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TOPOLOGY OPTIMIZATION OF AIRFOILS RIBS USING BIDIRECTIONAL EVOLUTIONARY ALGORITHM

Gustavo Corrêa Fazio
Carlos Eduardo de Souza
Marcos Daniel de Freitas Awruch
Universidade Federal de Santa Maria, Santa Maria, RS, Brazil
gusfazio15@gmail.com, carlos.souza@ufsm.br, marcos.awruch@ufsm.br

Abstract. *Detailed and complex components are being studied and built with different methodologies and materials since 3D printers are a reality in several industry fields. Aerospace applications usually require lightweight assembly parts and this work is inserted in this context as it models an aircraft airfoil rib subjected to aerodynamic forces in a fixed shape. This work discusses the topology optimization of a two-dimensional airfoil rib. Compliance or stress are minimized, considering a volume constraint. The optimization is performed through the bidirectional evolutionary structural optimization (BESO) method for a two-dimensional case. Initially, the compliance formulation is shown defining the minimization and constraint functions. Also, the sensitivity filtering scheme is analyzed. Then the stress minimization is presented based on the modified P-norm stress function, which provides values below the maximum stress on the structure. Finally, some simulations are performed on symmetrical and a NACA 2412 airfoils for different angles of attack. Those loads are obtained through the panel method on airfoils to achieve the pressure coefficients distribution. Comparisons are made on the compliance and stress minimizations results. The discussion analyzes the main effects for each method on the optimized topologies.*

Keywords: *airfoil, compliance, optimization, stress.*

1. INTRODUCTION

The search for detailed and complex structures have brought the need to develop new production techniques in various engineering applications, aiming the development of new materials and the improvement of manufacturing processes, especially regarding to additive manufacturing. Many industrial applications require the use of lightweight parts, enabling different methodologies to achieve the desired topology. The topological optimization problem consists on finding an optimal geometry, within a fixed domain, that satisfies an objective function allowing the change of material distribution in the structure. A typical aerospace application of such fundamental optimization method is on the development of light airfoil ribs that are resistant to the aerodynamic loads. Considerable effort has been put on finding optimal topologies of two-dimensional continuous structures based on the compliance and other global constraints.

Yang *et al.* (1999) developed the bidirectional evolutionary structural optimization (BESO) method, applying it to two-dimensional stiffness optimization problems. Its bidirectional capability allowed the gradual addition or removal of elements from the mesh of the structure. Based on the procedure, Nabaki *et al.* (2019) applied it to stress minimization of continuum structures using the modified p-norm stress approach, which provided values below the maximum stress of the structure, as well as the application in structures under critical fatigue stresses. Similarly, Bortoluzzi *et al.* (2019) evaluated the influence of the numerical parameters used in the stress-based BESO method for structural optimization, concluding that a smaller filter radius provided smaller values for the maximum von Mises stress, a lower evolutionary rate increased the computational cost and the variation of the P-norm factor directly affected the results of stresses and final topology.

Walker *et al.* (2015) conducted the topology optimization of a three-dimensional wing of the RV-4 experimental plane, seeking to improve its structural performance and reduce the component's mass. The airfoil was divided into a shell of a certain thickness outside the design space and the center of the airfoil, in which enabled the removal and addition of elements. Along the same lines, Cesconeto (2016) developed a method for discretization of unstructured meshes. The work focused on the structural optimization of the aerodynamic profile NACA 4412, simulating the atmospheric air flow and the pressure loads around it through the Navier-Stokes equations.

Zhu *et al.* (2015) reviewed recent advances in the application of topological optimization techniques in the design of aerospace structures such as pylons and ribs, emphasizing the development of light and smart structures, in addition to the application of composite materials. The work exposed capabilities and effectiveness of topology optimization in engineering applications on aeronautical and space field, demonstrating several opportunities and challenges for the development of new methods and algorithms.

In this context, the present study focuses on the topology optimization of an airfoil rib under a pressure loading considering the minimization of two objective functions: compliance or stress. The optimizations are conducted numerically, by using the finite element method to solve the equilibrium equation of a static structure, obtaining the global stiffness and the displacement field of the elements. The surface pressure distribution is obtained through the constant-strength doublet panel method, introduced by Katz and Plotkin (1991), which can calculate the flow over thick lifting airfoils.

