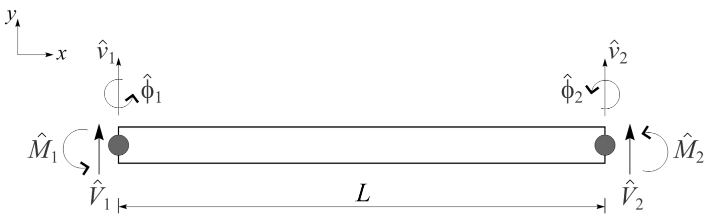


Figure 3: Two nodes beam spectral element.

The spectral nodal displacements and slopes of the finite beam element can be related to the displacement field as at node 1 ($x = 0$) and at node 2 ($x = L$)

$$d = \begin{Bmatrix} \hat{v}_1 \\ \hat{\phi}_1 \\ \hat{v}_2 \\ \hat{\phi}_2 \end{Bmatrix} = \begin{Bmatrix} \hat{v}(0) \\ \hat{v}'(0) \\ \hat{v}(L) \\ \hat{v}'(L) \end{Bmatrix} = \begin{Bmatrix} e(0,\omega) \\ e'(0,\omega) \\ e(L,\omega) \\ e'(L,\omega) \end{Bmatrix}, \tag{13}$$

where $\mathbf{a} = \mathbf{H_B}(\omega)^{-1}\mathbf{d}$, and

$$\mathbf{H_B}(\omega) = \begin{bmatrix} 1 & 1 & e^{-ikL} & e^{-kL} \\ -ik & -k & ike^{-ikL} & ke^{-kL} \\ e^{-ikL} & e^{-kL} & 1 & 1 \\ -ike^{-ikL} & -ke^{-kL} & ik & k \end{bmatrix}.$$

The frequency-dependent displacement within an element is interpolated from the nodal displacement vector, it is expressed as

$$\hat{v} = e(x,\omega)H_B^{-1}(\omega)\mathbf{d}. \quad (14)$$

Shear forces and bending moments defined for the beam is related to the defined forces and moments in a spectral nodal form as

$$f = \begin{Bmatrix} \hat{V}_1 \\ \hat{M}_1 \\ \hat{V}_2 \\ \hat{M}_2 \end{Bmatrix} = \begin{Bmatrix} -V(0) \\ -M(0) \\ V(L) \\ M(L) \end{Bmatrix} = \begin{Bmatrix} -\hat{v}(0)''' \\ -\hat{v}(0)'' \\ \hat{v}(L)''' \\ \hat{v}(L)'' \end{Bmatrix}. \quad (15)$$

where by applying boundary conditions it has,

$$f = EI\begin{Bmatrix} -ik^3 & k^3 & ie^{-ikL}k^3 & e^{-kL}k^3 \\ k^2 & -k^2 & e^{-ikL}k^2 & -e^{kL}k^2 \\ ie^{-ikL}k^3 & -e^{kL}k^3 & -ik^3 & k^3 \\ -e^{-ikL}k^2 & e^{-kL}k^2 & -k^2 & k^2 \end{Bmatrix}\mathbf{a} = G(\omega)\mathbf{a}. \quad (16)$$

By relating the nodal forces to the nodal displacement, one has

$$\mathbf{f} = \mathbf{G}(\omega)\mathbf{H}_B^{-1}(\omega)\mathbf{d} = \mathbf{S}(\omega)\mathbf{d} \quad (17)$$

where $\mathbf{S}(\omega) = \mathbf{G}(\omega)\mathbf{H}_B^{-1}(\omega)$ is the dynamic stiffness matrix of the Euler-Bernoulli beam spectral element.

3.2 Beam spectral element connected by bolt

The beam spectral element connected by a bolt considered two beams coupled from one bolt is illustrated in Fig.4(a), and the spectral bolt model, presented in (Lee, 2001; Machado *et al.*, 2019), is equivalent to a lumped mass and a spring-system. The lumped mass has its mass m and the mass moment of inertia I, the spring-system consists of a linear spring kv and a torsional spring kt, as shown in Fig.4(b) The lumped mass and the spring-system are connected in series.

