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AN EXTENDED AXIAL MOMENTUM ANALYSIS FOR STEAM TURBINES

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Abstract. *Classically steam turbines are analyzed using cascade methods, which can be applied to impulse or reaction rotors. On both kind of turbines, the stator is indeed necessary as it may change pressure and velocity on the rotor blades, making the turbine efficiency about 70%. However, the stator turns the steam turbine more complex to design, even for one stage. In this work, an extended axial momentum analysis applied to steam turbine is developed. The approach takes into account a combination of the kinect energy transported by the steam flow and the effect of enthalpy drop throughout the rotor, affecting the turbine efficiency. Methods based on axial momentum theory are important because they open the possibility of designing steam turbine blades without the need of stator. As a result, the proposed approach can exceed the turbine efficiency beyond Betz limit (59.3%), reaching about 90% or more depending on the impact of the enthalpy variation through the steam rotor. Also, the coupling of the extended axial momentum analysis with the blade element theory can result in a new expression to optimize steam turbine blades.*

Keywords: *Steam turbines, Axial momentum theory, Turbine efficiency.*

1. INTRODUCTION

It is well known that steam turbines are used in large industries, such as thermal system plants. They are useful to process manufactured product or even for producing energy. However, the power thermal systems can be very expensive, and so well-designed steam systems could provide significant cost savings (Li et al., 2014). A steam turbine is primary an equipment that converts thermal energy from steam to mechanical power while distributing the fluid to different pressure levels. They are designed to work in stable operating conditions of a steam system (Hamzaoui et al., 2015, Li et al., 2014).

The process of choosing the best turbine for a given application involves juggling several parameters that may be of equal importance; for instance, rotor stress, weight, outside diameter, efficiency, noise, durability, and cost, so that the final design lies within acceptable limits for each parameter (Dixon & Hall, 2010). Modeling and optimizing steam systems, including boilers, steam turbines, multiply steam headers, and condensers, have been widely investigated through thermodynamic associated to computational methods. Thermodynamic methods are generally used to analyze the energy recovery or usage conditions of the steam system in an industrial process (Li et al., 2014).

Classically steam turbines are analyzed using cascade methods, which can be applied to impulse or reaction rotors. On both kind of turbines, the stator is indeed necessary as it may change pressure and velocity on the rotor blades, making the turbine efficiency about 70%. However, the stator turns the steam turbine more complex to design and building, even for one stage (Lews, 1996, Dixon and Hall, 2010). Analysis of steam turbine is usually done through physical modeling, considering the thermodynamic aspects of the system. The major portion of this modeling consists of the application of the mass and energy balance equations (Gajehkaviri et al., 2015). The disadvantage of conventional steam turbine energy analysis is that the extracted energy from the flow is not equal in real (polytropic) and ideal (isentropic) expansion processes (Viola et al., 2020).

For this paper, it is proposed a modeling based on the axial momentum theory (AMT) applied to steam turbine, using the energy balance equation including the enthalpy variation crossing the rotor. The AMT is also known as the actuator disc theory, which was initially proposed by Rankine (1865) and Froude (1878), representing a simple model for the design of horizontal axis turbines (Vaz and Wood, 2016). This theory is widely used to design horizontal axis wind and hydrokinetic turbines with or without diffuser (Vaz and Wood, 2016, Rio vaz et al., 2018, Ma et. al., 2018). AMT importance lies on the possibility of couple it to the blade element theory (BET), which consider the influence of lift and drag forces on each blade section of the steam rotor. Figure 1 demonstrates the annular control volume for a typical horizontal axis steam turbine. As a result, the proposed approach can exceed the turbine efficiency beyond Betz limit (59.3%), reaching about 90% or more depending on the impact of the enthalpy variation through the steam rotor. Also, the coupling of the extended axial momentum analysis with the blade element theory can result in a new expression to optimize steam turbine blades. Therefore, methods based on AMT are important because they open the possibility of designing steam turbine blades without the need of stator and they can be used for optimizing low pressure horizontal axis steam turbines applied to small industries.