2. BESO PROBLEM BASED ON COMPLIANCE

The basic idea behind topology optimization problems based on compliance is to obtain a solid-void design with maximum stiffness, where the mean compliance is minimized for a given volume material (Huang and Xie, 2009). The optimization problem is stated as

$$\text{Minimize: } C = \frac{1}{2}\mathbf{f}^{\mathrm{T}}\mathbf{u} \qquad \text{Subject to: } \begin{cases} V^* - \sum_{i=1}^{N_e} V_i x_i = 0 \\ x_i = x_{min} \ \text{ or } \ 1 \end{cases} \tag{1}$$

where C is the mean compliance and $\mathbf{f}$ and $\mathbf{u}$ are the applied load and displacement vectors of the structure, respectively. The prescribed total structural volume and the volume of the ith element are V^* and V_i, and N_e is the total number of elements in the system. x_i is the discrete design variable of the ith element, where $x_i = 1$ denote a solid element and $x_i = x_{min}$ denote a void element. To avoid the complete removal of an element from the design domain, x_{min} is set to 0.001, what is called a soft-kill method.

When the specified volume (V*) is less than the current volume (Vk), the target volume for the next iteration is given by:

$$V_{k+1} = V_k(1 - ER) \tag{2}$$

where k is the current iteration number and ER denotes the evolutionary rate of structural volume. If the specified volume (V^*) is larger than the next iteration volume, V_{k+1} is set to be V^*;

2.1 Finite element analysis

In finite element (FE) analysis, the equilibrium of a static structure is defined by

$$\mathbf{Ku} = \mathbf{f} \tag{3}$$

where $\mathbf{K}$ is the global stiffness matrix of the structure. The material interpolation based on the solid isotropic microstructure with penalization (SIMP) model (Bendsøe and Sigmund, 1999) can be expressed by

$$E(x_i) = E^0 x_i^p \tag{4}$$

where the Young's modulus of solid elements is denoted by E^0 and p is the penalty exponent. In this paper $p = 3$ is used. The global stiffness matrix, $\mathbf{K}$, can be calculated by the superposition of the elemental stiffness matrix of the solid element, $\mathbf{K}_i^0$ and design variables, x_i, as

$$\mathbf{K} = \sum_{i=1}^{N_e} x_i^p \mathbf{K}_i^0 \tag{5}$$

2.2 Sensitivity analysis

According to Huang and Xie (2010), the sensitivity number for the BESO method can be defined by the relative ranking of the sensitivity of an individual element (α_i). Therefore, by using the material interpolation model, the sensitivity number should be calculated based on the gradient of the compliance with respect to the change in the design variables as

$$\alpha_i = -\frac{1}{p}\frac{\mathrm{d}C}{\mathrm{d}x_i} = \frac{1}{2}x_i^{p-1}\mathbf{u}_i^{\mathrm{T}}\mathbf{K}_i^0\mathbf{u}_i \tag{6}$$

2.3 Convergence criterion

The convergence criterion for compliance minimization considers the absolute change of the mean compliance (C) between iterations as

$$\frac{\left|\sum_{i=1}^{N} C_{k-i+1} - \sum_{i=1}^{N} C_{k-N-i+1}\right|}{\sum_{i=1}^{N} C_{k-i+1}} \leq 0.01 \tag{7}$$

where N is the evaluation point for the iterations and k the current iteration. In this paper, $N = 5$ is used in all evaluated cases since the variation of the mean compliance over the last ten iterations is acceptably small (Huang and Xie, 2007).