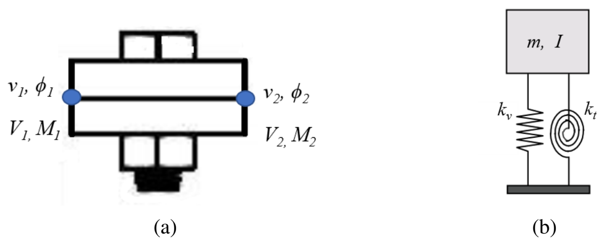


Figure 4: Representation: (a) Physical structure; (b) Spectral element model.

The symmetric dynamic spectral element matrix or the equivalent bolt-joint model is expressed as (Lee, 2001),

$$\mathbf{S}_b(\omega) = \begin{bmatrix} k_v & 0 & -k_v & 0 \\ 0 & -k_t & 0 & k_t \\ -k_v & 0 & -m\omega^2 + k_v & 0 \\ 0 & k_t & 0 & -I\omega^2 - k_t \end{bmatrix}. \quad (18)$$

where ω is the circular frequency. One should note that the spectral element matrix $\mathbf{S}_b(\omega)$ includes equivalent bolt-joint model parameters. Analogous to FEM, the SEM can be assembled to form a global structure matrix system.

4. NUMERICAL RESULTS

For the numerical analysis, a similar structure presented in (Kim *et al.*, 2007a) was assumed to compare and validate the results obtained in the work. The specimen used in the modal test is made of two beams with a width of 31.6 mm, a thickness of 8.46 mm, and a length of 320 mm, joined with an M10 bolt. Beams and bolt mechanical properties are assumed Young's modulus of 200 GPa, density of 7850 Kgf/m^3, and Poisson's ratio of 0.3. Receptance responses and modal parameters were estimated using SEM and FEM. Figure 5 shows a 3D model of identical beams jointed by a bolt and their dimensions. The structure is considered in free-free boundary condition and excited with a unitary force applied in point A and the response is measured at point B. It was modelled in SolidWorks and used ANSYS software to perform a Modal Analysis based on the finite element method.

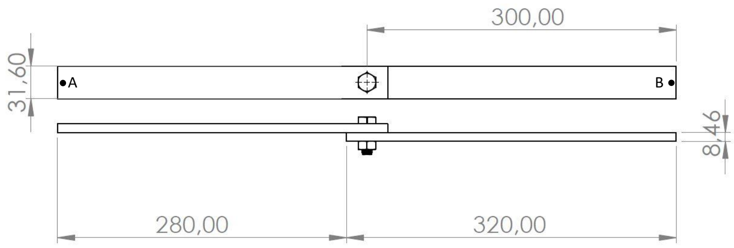


Figure 5: Model bolt-joint (unit: mm).

SEM model used three elements in the mesh, a spectral bolt element (element 2) and two beam elements as shown in Fig. 6. The beam elements 1 and 3 are similar to the mechanical and geometrical parameters specified above. The bolt element has $k_v = 10^9$ N/m, $k_t = 10^4$ N/rad, $m = 0.0185$ Kgf/m^3 and $I_{bolt} = 4.9087^{-10}$.

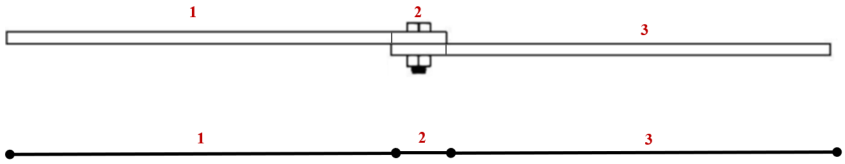
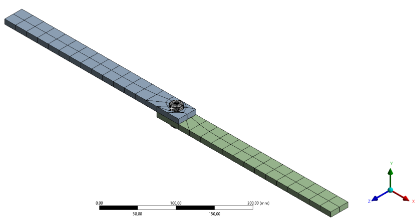
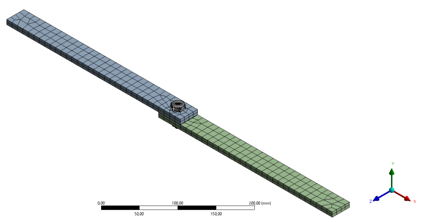


Figure 6: Representation of the physical structure and by SEM.