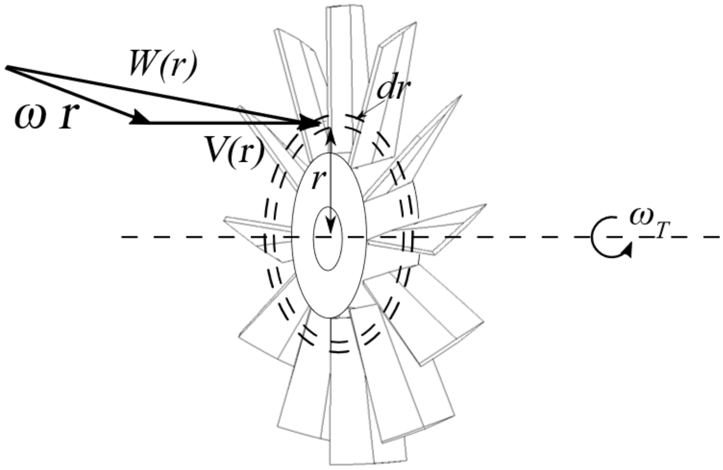

Figure 1. Annular control volume for a typical horizontal axis steam turbine.

2. METHODOLOGY

Here, the AMT combined with the classical thermodynamic is implemented. As an initial study for the development of a new optimization model for steam turbine, upstream and downstream velocities relationship crossing the rotor, and the variation of enthalpy are employed. In addition, the steam turbine efficiency considering the axial induction factors on the rotor is demonstrated.

2.1. Axial momentum analysis for steam turbines

The simple AMT provides information about the flow around the rotor, which is different from the free stream flow. In this model, the rotor is considered as a homogeneous disk, in which the representation of the turbine is possible if the rotor is considered with an infinite number of blades, capable of extracting energy from the fluid. Also, the steam flow is frictionless, ignoring the rotational velocity component (Vaz and Wood, 2016).

It is important to emphasize that this model is derived from the application of the equation of conservation of momentum on the control volume shown in Figure 2 that contains the rotor (Hasen, 2008, Vaz and Wood, 2016). Thus, for the development of this theory, it is important to consider the following hypotheses (Vaz and Wood, 2016):

(1) One-dimensional, compressible, non-viscous and steady-state flow;

(2) Free flow upstream and downstream of the rotor plane;

(3) The uniform velocity field at the control volume inlet is reduced when passing through the actuator disc due to the removal of kinetic energy from the steam flow and pressures, which are different at each position (0 to 3).

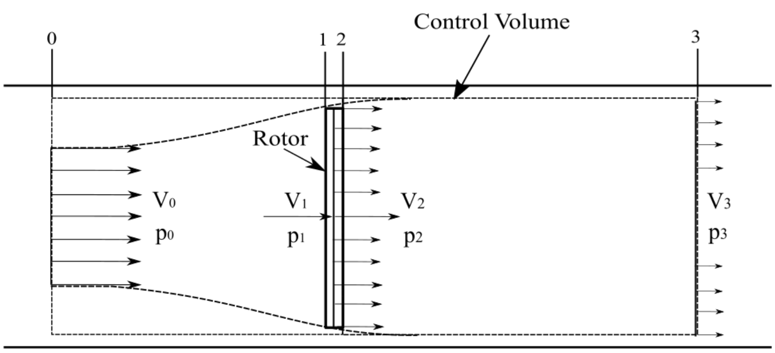

Figure 2. Schematic drawing of the control volume for an ideal steam turbine.

The energy balance is applied using positions 0, 1, 2 and 3. Position 0 is assumed to be the free stream flow with a flow velocity [V_0] and a static pressure [p_0] with density [ρ]. Position 1 is the inlet of the steam turbine with a flow velocity [V_1] and static pressure [p_1]. Position 2 is the outlet of the turbine with a flow velocity [V_2] and static pressure [p_2]. Then, the steam flow decreases to the wake velocity [V_3] where the static pressure is [p_3].