3. BESO PROBLEM BASED ON STRESS

Presented by Nabaki *et al.* (2019), the BESO based on stress aims to find an optimal topology of a structure minimizing its maximum stress subject to a specified volume. The optimization problem is stated as

$$\text{Minimize: } \sigma_G^{PN}(x) = \left(\frac{1}{N_e}\sum_{i=1}^{N_e}\left(\sigma_i^{vm}(x)\right)^P\right)^{\frac{1}{P}} \qquad \text{Subject to: } \begin{cases} V^* - \sum_{i=1}^{N_e} V_i x_i = 0 \\ x_i = x_{min} \ \text{ or } \ 1 \end{cases} \tag{8}$$

where $\sigma_G^{PN}(x)$ is the function of the modified P-norm stress in which provides approximations below the maximum value of stress (Duysinx and Sigmund, 1998). σ_i^{vm} is the von Mises stress in the centroid of elements and P the P-norm factor. In this paper, $P = 4$ is used.

3.1 FE analysis and stress computation

To solve the stress-based optimization problem, a static linear analysis is used, in order to evaluate the displacements and stresses of the elements, being the equilibrium of the structure described according to Eq. (3). The von Mises stress is obtained from the stress tensor ($\boldsymbol{\sigma}_i$) . This tensor is calculated in the FE analysis as

$$\boldsymbol{\sigma}_i = \mathbf{DBu}_i = \{\sigma_{xx}, \sigma_{yy}, \tau_{xy}\}^{\mathrm{T}} \tag{9}$$

where $\mathbf{D}$ is the constitutive matrix of the material and $\mathbf{B}$ the strain displacement matrix. Assuming a plane stress problem, the constitutive matrix is given by

$$\mathbf{D} = \frac{E^0 x_i}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \tag{10}$$

where ν is the Poisson coefficient.

As the stress evaluation is carried out at the center of each element, just one integration point is used. Thus, one stress value is necessary considering that the sensitivity number is only one per element. Therefore, the von Mises stress (σ_i^{vm}) is defined as

$$\sigma_i^{vm} = \sqrt{\sigma_{ix}^2 - \sigma_{ix}\sigma_{iy} + \sigma_{iy}^2 + 3\tau_{ixy}^2} \tag{11}$$

3.2 Sensitivity analysis

For stress minimization, the sensitivity number for the BESO method should be calculated based on the derivative of the modified P-norm stress with respect to the change in the design variables. According to Nabaki *et al.* (2019), due to the minimization, the sensitivity number should be the negative gradient of the modified P-norm stress expressed by

$$\alpha_i = -\frac{\partial \sigma_G^{PN}(x)}{\partial x_i} = -\left[\frac{\partial \sigma_G^{PN}(x)}{\partial \sigma_i^{vm}}\left(\frac{\partial \sigma_i^{vm}(x)}{\partial \boldsymbol{\sigma}_i}\right)^{\mathrm{T}}\frac{\partial \mathbf{D}(x)}{\partial x_i}\mathbf{Bu} - \boldsymbol{\lambda}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}(x)}{\partial x_i}\mathbf{u}\right)\right] \tag{12}$$

The adjoint variable, $\boldsymbol{\lambda}$ is defined by

$$\boldsymbol{\lambda}^{\mathrm{T}} = \frac{\partial \sigma_G^{PN}(x)}{\partial \sigma_i^{vm}}\left(\frac{\partial \sigma_i^{vm}(x)}{\partial \boldsymbol{\sigma}_i}\right)^{\mathrm{T}}\mathbf{DBK}^{-1} \tag{13}$$

To compute lambda, an adjoint system is assembled as:

$$\mathbf{K}\boldsymbol{\lambda} = \frac{\partial \sigma_G^{PN}(x)}{\partial \sigma_i^{vm}} \mathbf{B}^{\mathrm{T}} \mathbf{D}^{\mathrm{T}} \frac{\partial \sigma_i^{vm}(x)}{\partial \boldsymbol{\sigma}_i} \tag{14}$$

3.3 Convergence criterion

The convergence criterion for stress minimization takes into account the absolute change of the maximum von Mises stress between iterations such that the following condition must be met:

$$\frac{\left| \sum_{i=1}^{N} \sigma_{max,k-i+1}^{vm} - \sum_{i=1}^{N} \sigma_{max,k-N-i+1}^{vm} \right|}{\sum_{i=1}^{N} \sigma_{max,k-i+1}^{vm}} \leq 0.01 \tag{15}$$