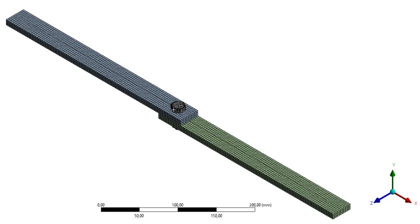
In ANSYS-Workbench, solid elements SOLID186 was used in the mesh, the CONTAC174 element to model the surface-to-surface contact between the head and nut, and the TARGE170 element to model the flange interfaces. The element sizing is important to secure a good accuracy of the response maintains the optimal computational time. Elements are sizing 3, 10, and 20mm was tested in the mesh to demonstrate the convergence (Fig. 7). Table 1 three discretization meshes were used, this study shows a finer mesh takes a more elapsed CPU time. Reduction of finite element size leads to more elements, which in turn leads to more nodes in the model.



(a) 20 mm.



(b) 10 mm.



(c) 3 mm.

Figure 7: Meshed beams bolt-joint.

Table 1: Analysis meshing of linear model.

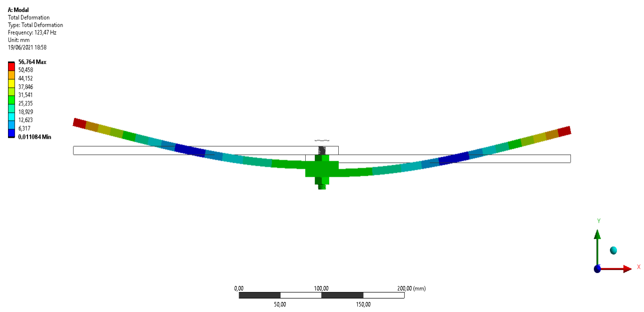
Element size [mm]	**20**	**10**	**3**
No. of nodes	37433	40008	88322
No. of elements	22548	22896	39372
Elapsed CPU Time [s]	8	7	20

The first six modes are listed in Tab. 2 associated with element size in the mesh and were observed did not show significant variation in natural frequencies during the simulation. The results indicate there is no need to do more simulations and use a mesh that is too refined because this does not bring any extra information relevant to the simulation. Therefore, a 3 mm mesh was used to indicate a smaller mesh is not necessary to obtain more accurate results and that an element size of 10mm or 20mm would already be a good response from the model.

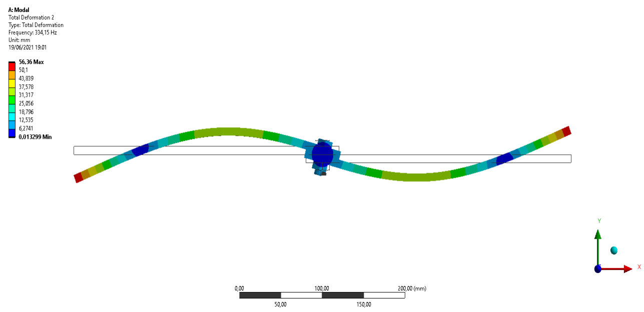
Table 2: Natural Frequencies under different meshing.

Frequency	For 20 Element size [Hz]	For 10 Element size [Hz]	For 3 Element size [Hz]
1	123.63	123.7	123.45
2	334.25	334.2	334.15
3	437.6	437.4	437.04
4	655.9	656.2	654.93
5	1069.84	1069.18	1068.78
6	1223.86	1223.36	1223.2