Figure 1 considers the relationship between positions 0 and 1, as well as between 2 and 3. The energy balance expressions are obtained as in Eqs. (1) and (2), taking into account the previous hypotheses. The flow velocity [V_1] approaching the rotor plane is equal to the flow velocity [V_2] leaving the rotor.

$$\boldsymbol{p_1 = p_0 + \frac{1}{2}\rho\,(V_0^2 - V_1^2)}, \tag{1}$$

$$\boldsymbol{p_2 = p_3 + \frac{1}{2}\rho\,(V_3^2 - V_2^2)}, \tag{2}$$

The difference between Eqs. (1) and (2) results in (3).

$$\boldsymbol{\Delta p_R = \Delta p_f + \frac{1}{2}\rho\,(V_3^2 - V_0^2)}, \tag{3}$$

where $\boldsymbol{\Delta p_f = p_3 - p_0}$. The simple AMT considers a flow without frictionless and rotation in the wake. For a one-dimensional analysis, it is necessary to consider the control volume for an ideal steam turbine, as shown in Figure 1. In this case, the thrust force [T] on the rotor is given by the product of the pressure difference [Δp_R] between positions 1 and 2 and the swept area of the rotor [A], as in Eq. (4). The negative sign means the opposite direction of the thrust in relation to the flow passing through the wake.

$$\boldsymbol{T = -\Delta p_R A}, \tag{4}$$

Combining Eqs. (3) and (4) yields:

$$\boldsymbol{T = \frac{1}{2}\rho A(V_0^2 - V_3^2) - \Delta p_f A}\,, \tag{5}$$

Now, using the simple axial momentum balance.

$$\boldsymbol{T = -\int_s \vec{V}\, d\dot{m} = \rho A\, V_2(V_0 - V_3)}\,, \tag{6}$$

where [$d\dot{m}$] is the elemental mass flow. Equating both thrust expressions, Eqs. (5) and (6), results in.

$$\boldsymbol{V_3 = V_2 \pm \sqrt{(V_2 - V_0)^2 - \frac{2\Delta p_f}{\rho}}}, \tag{7}$$

The root physically consistent in Eq. (7) is.

$$\boldsymbol{V_3 = V_2 - \sqrt{(V_2 - V_0)^2 - \frac{2\Delta p_f}{\rho}}}, \tag{8}$$

Applying the classical thermodynamic and making enthalpy variation [Δh] between positions 0 and 3, Eq. (8) yields

$$\boldsymbol{V_3 = V_2 - \sqrt{(V_2 - V_0)^2 - 2\Delta h},\ \ \Delta h = h_3 - h_0} \tag{9}$$

where $\boldsymbol{\frac{\Delta p_f}{\rho} = \Delta h}$. For the mechanical power of a steam turbine, the first law of thermodynamics is applied to steady-state regime through the control volume in Figure 1. So, the steady-state flow energy equation can be written as:

$$\boldsymbol{\dot{Q} - \dot{W}_s = \int_s \left(h + \frac{V^2}{2} + g_Z\right)\rho\vec{V}.d\vec{A}}, \tag{10}$$

According to Dixon and Hall (2010), energy is transferred from the steam to the blades, positive work being done, through the rotor. The mechanical power is given by [$\dot{W}_s$]. In general, positive heat transfer takes place through $\boldsymbol{\dot{Q}}$, from the surroundings to the control volume. The specific enthalpy is [h], $\boldsymbol{\frac{V^2}{2}}$ is the kinetic energy per unit mass, and $\mathbf{g_z}$ is the potential energy per unit mass. Applying Eq. (10) to the control volume in Figure 1, considering the mass conservation $[\rho V_0 A_0] = [\rho V_2 A_2] = [\rho V_3 A_3]$, it gives.

$$\dot{W}_s = \dot{Q} + \rho A V_2 \left(\frac{V_0^2 - V_3^2}{2} - \Delta h\right), \tag{11}$$

Equation (11) is very important to analyze the mechanical power of the steam turbine, considering upstream and downstream velocities relationship crossing the rotor, as well as the variation of enthalpy. The steam turbine efficiency [η] can be calculated by Eq. (12).