4. OVERALL PROCEDURE FOR BIDIRECTIONAL EVOLUTIONARY STRUCTURAL OPTIMIZATION

The summary of the evolutionary optimization interactions procedure can be presented as follows:

1. Discretize the design domain with finite elements;

2. Specify initial parameters such as penalty exponent (p), evolutionary rate (ER), prescribed volume (V^*) and P-norm factor (P);

3. Perform finite element analysis and compute the elemental sensitivity numbers according to Eq. (6) for minimizing compliance or Eq. (12) for minimizing stress;

4. Filter the element sensitivity numbers using

$$\hat{\alpha}_i = \frac{\sum_{j=1}^{M} w(r_{ij}) \alpha_j}{\sum_{j=1}^{M} w(r_{ij})} \tag{16}$$

   where $\hat{\alpha}_i$ is the filtered sensibility, M the number of elements connected to the jth evaluation point and r_{ij} the distance between the center of the element i and element j. The linear weight factor ($w(r_{ij})$) is given as

$$w(r_{ij}) = \begin{cases} r_{min} - r_{ij} & \text{when} \quad r_{ij} < r_{min} \\ 0 & \text{when} \quad r_{ij} \geq r_{min} \end{cases} \tag{17}$$

   where r_{min} denotes the filter radius. In order to improve the convergence of the BESO optimization problem, Huang and Xie (2007) propose the averaging of sensitivity numbers with its historical information given as

$$\alpha_i = \frac{\hat{\alpha}_i^k + \hat{\alpha}_i^{k-1}}{2} \tag{18}$$

   Then, letting $\hat{\alpha}_i^k = \alpha_i$ for the next iteration, the historical information is preserved;

5. Determine the target volume for the next iteration;

6. Add and remove elements as follows: solid elements ($x_i = 1$) are removed if $\alpha_i \leq \alpha^{th}$ and void elements ($x_i = x_{min}$) are added if $\alpha_i > \alpha^{th}$, where α^{th} is the threshold sensitivity number;

7. Repeat steps 3-6 until the prescribed volume (V^*) is achieved and the convergence criterion (Eq. (7) for minimization of compliance or Eq. (15) for minimization of stress) is satisfied.

5. RESULTS AND DISCUSSION

This section presents the topology optimization of a symmetrical (NACA 0010) and a cambered (NACA 2412) airfoil ribs, covering the two minimization problems introduced previously. For the theory implementation and the simulation environment, the software MATLAB® is used. Analyzes are performed by evaluating aerodynamic loads for angles of attack (α) 0° and 5°. Next, the results are compared and discussed.

5.1 Initial considerations

The considered design domain is illustrated in Figure 1. In blue, the design space is defined, where it is possible to add or remove elements. Highlighted in red are the non-design space regions: a fixed shell intended to propagate the load while maintaining the aerodynamic shape, and a hole located a quarter of the airfoil chord with a radius proportional to the number of elements used, with the purpose of representing the space of a simple cylindrical spar and serving as a boundary condition for the static analysis.

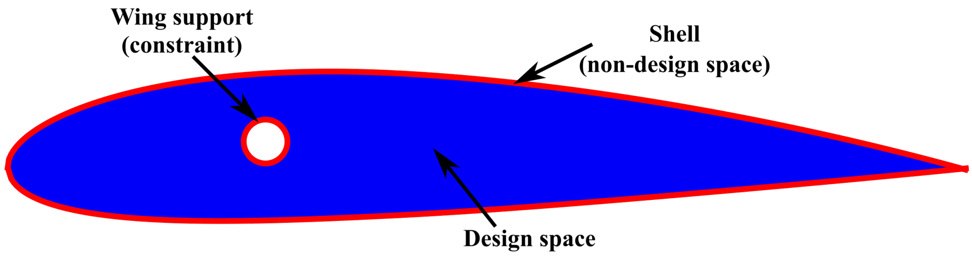

Figure 1. Design space example for the airfoil topology optimization.

