COBEM 2021 26th International Congress of Mechanical Engineering
ABCM Associação Brasileira de Engenharia e Ciências Mecânicas
November 22-26, 2021, Virtual Congress, Brazil

COBEM-2021-0037
A NEW MULTI-OBJECTIVE OPTIMIZATION ALGORITHM INSPIRED BY LICHTENBERG FIGURES APPLIED TO ISOGRID STRUCTURES

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Abstract. *Classic optimization methods had their importance in the past, but they lost space for new algorithms that emerged with the advancement of computing, better able to deal with a greater number of variables, objectives and nonlinearities. Evolutionary and meta-heuristic algorithms are exponents in the literature as the main tool for solving complex multi-objective problems. This work presents the first multi-objective meta-heuristic registered as a computer program in Brazil. The Multi-objective Lichtenberg Algorithm is an optimizer inspired by the physical phenomenon of radial intra-cloud lightning and Lichtenberg Figures. This algorithm will be applied to isogrid structures and compared with one of the most used meta-heuristics in the literature: the genetic algorithm. The present work aims to find the optimal design parameters for the lattice structure in order to minimize the Tsai-Wu failure index and the mass of the structure under compression loads. A Response Surface Methodology (RSM) was used for the purpose of setting up a series of experiments for adequate predictions of the responses and the generated models. The Brazilian meta-heuristic proved to be powerful for solving complex multi-objective problems.*

Keywords: *Multi-objective optimization, Meta-heuristics, Lichtenberg Algorithm, Design, Isogrid Structures.*

1. INTRODUCTION

With the advancement of computing and data science, more robust and efficient algorithms have shown more capacity to find optimal solutions compared to classic optimization methods (YANG, 2014). The meta-heuristics are an exponent of Artificial Intelligence in optimization problems and are currently the most used tools in complex engineering problems (MIRJALILI *et al*., 2016). These types of algorithms are better able to deal with non-linear, multi-modal, discontinuous and with a large amount of input variables and objective functions in the same problem. When the problem has more than one objective, it is called Multi-objective and the number of algorithms capable of solving them is even smaller. Most of the works found in the literature using multi-objective meta-heuristics use evolutionary or swarm algorithms. The most used are SPEA, NSGA-II, MOEA/D and PAES (MIRJALILI *et al*., 2017). According to the no-free-lunch (NFL) theorem (JOYCE & HERMAN, 2018), none of them can be excellent at solving any type of problem.

This study brings one of the first meta-heuristics registered in Brazil, the Lichtenberg Algorithm (LA), for application in a complex problem in Mechanical Engineering. This algorithm is inspired by the physical phenomena of lightning storms and and Lichtenberg figures. The details of the creation, validation and some applications can be found in Pereira *et al*. (2021). The optimizer has been successfully applied in the identification of cracks (PEREIRA *et al*., 2020), damage in composites (PEREIRA *et al*., 2021) and in the design optimization of carbon fiber isogrid lower limb prosthesis considering single-objective optimization (FRANCISCO *et al*., 2020). One of the reasons for LA's success is that it combines in a single optimizer a stochastic search based on population and trajectory, with shots of Lichtenberg Figures (LF) in the best solution from the previous iteration. At each iteration, this figure (known as a peculiar natural pattern - fractals) is fired with different sizes and rotations, being able to sweep from tiny regions to the entire search space.

The objectives of this work are to present the LA for the first time in a Congress and to compare it with the most used algorithm in the literature today for multi-objective optimization, the NSGA-II. Both will be used in this work to optimize an isogrid structure considering simultaneously the mass and the failure criterion of the mechanism. The metamodel generated by the Response Surface Methodology (RSM) through a Design of Experiments of the input variables of the structure will be considered as objective functions. The finite element method was used to evaluate the DOE experimental matrix in the objectives of the optimization problem.

The manuscript is organized as follows: Section 2 brings a general theoretical nackground about Lichtenberg Algorithm and RSM. Section 3 presents the methodological procedure. Section 4 brings the results and discussions and section 5 brings the conclusions.