$$\eta = \frac{\dot{W}_s}{\dot{Q} + \frac{1}{2}\rho A V_0^3}, \tag{12}$$

Rankine and Froude demonstrated that the induced velocity at the rotor plane and in the free wake are directly proportional to the free stream velocity [V_0], leading to $\boldsymbol{u} \propto \boldsymbol{V_0} = \boldsymbol{aV_0}$ and $\boldsymbol{v} \propto \boldsymbol{V_0} = \boldsymbol{bV_0}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the axial induction factors on the rotor plane and wake, respectively (Hansen, 2008, Vaz and Wood, 2016). The flow velocities are

$$V_2 = (1 - a)V_0, \tag{13}$$

$$V_3 = (1 - b)V_0, \tag{14}$$

using Eqs. (13) and (14) into (5), and making the thrust coefficient as $C_T = \frac{T}{\frac{1}{2}\rho V_0^2 A}$, for the ideal steam turbine yields.

$$C_T = 2(1 - a)b, \tag{15}$$

The same ideal condition for the axial induction factors can be considered to the enthalpy variation and the freestream velocity at the position 0 (Figure 1). Substituting Eqs. (13) and (14) within (9), yields

$$b = a + \sqrt{a^2 - \frac{2\Delta h}{V_0^2}}, \tag{16}$$

Now, the proposed analysis can be done on the steam turbine efficiency considering the axial induction factors $\boldsymbol{a}$ and $\boldsymbol{b}$ into Eq. (12), resulting in

$$\eta = (1 - a)\left[(2 - b)b - \frac{2\Delta h}{V_0^2}\right], \tag{17}$$

In order to demonstrate the optimization of steam turbine blades, the AMT analysis needs to be coupled to the BET. This can be done through the BET expression to calculate the chord of a steam turbine blade (Silva et al., 2017):

$$c = \frac{2\pi r}{B}\frac{C_T}{C_n}\left(\frac{V_0}{W}\right)^2, \tag{18}$$

where [c] is the chord at a blade section, [r] is the radial position on the steam rotor, [B] is the number of blades, [C_n] is the normal force coefficient acting on the blade section, and [W] is the relative velocity of the steam on a blade section. Substituting Eq. (15) in (18), it gives

$$c = \frac{4\pi r}{B}\frac{(1-a)}{C_n}\left(a + \sqrt{a^2 - \frac{2\Delta h}{V_0^2}}\right)\left(\frac{V_0}{W}\right)^2, \tag{19}$$

To optimize the chord along the blade, it is necessary to find the optimum axial induction factor [a_{opt}], which is obtained making [$\frac{d\eta}{da} = 0$] in Eq. (17), which results in

$$2(a_{opt} - 1)\left[3a_{opt}^2 + a_{opt}\left(3\sqrt{a_{opt}^2 - \frac{2\Delta h}{V_0^2}} - 1\right) - \sqrt{a_{opt}^2 - \frac{2\Delta h}{V_0^2}} - \frac{4\Delta h}{V_0^2}\right] = 0, \tag{20}$$

Note that [a_{opt}] can be calculated solving Eq. (20) iteratively. Therefore, the expression for the optimum chord becomes

$$c_{opt} = \frac{4\pi r}{B}\frac{(1-a_{opt})}{C_n}\left(a_{opt} + \sqrt{a_{opt}^2 - \frac{2\Delta h}{V_0^2}}\right)\left(\frac{V_0}{W}\right)^2, \quad (21)$$