The flight regime is based on the Cessna Skyhawk 172 (Fig. 2). A normal cruise flight condition is adopted, with a speed of 72 m/s and an altitude of 2438.4 m (Jackson, 2004). In addition, the aircraft's aerodynamic mean chord dimension of approximately 1.5 m is used as default for all the simulations. The material used in the ribs is an Alclad 2024-T3 aluminum (ASTM and SAE, 2008). All aircraft specifications and BESO input parameters used in the optimizations are illustrated in the Table 1. The pressure coefficient distribution along both airfoils for the evaluated angles of attack is presented in Figure 3.

Figure 2. Cessna Skyhawk 172 aircraft layout.

Table 1. List of the main input parameters of the optimization.

Input parameter	**Value**	**Unit**
Young's modulus	73.1	GPa
Poisson's ratio	0.33	-
Mesh size (NACA 0010 / 2412)	11091 / 13268	-
Specified volume (V^*)	0.75	-
Evolutionary rate (ER)	0.01	-
Filter radius (r_{min})	3	-
Cruise speed	72	m/s
Flight altitude	2438.4	m
Airfoil chord	1.5	m
Angle of attack (α)	0 / 5	°

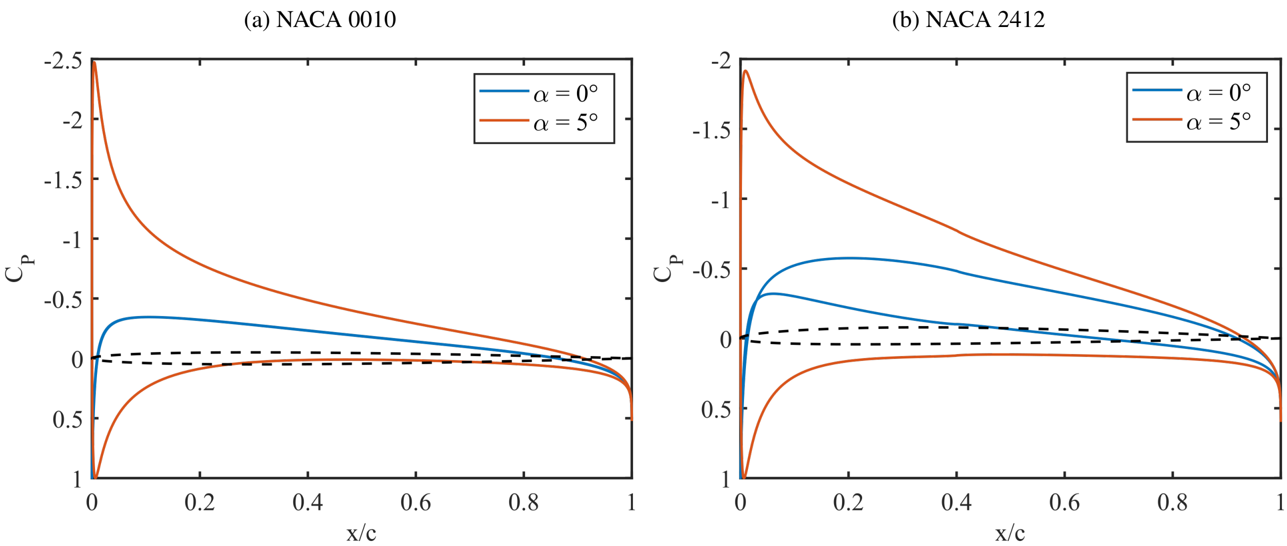

Figure 3. Pressure coefficient distribution for different angles of attack.

5.2 Airfoil rib optimization

The results of optimizations to minimize compliance or stress considering the airfoil ribs are presented below. Figure 4 shows an example of the evolutionary history of the mean compliance and volume fraction of the compliance-based problem for the NACA 2412 and $\alpha = 0°$.