2. BACKGROUNDS

2.1 Lichtenberg Algorithm

A new meta-heuristic inspired by the physical phenomena of lightning storms and Lichtenberg Figures was recently created by Pereira *et al*. (2021), which presents all the details of the creation of the Lichtenberg Algorithm (LA). Lichtenberg was the first to study the phenomenon of propagation of electric discharges in dielectric material, which leads the figure to have branched and tortuous aspects. According to Merrill (1939), the impossibility of determining the resistances at each point, having the heterogeneities of the material, determines the random growth of the figure for each case, even for the same material and electrostatic conditions. Turner (2019) suggests that LF can be built through a random growth process with many particles, forming a cluster. Due to its stochastic model, each execution of the algorithm can generate different figures. Therefore, the construction of the Lichtenberg Figure is entirely numerical.

Among some growth models found in the literature, the Diffusion Limited Aggregation (DLA) theory, proposed by Witten and Sander (1981 & 1983), was used. A binary matrix (0 and 1) is built as a map and, in the center, a particle represented by the number one is fixed. The cluster is built by the values of the matrix that is one and the empty spaces have zero value. Each matrix element of value one is a particle of the cluster and the number of them (N_p) in the cluster is defined at the beginning of the program. The space for construction of the figure is defined by the creation radius (R_c) and from it the matrix is generated with line and column numbers equal to twice R_c (diameter).

Particles are randomly released across the matrix and if they reach the cluster that in the beginning was just a particle in the center, they have an S probability of fixing, also called the stickiness coefficient. This parameter controls the density of the cluster. The particle walks are plotted randomly, radially, and as if they were on a Cartesian plane from the center and settling down anywhere on the map, rounding off the position to a matrix element with line and column. At this point, it can be added only if there is another particle next to it confirmed by a lateral check. If it reaches a radius slightly larger than the R_c, it is exterminated and another one starts the random walk again. This happens until all the particles determined at the entrance N_p are contained in the cluster or until it reaches its limit of construction.

Each particle in the cluster can be transformed into locations on a Cartesian plane and the LF can be plotted at any size, slope or starting point. Then, the extracted figure is plotted in the exact size of the search space and its center in the center of it. At each iteration, this figure can be plotted with different sizes and rotations, selected at random. This is done as a measure to improve the exploration and exploitation capabilities of the algorithm, in addition to preventing a flawed reading of the search space.

Another parameter of the optimizer is the refinement (*ref*), an input parameter that can be from 0 to 1 and is a creator of a second LF (red) every iteration from 0 to 100% with the same size of the main LF (blue) (see Figure 3). This smaller scale figure improves local search. If *ref* = 0, only the global LF (blue) acts on the optimizer every iteration, the global one.

Not all LF points are used to compute the objective function(s), as the number of points used for this purpose or population (*pop*) is defined at the beginning of the algorithm. The LF points (that represents the population) are chosen throughout the LF structure and are represented graphically by black dots, all of which are always within the search space by means of a check. This form makes LA a hybrid algorithm as it merges two types of algorithms found in the literature: population and trajectory. This hybrid routine, not found in any meta-heuristics, brought to the algorithm a great capacity for exploitation and exploration.

The sixth parameter is switching factor (M), a parameter for changing the LF in the optimizer input data. This parameter can be set as zero, one or two. If zero, a figure is generated when starting the program and the same figure is used in all the iterations of the execution. If it is 1, a new figure is generated at each iteration. If M is worth 2, a previously saved figure is used in the optimizer and no figure is generated. It is important to note that the generation of a Lichtenberg Figure can take about two minutes. However, if no figure is generated, the optimizer generates final solutions in less than a second. Finally, the number of iterations (N_{iter}) is also defined as an initial configuration parameter of the algorithm LA, being equal to 100 iterations, a quite common number for many algorithms found in the literature.

All points evaluated in the search space generate solutions in the objective space and these solutions are compared using the Pareto dominance relationship, where the non-dominated solutions are kept in the solution space and the non-dominated ones are excluded. The set of non-dominated solutions for each iteration forms the current Pareto front of the problem, which tends to approach the real one through the iterations. The algorithm works considering all points of the current Pareto front as candidate points to plot LF's. At each iteration, one of these points are selected at random to plot a LF, generating in the variable space a forced search in the regions that present better values of objective functions.

Table 1 shows some recommendations for the parameter ranges to be used. As explained earlier, there are parameters for flat and spatial Lichtenberg figures according to the dimension of the problem. Figure 1 shows the LA acting in the search space for a two-dimensional problem.