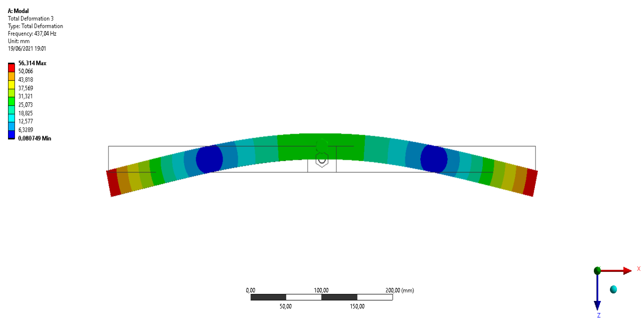
The dynamic analysis investigates modal parameters (natural frequencies and modal shape) and harmonic analysis of the bolted structure. In the modal analysis performed in ANSYS, we estimated the firsts six natural frequencies and modal shapes, where the 1st, 2nd, 4th and 5th are flexural modes (out plane) and the 3rd and 6th are in-plane vibrations. Figure 8 illustrates the vibrational and natural frequencies obtained in the simulation, where the flexural modes are occurring at frequencies 123.45 (Fig. 8a), 334.15 (Fig. 8b), 654.93 (Fig. 8d) and 1068.8 Hz(Fig. 8e). The in-plane vibration modes happened at 437.04 (Fig. 8c) and 1223.2 Hz (Fig. 8f). Those natural frequencies and modal shape follow the results published in (Kim *et al.*, 2007a), used as a reference in this work.



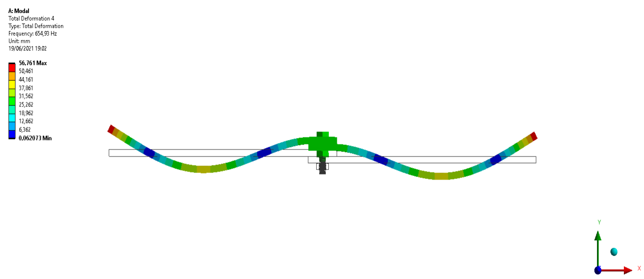
(a) 1st mode at 123.45 Hz.



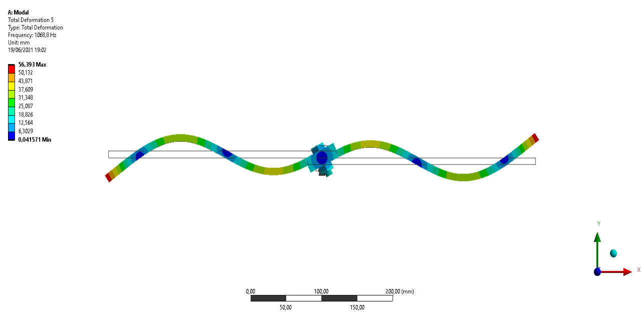
(b) 2nd mmode at 334.15 Hz.



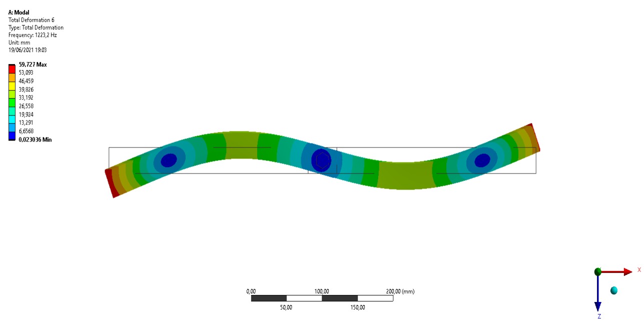
(c) 3rd mode at 437.04 Hz.



(d) 4th mode at 654.93 Hz.



(e) 5th mode at 1068.78 Hz.



(f) 6th mode at 1223.2 Hz.

Figure 8: Modes Shape of the structure using a solid bolt model.

The SEM yields the transcendental eigenvalue problems, while the conventional FEM yields the linear eigenvalue problems (Lee, 2009). Transcendental eigenvalue problems are not direct and imply a proper solution method for the eigenvalue problems derived by the SEM. Hence, the estimation of the modal parameters of the system is indirect. Therefore, harmonic analysis is applied in this case, and only the flexural modes are considered. A unitary force exciting the beam at points A and the receptance frequency response function(FRF) estimated at point A and B in a frequency range of 0-1200Hz, both points are shown in Fig. 5.