3. RESULTS AND DISCUSSION

Typically, the steam turbine efficiency is based on thermodynamic analysis as shown in Eq. (12), relating the input steam thermal energy (the heat transfer rate $\dot{Q}$) with the output mechanical power [$\dot{W}_s$] or by steam enthalpy variation rate at the inlet and outlet of the turbine (Dixon and Hall, 2010). Hence, the hypotheses described in subsection 2.1 are assumed, considering the steam inlet temperature equal to 418.15 *K* and $\dot{Q}$ = 0. The values for the optimized steam turbine adopted here are: **R** = 0.138 *m* (rotor radius), **A** = 0.0598 m^2 (swept area), $\mathbf{V_0}$ = 10 *m/s* and Δh = {0.0, -5.0, -10} *kJ/kg*; at a flow rate [q] of 0.5983 m^3/s. Figure 3 shows the steam turbine efficiency as a function of the thrust coefficient in three different enthalpies variation Δh. The turbine efficiency is calculated through Eq. (17), while the thrust coefficient through Eq. (15). When Δh = 0.0 *kJ/kg*, it is observed that the optimum efficiency [η] is equal to 0.592593 for a C_{Topt} = 0.888889 reaching the Betz limit (59.3%), which, theoretically, is the maximum energy extracted by horizontal axis turbines. For Δh = -5.0 *kJ/kg*, there is an increase in the efficiency, resulting in η_{opt} = 0.736492 and C_{Topt} = 0.949627 without stator. When the enthalpy variation assumes a value of -10.00 *kJ/kg*, the turbine efficiency reaches 90.5% for a C_{Topt} equal to 0.992202. If $\Delta h \neq 0$, and $C_T \to 0$, the turbine efficiency tends to infinite, i.e., $\eta \to \infty$. This result shows that an ideal steam turbine has a limited region of operation in terms of thrust coefficient, which depends on the enthalpy variation across the rotor. Another important result is, depending on Δh, there is an optimum value for η. This open the possibility of steam turbine blade optimization, because, as previously stated, the AMT can be coupled to the BET analysis, which is a good technique for blade optimization (Vaz and Wood, 2016, do Rio Vaz *et al.*, 2018).

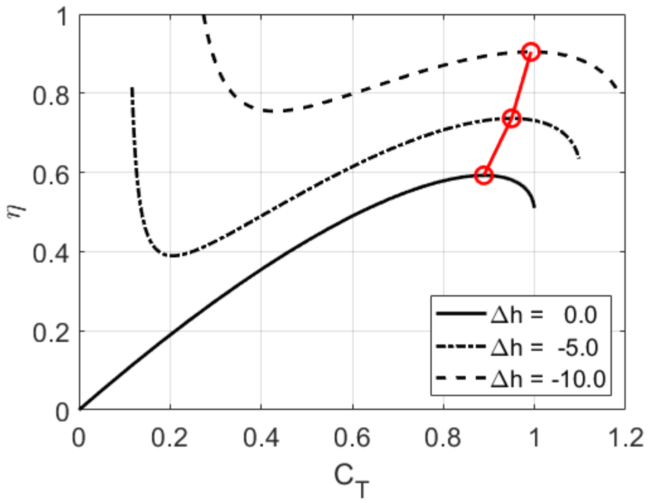

Figure 3. Steam turbine efficiency as a function of the thrust coefficient.

The optimum turbine efficiency given by Eq. (17), varying the axial induction factor [*a*] in Eq. (16), is depicted in Figure 4. In this case, the behavior of the optimum axial induction factor [a_{opt}] of the turbine for different enthalpy variation is evaluated through Eq. (20). Also, Figure 3 shows the optimum axial induction factor [a_{opt}] varying the enthalpy. The Δh = [0.0, - 5.0, -10.0] *kJ/kg* corresponds to a_{opt} = [0.333333, 0.224440, 0.088304], respectively. It is observed that as the enthalpy variation decreases through the steam rotor, the optimum axial induction factor decreases, not limited to the value 0.3333, which corresponds to the optimum efficiency of 0.592593 according to Betz limit. This shows that the optimum value of the axial induction factor changes in relation to Δh, being it very different from the classical AMT applied to horizontal axis turbines. Hansen (2008) explain that the AMT is not valid for values of *a* greater than approximately 0.4, as the free shear layer at the edge of the wake becomes unstable when the velocity jump becomes too high and eddies are formed which transport momentum from the outer flow into the wake, called turbulent-wake state. However, this condition may be modified for a steam turbine due to the dependency of Δh, as in Figure 4.