Table 1 - Recommended LA Parameters (Pereira *et al*., 2021)

Parameters	Any Dimension	Three-dimensional
R_c	50 to 200	50 to 150
N_p	$>10^3$ and $<10^6$	$>10^5$ and $<10^6$
S	0 to 1	0 to 1
Pop	(10 a 40) × d	(10 a 40) × d
ref	0 to 1	0 to 1
M	0, 1 or 2	0 or 1
N_{iter}	$>10^2$ and $<10^3$	$>10^2$ and $<10^3$

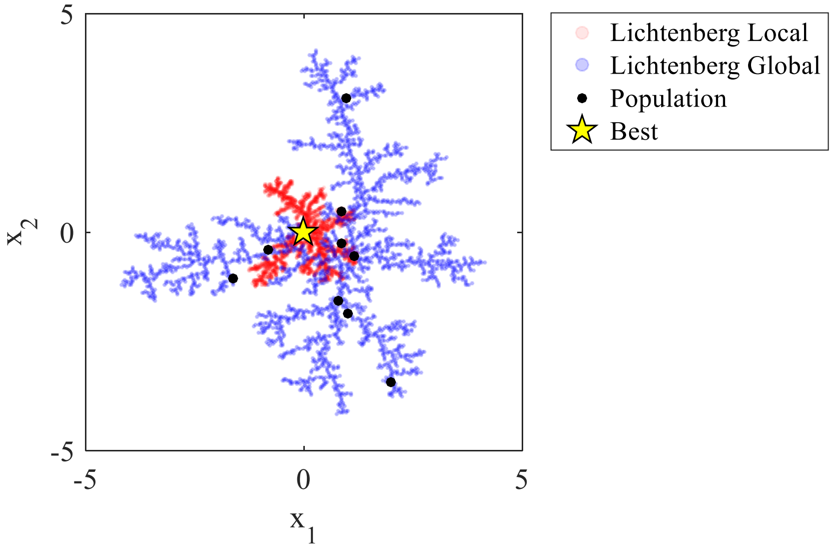


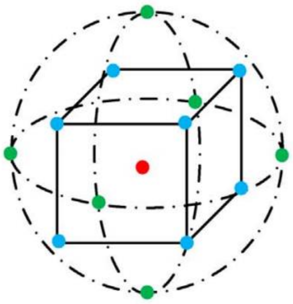
Figure 1 – Population distribution in the Lichtenberg Figures (pop = 10 and ref = 0.3) (Pereira *et al*., 2021)

2.2 Response Surface Methodology

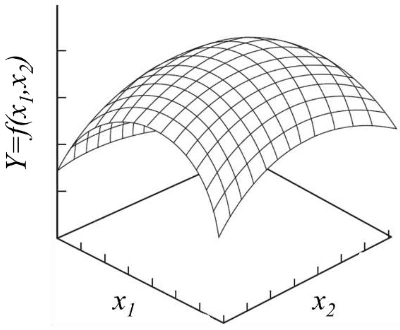
According to Brendon *et al*. (2020b), the RSM is statistical which must describe the behavior of a data set. Thus, always the system reveals a curvature, is necessary to use a second-order model as shown in Equation 1 (Montgomery, 2017).

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon \quad (1)$$

The Equation 1 demonstrates the response surface model considering the curvature in the system, where k represents the number of design parameters. The Figure 2a represents a central composite design (CCD), that is a type of design used in response surface suitable to generate complete quadratic model using all design factors related to the experiment. Figure 2b shows a plot in a 3-D space, where the factors are plotted depending on a response often aiming a fitted surface.



(a)



(b)

Figure 2 – Typical response surface: (a) Central composite design (Lian *et al*., 2017) and (b) three-dimensional (adapted from Montgomery, 2017).

3. METHODOLOGY

The objective of this work is to apply the Lichtenberg Algorithm (LA) in the multi-objective optimization of an isogrid tube made of fiber reinforced polymer composite (CFRP). For this, the metamodels found by Brendon *et al*. (2020) that were generated from the RSM and a Finite Element Analysis (FEA). To validate the capacity of LA, one of the most used algorithms in the Literature on multi-objective optimization will also be applied in the problem, the Non-Sorting Genetic Algorithm II (NSGA-II). A brief discussion of some non-dominated solutions found will be discussed. The non-dominated solutions are those that no other solution can reduce some objective without causing a simultaneous increase in at least one other objective and the set of this solutions is called Pareto front and present several optimal solutions (Chiandussi *et al*., 2012). All solutions in Pareto front are equally optimal solutions and can be chosen according to the decision and priority of the operator of the problem.