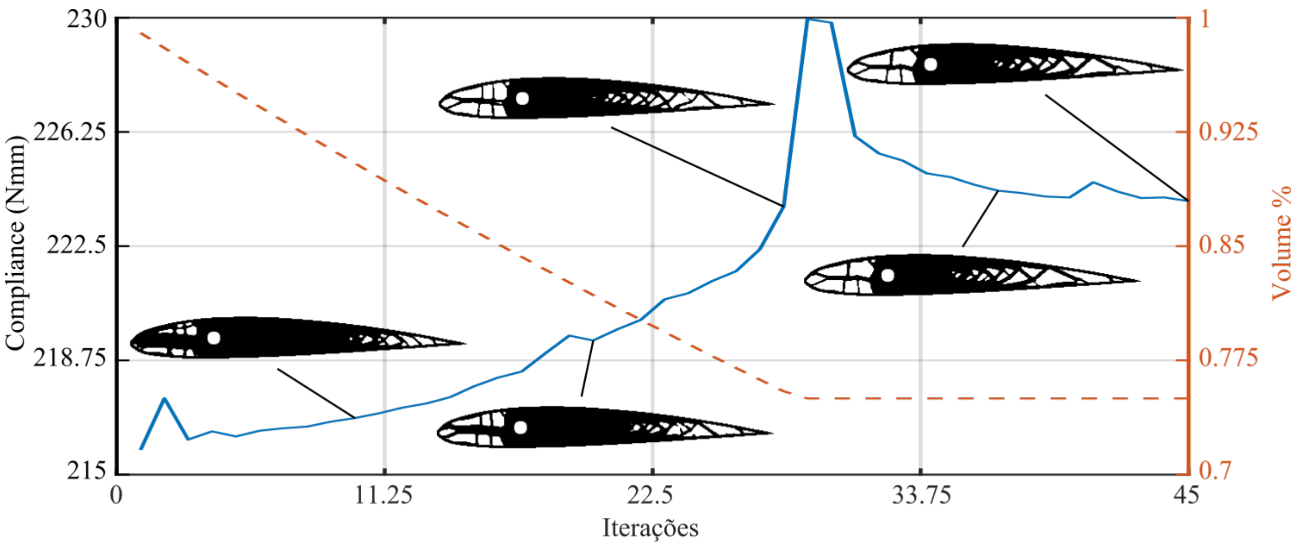

Figure 4. Evolutionary history of mean compliance, volume fraction and topology for the optimizations.

The topologies with the lowest mean compliance or modified P-norm values obtained for each optimized mesh for the NACA 0010 are shown in Figure 5, where the von Mises stress gradient for each generated shape can also be observed. Here, $iter$ is the iteration number. The same evaluation is made for the NACA 2412. The optimized topologies are presented in Figure 6, as well as the von Mises stress gradient for each generated shape.

From the results, the following observations and comparisons are made:

- For the NACA 0010 and $\alpha = 0°$, there is a great similarity between the topologies obtained for both minimization methods. The values of maximum von Mises stress and compliance in the cases are very close;
- For the NACA 0010 and $\alpha = 5°$, the optimized topology of the leading edge on both minimizations are close, however the middle section and trailing edge have different topologies. There is a more trussed formation on the compliance minimization problem, compared to the other one. Besides that, stress minimization features a lower value for the maximum von Mises stress and compliance;