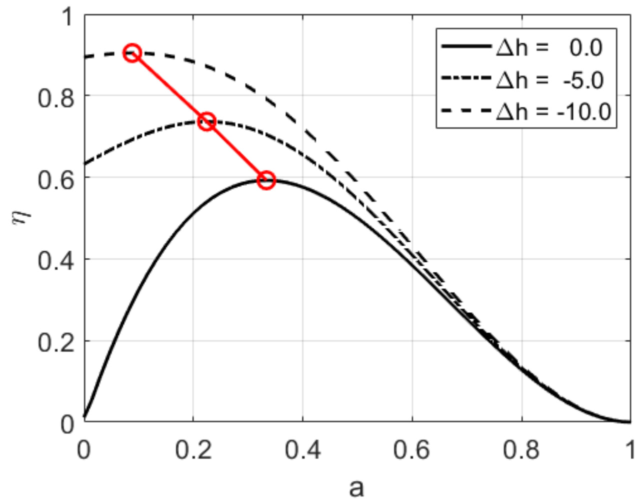

Figure 4. Steam turbine efficiency as a function of the axial induction factor.

Figure 5 shows the relationship between the axial induction factors at the rotor plane [*a*] and in the free wake [*b*] of an ideal steam turbine. This result presents an interesting behavior between ***a*** and ***b***, varying in relation to the enthalpy drop throughout the rotor plane. The big changing in *b* leads to a big increase in the steam turbine efficiency, as previously depicted in Figure 3, which can exceed 80% of efficiency without stator.

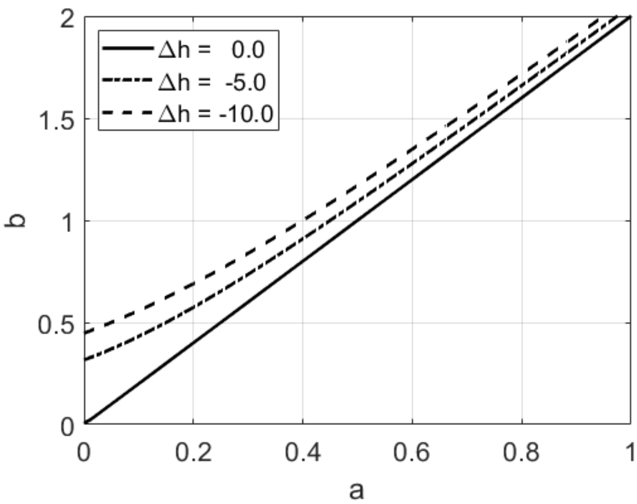

Figure 5. Relationship between the axial induction factors at the rotor plane [*a*] and at free wake [*b*].

To demonstrate the behavior of Eq. (21), Figure 6 shows the optimum chord distribution along the steam turbine blade varying the enthalpy difference between rotor inlet and outlet. It is worth noting that the tangential induction factor is neglected in this work, i.e., only the contribution of the axial induction component of the flow velocity is considered. Note that small Δh decreases the chord distribution, mainly close to the blade root. This is due to a small optimum axial induction factor (a_{opt} = 0.088304) for Δh = -10.0, as shown in Figure 3. However, close to the blade tip, the optimum chord distribution has no significant difference when compared to the chord distributions calculated for Δh = [0.0, - 5.0]. This result is important because the main contribution of the turbine torque comes from the forces acting on the blade tip, and they are dependent on the chord for horizontal axis turbines, as recently described by Ribeiro *et al*. (2021). Figure 7 shows the blade shapes for Δh = 0.0 (Figure 7a), and for Δh = -10.0 (Figure 7b). The shapes are modified depending on the enthalpy variation.