Brendon *et al.* (2020b) initially considered 8 input variables. But using Analysis of Variance (ANOVA) and RSM, the problem has been reduced to the same one that will be considered here. The input variables are the Angle of Helical ribs with respect to the axial axis of the structure (φ), width of circular (δ_c) and helical (δ_H) crosspieces (See Figure 3). As objective functions, metamodels representing the mass (M) (for minimization) and the Tsai-Wu failure index (T_{WC}) (minimization) of the structure under compression forces were considered. Table 2 shows the metamodels found by the author that had good Adjusted R^2 values, being 99.19% for mass and 81.52% for TWC. Adjusted R^2 shows how well the data fits the model. The higher, the better the data fit.

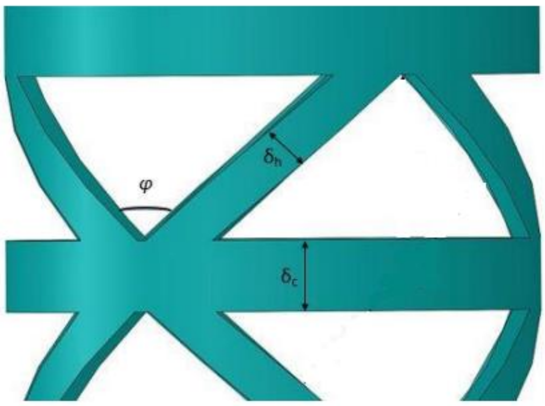


Figure 3 – Geometrical parameters of isogrid structure (adapted from Brendon *et al.*, 2020b).

Table 2 – Equations for Metamodels (adapted from Brendon *et al.*, 2020b).

Response	X_1	X_2	X_3	X_2^2	X_3^2	$X_1 X_2$	$X_1 X_3$	$X_2 X_3$	C
.M	0.0003	0.339	3.052	-	-	0.05	0.03	-	-0,08
T_{WC}	0.031	-0.127	-1.142	-	0.136	-	-	-	2.536

4. RESULTS AND DISCUSSIONS

The multi-objective optimization of isogrid structures consists on finding the non-dominated solutions of the conflicting objectives mass (M) and Tsai-Wu failure index for compression (T_{WC}) through the variations of the Angle of Helical ribs with respect to the axial axis of the structure (φ), width of circular (δ_c) and helical (δ_H) crosspieces. The ranges that determine the search space for the decision variables for this problem are the same as those used in the Design of Experiments. These boundary conditions used were based on a standard that deals with structural testing of lower-limb prostheses (NBR ISO 10328). The only constraint of the problem is inequality and is related to the DOE constraint for coded variables. The structure was embedded in one end and a compression force of 4800 N was applied. The optimization problem is represented in Equation 2, where $\boldsymbol{X} = \{ \varphi , \delta_c , \delta_H \}$

$$
\begin{aligned}
&\min F(\boldsymbol{X})=\{ \boldsymbol{M}(\boldsymbol{X}), \boldsymbol{T_{WC}}(\boldsymbol{X}) \} \\
&\textit{subject to:} \\
&20 \leq \varphi \leq 40 \\
&2 \leq \delta_C \leq 6 \\
&2 \leq \delta_H \leq 6 \\
&\varphi^2 + \delta c^2 + \delta c^2 \leq 1{,}68
\end{aligned}
\tag{2}
$$

The optimizations were performed using the Lichtenberg Algorithm (LA) and the Non-sorting Genetic Algorithm II (NSGA-II) considering the metamodels (Equations 2 and Table 2). For both Algorithms, the same number of iterations and population were considered, equal to 100. Therefore the parameters of the algorithms are: *i*) LA: $Pop = 100$; $N_{iter} = 100$; $R_c = 200$; $N_p = 10^6$; $S = 1$; $ref = 0.4$ *e* $M = 0$. *ii) NSGA-II:* $Pop = 100$; $N_{iter} = 100$; $P_c = 0.8$ *(cross over probability)* and $P_m = 0.1$ (mutation probability). The results can be seen in Figure 4.