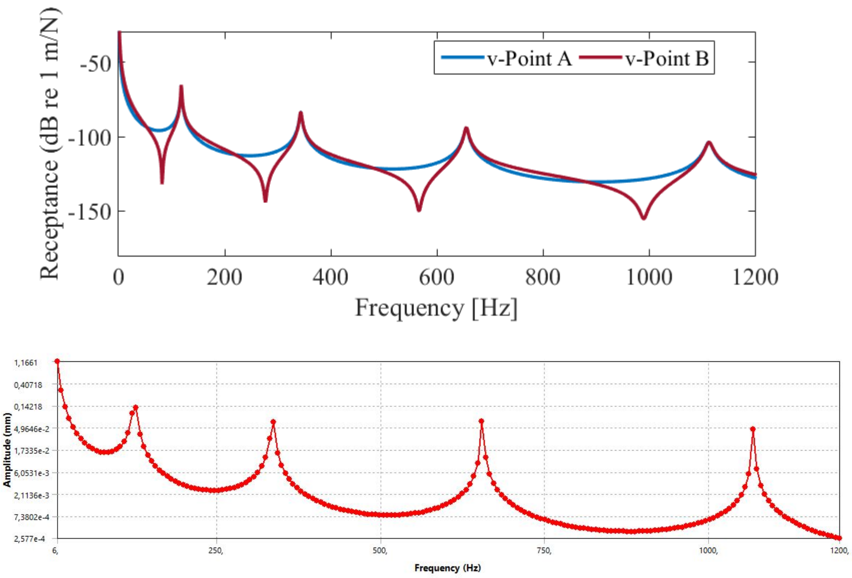


Figure 9: Receptance FRF obtained with SEM(Top) and FEM(bottom).

Figure 9 shows the receptance response obtained using FEM at point B and SEM at points A and B. The first four resonance peaks are related to the flexural modes. By comparing responses obtained using both methods, the amplitude and modal shapes are similar and close resonance peaks over the frequency range. Aside from the comparison over the FRFs, the spectral model of bolted beam resonance frequencies is compared to the data of Kim et al.(Kim *et al.*, 2007a) work.

Table 3: Natural frequencies obtained via SEM, FEM, and Ref.(Kim *et al.*, 2007a) [unit: Hz].

ω_n/**Modes no.**	**1**	**2**	**4**	**5**
Experiment(Kim *et al.*, 2007a)	120	332	640	1060
Solid and Spider bolt model(Kim *et al.*, 2007a)	119	334.1	632.5	1068
SEM	118	343	649	1113
FEM	123.45	334.15	654.93	1068.78

Table 3 displays experimental and numerical natural frequencies presented in (Kim *et al.*, 2007a) and the natural frequencies obtained via SEM and FEM. Close results are found by comparing the experimental data with the numerically simulated via FEM and SEM, although with a certain error associated. The error can be due to some assumptions made in this work as the parameters of the bolted joint interface and the bolt linear and rotational spring. The advantage of using SEM is the easy access to the matrix and general formulation, reduced number of elements in the mesh propitiating inclusion of uncertainties in parameters and non-linearities without losing accuracy and low computational cost.

5. FINAL REMARKS

This work explores the numerical model via SEM and FEM of two Euler–Bernoulli beams connected by a bolted joint in the mid-span. The validation of the models was performed by comparing the results with the work of Kim *et al.* (2007a). For the FEM analysis, we used ANSYS-Workbench combining in the mesh the SOLID186, CONTAC174, and the TARGE170 elements. SEM model employed the use of a beam spectral element and the spectral bolt element. An approximate solution for the dynamical behaviour of the assembled structure was obtained using both methods. The solution provides the frequency response function of the connected system and its modal parameters. The obtained resonance frequencies were compared with the corresponding experimental counterparts presented by Kim *et al.* (2007a). Close results are found by comparing the experimental data with the numerically simulated via FEM and SEM. The error associated with the estimations can be due to some assumption made by the parameters of the bolted. It must be investigated, and some adjust implemented as in the parameters of the bolted joint interface and the bolt linear and rotational spring. However, using SEM is easy access to the matrix and general formulation, reduced number of elements in the mesh propitiating inclusion of uncertainties in parameters and non-linearities without losing accuracy and low computational cost. Although this work was applied to a simple problem composed of two bars, SEM model can also apply to problems with several bars and bolted joints.

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