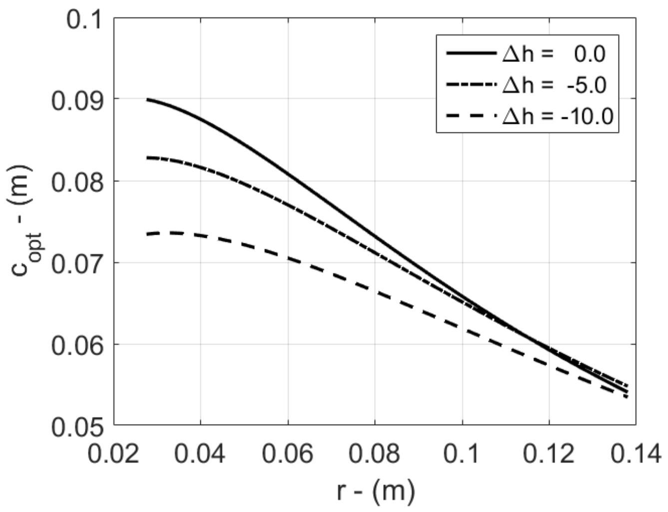

Figure 6. Optimum chord distribution along the blade.

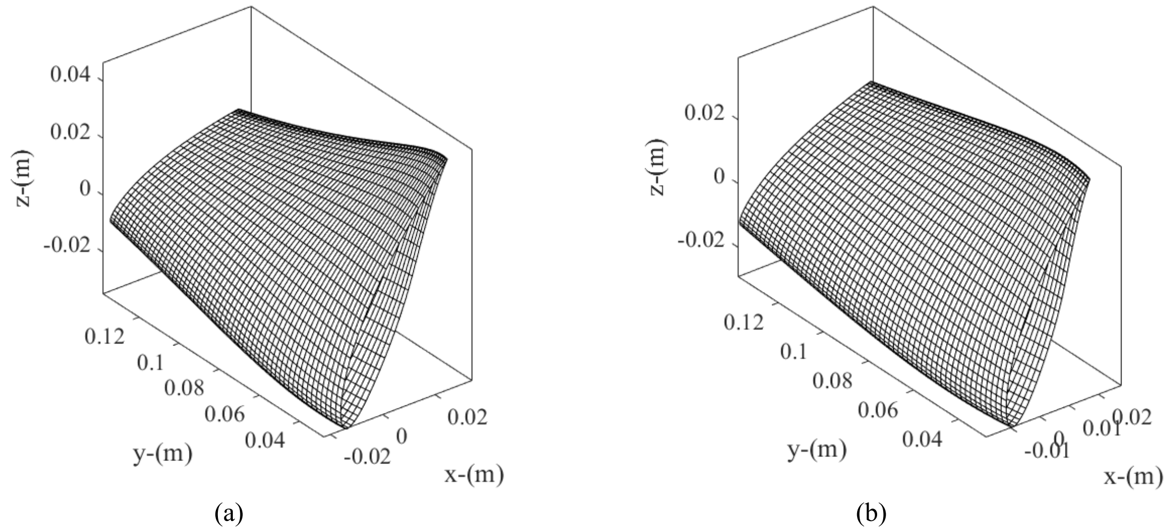

Figure 7. (a) Blade shape for $\boldsymbol{\Delta h}$ = 0.0. (b) Blade shape for $\boldsymbol{\Delta h}$ = -10.0.

4. CONCLUSIONS

This paper presents an extended axial momentum analysis applied to steam turbine. The approach is coupled to the BET model, in order to obtain a new expression to optimize chord distribution along the blade of a steam turbine. This technique opens the possibility of a new optimization procedure for the design of steam turbine blades without the need of stator. All results show a good behavior of the efficiency of a steam turbine, depending on the effect of enthalpy drop throughout the rotor.

The optimum turbine efficiency beyond Betz limit (59.3%), reaching about 90.5% to the enthalpy variation of -10.0 kJ/kg can be used to the hydrodynamic optimization of steam blades. It is necessary to consider some limitation of the axial momentum theory, such as the frictionless and non-rotating flow in the wake. Also, it presents a rotor with infinite number of blades, where the axial momentum theory does not establish a relationship between rotor geometry and turbine performance. This subject will be developed in a future work, in which the blade element analysis will be implemented considering the effect of the tangential component of the induction flow velocity crossing the rotor plane, establishing a relation to the rotor blade geometry without the need of stator as well.

5. ACKNOWLEDGEMENTS

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