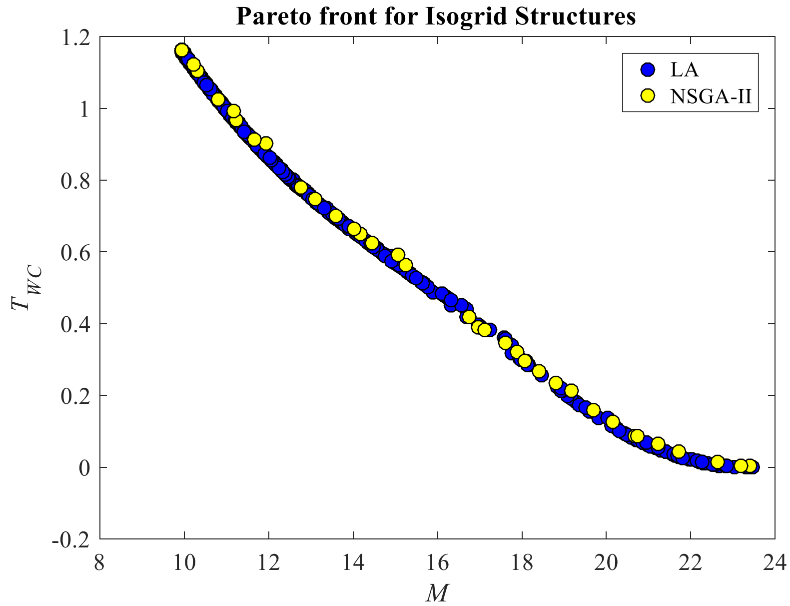


Figure 4– Pareto fronts found by all Algorithms

No Pareto front comparison metric is necessary to visualize that the Lichtenberg Algorithm found a Pareto front with more solutions, less distance between them (more continuity), slightly more distant end points and a Pareto curve a little more convergent than the NSGA-II. Therefore, the Pareto front of LA will be chosen for discussion of some interesting points from the Pareto front. It is worth remembering that on a Pareto front, there is no better solution than another, they are all equally important and selected as the operator of the problem may prefer some solutions.

Two points from PF will be highlighted through Figure 5 and Table 3: *i*) a point that is closer to the ideal point (point that consists of the individual minimum of each objective function) and the farthest from the worst (the opposite of the ideal point), selected through Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (BYUN & LEE, 2005) and finally, *ii)* the point that is only the closest to the ideal. The decision variables found will be entered into the Finite Element Analysis (FEA) software to compare the answers found.

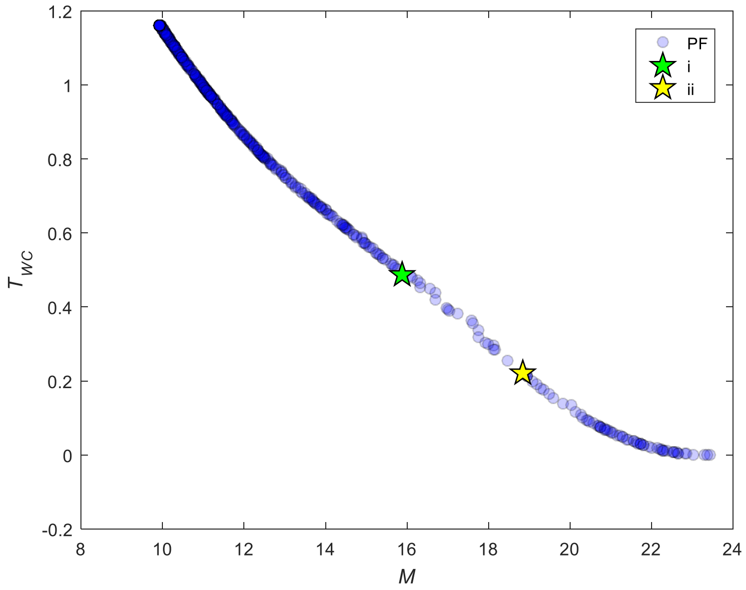
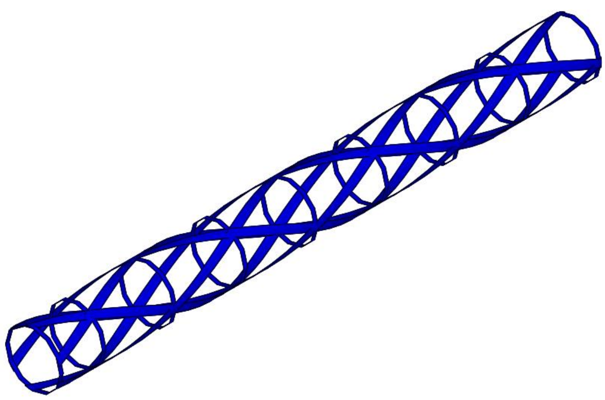