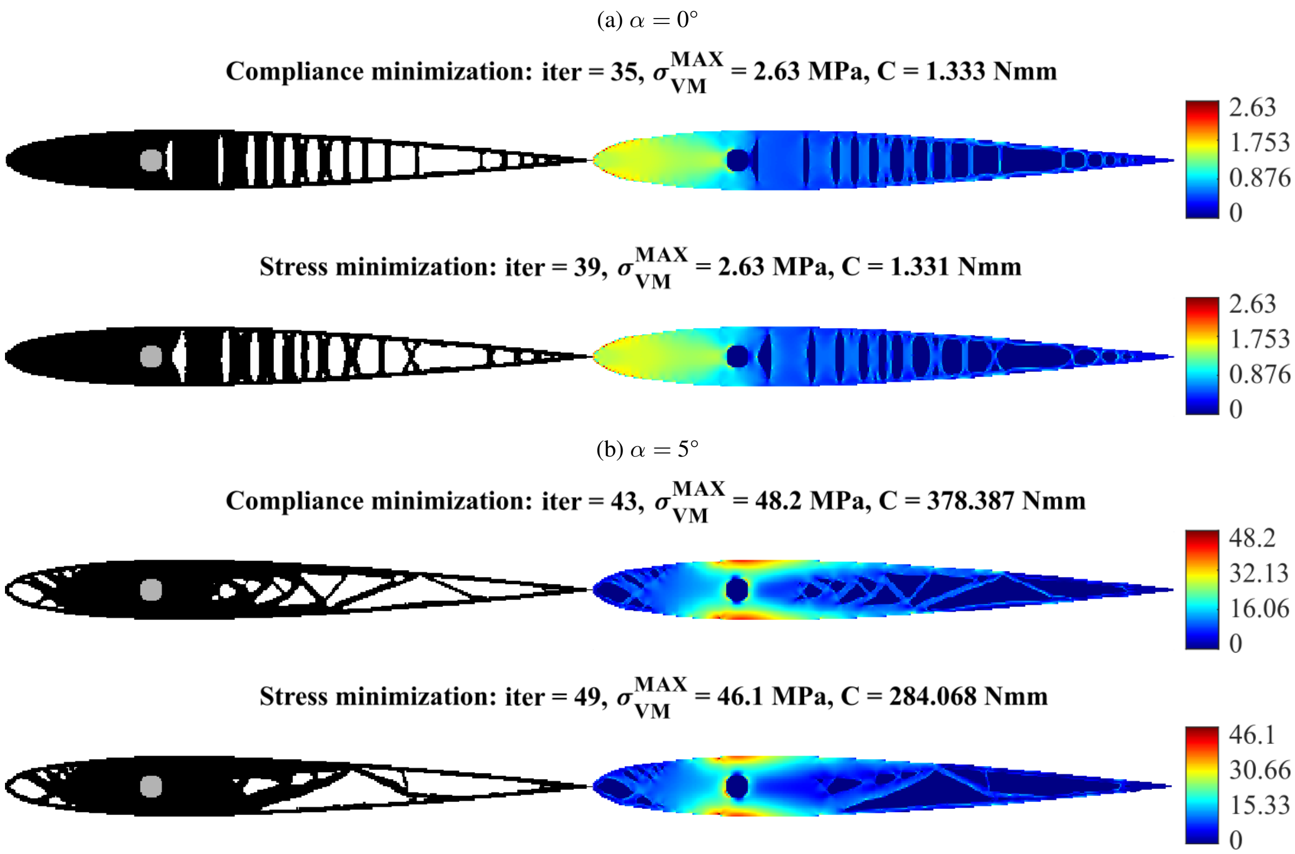

Figure 5. Best NACA 0010 optimized topologies for the minimization problems.

- For the NACA 2412 and $\alpha = 0°$ and $5°$, both optimized structures have different resulting topologies. On the compliance minimization problem, there is the formation of trussed like structures with the value of compliance being the lowest among the analyses. In contrast, on the other method the result shows a less predictable formation and the value for the maximum von Mises stress is lower;

- Among the stress-minimized topologies, it is notable that there is a preference for the formation of thicker structural reinforcements and larger holes. On the other hand, compliance minimization provides a more detailed and branched structure;

- The evolutionary iterative process of compliance minimization happens to be more stable when compared to the stress one. However, on the stress minimization, the modified P-norm stress values varies less in magnitude. It is noteworthy to say that the compliance or modified P-norm stress jumps observed along the evolutions are directly associated with sudden changes in the topologies of some region within the design domain.

6. CONCLUSION

In this work, the theory and implementation of the bidirectional evolutionary structural optimization method for compliance and stress minimization applied to an airfoil rib were developed. Topological optimizations were performed using the software MATLAB® for the NACA 0010 and 2412 airfoils, considering a reference flight condition based on the Cessna Skyhawk 172 aircraft.

On the overall, results showed a regular pattern on the formation of trussed like structures for compliance minimization, in contrast to a less predictable formation of the stress-minimized topologies. Furthermore, with the exception of the symmetrical airfoil ribs, compliance-based minimizations resulted on lower mean compliance values. Likewise, stress-based minimizations resulted on lower maximum von Mises stress values.