Figure 5– Some of the non-dominated solutions found

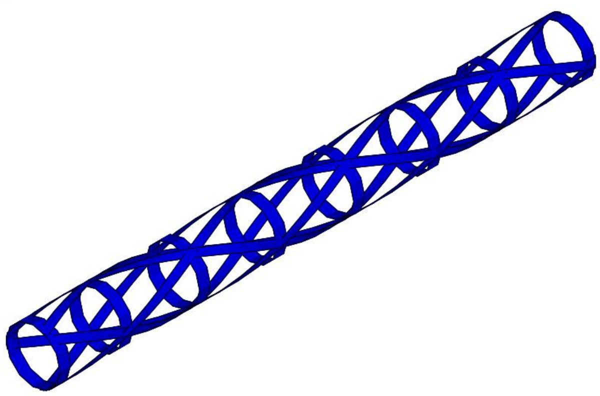
Table 3 – Quantitative description of highlighted solutions

Solution	Decision Variables			Metamodel response		FEM response		Error	
	φ (°)	δ_C (mm)	δ_H (mm)	M (g)	T_{WC}	M (g)	T_{WC}	M %	T_{WC} %
i	21.8423	2	4.0076	15.8840	0.4879	13.76778	0.38867	15.37	25.53
ii	20	6	3.9737	18.8256	0.2224	22.94814	0.18690	-17.96	18.99

Although the Lichtenberg Algorithm has been shown to be a slightly better optimizer than the NSGA-II when optimizing using metamodeling, it was found that the approximation of a complex engineering problem through this methodology in a multi-objective problem generates some errors between the answers of the metamodel and the software. However, the results are reliable and the isogrid structure does not fail for any of the situations, having results with reduced mass and failure criteria. It is also noted that the TOPSIS point generated a structure with less mass and acceptable failure criteria. The closest point to the ideal generated a more robust design. Figure 6 shows the geometry of the isogrid structure generated for the two solutions.



a) Project *i*



b) Project *ii*

Figure 6– Final Isogrid Structures

The biggest difference between the two solutions is in the width of circular, with TOPSIS having found the smallest and, consequently, the smallest mass. Both designs are optimized and the failure criterion is far from 1, indicating that both are suitable for the structural application they were designed for.

5. CONCLUSION

The advancement of computing allowed for countless calculations in a very short time, allowing optimization techniques that test and learn from the problem to gain more space among optimization tools. Starting with evolutionary algorithms, meta-heuristics are an exponent of Artificial Intelligence in Optimization that are based on optimal processes inspired by physical, biological or social phenomena, which aim to obtain excellent answers to optimization problems with lower computational cost and precision.

This manuscript brings one of the first Brazilian meta-heuristics that was inspired by a physical phenomenon typical of tropical countries like ours, the lightning. Honoring one of the first scientists to study the propagation of electrical discharges in dielectric media, the Lichtenberg Algorithm has been applied in the multi-objective optimization of the isogrid structure. The algorithm found a more convergent, coverage and continuous Pareto front than one of the most used and powerful algorithms in the literature, the Non-Sorting Genetic Algorithm -II. Some interesting points of the Pareto Front of LA had the decision vectors highlighted for the analysis of the Project variables. It was noticed that the optimization of an isogrid structure is highly complex and the metamodeling generates errors in the outputs that reached 25.53%. It is believed that a methodology that connects optimization answers directly to the FEM can increase simulation time, but improve accuracy. Two designs that meet the failure criteria were found, one lighter and one more robust. The first being based on TOPSIS and the other being the closest solution to the ideal.

This newly created meta-heuristic has been increasingly applied and is proving to be a fast and accurate optimization tool for solving non-linear, multimodal, discontinuous optimization problems with many decision variables or many objective functions.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support from the Brazilian agency CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) and FAPEMIG (Fundação de Amparo à Pesquisa do Estado de Minas Gerais - APQ-00385-18). The authors would also like to acknowledge of the Tutorial Education Program (PET – Programa de Educação Tutorial).

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