Although the results presented lack validation, with the computational routine developed, several numerical studies became possible, such as the structural analysis of different airfoils for varied flight conditions. A more realistic analysis would be to take into account the complete flight envelope of an aircraft, evaluating its critical flight points to obtain the optimized topology. Also, considerate multiple aerodynamic loads in order to optimize the airfoil rib for the entire aircraft flight regime.

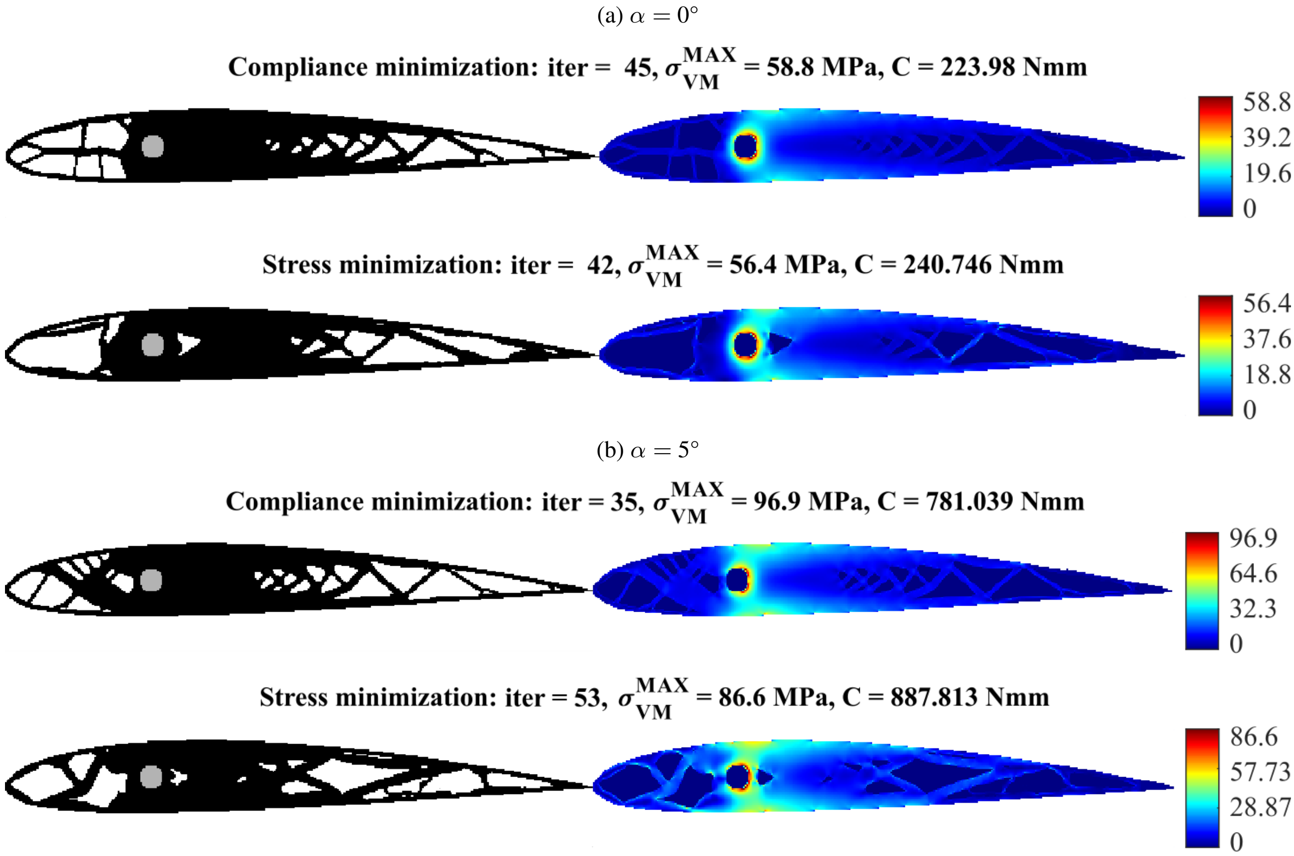

Figure 6. Best NACA 2412 optimized topologies for the minimization problems.

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