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INVESTIGATION OF LIFT AND DRAG COEFFICIENTS FOR A LOW REYNOLDS NUMBER AIRFOIL APPLIED TO SMALL URBAN WIND TURBINES

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Abstract. *Low Reynolds number airfoils are important for designing small wind turbines as they typically are designed to operate at low wind speed. This makes the determination of lift and drag coefficients a challenge that is overlooked in the literature as usually these aerodynamic parameters can be significantly unstable at low wind speed, compromising the starting behavior of small rotors. Hence, in this work, the low Reynolds number airfoil SG6040 is assessed through performance of an optimized small urban wind turbine. The turbine performance analysis is done for Reynolds numbers of 100000, 150000 and 200000. The aerodynamic data of SG6040 are used to design a small 4-bladed wind turbine with 2.0 m rotor diameter in which the Reynolds number is varied along the rotor blade. Such a variation aims to verify the impact of lift and drag forces on the optimization of small wind blades for urban purposes where the wind speed is in general lower. As a result, a small urban wind turbine with efficiency about 49% will be obtained whereas generally small turbines have 20-25% in the literature.*

Keywords: Small wind turbine, Low Reynolds number airfoil, Blade optimization.

1. INTRODUCTION

In the last few years, wind energy utilization has been increased, and electricity generation using small wind turbines have been largely utilized worldwide, mainly for small demand in isolated regions (do Rio Vaz et al., 2018). In this context, urban wind turbine designs are important to study, as generally, they require small dimensions when operating at low Reynolds numbers (Vaz and Wood, 2016, Singh et al., 2012.). These aspects create a challenge for the aerodynamic design of small wind blades as lift and drag coefficients, in this situation, normally need to be known at low wind velocity conditions (Vaz and Wood, 2018, Vaz and Wood, 2016).

Therefore, in this work, the SG6040 airfoil is utilized considering low Reynolds numbers of 100000, 150000 and 200000 in order to evaluate the performance of an optimized urban turbine operating at low wind velocity. Experimental data from Giguère and Selig (1998) are used for the proposed analysis. Also, these measurements are implemented to design a small urban wind turbine. The aerodynamic data are used to design a small 4-bladed wind turbine with 2.0 m rotor diameter, in which the Reynolds number is varied along the rotor blade. Such a variation aims to verify the impact of lift and drag forces on the optimization of small wind blades for urban purposes where the wind speed is in general low. As a result, a small urban wind turbine with efficiency about 49% can be obtained, as small turbines generally have 20-25% of efficiency in the literature. Thus, the main contribution of this paper is the estimation of wind speed for a good operating condition in terms of aerodynamic characteristics, such as lift and drag, of a small urban wind turbine.

2. THE OPTIMIZED WIND TURBINE BLADE

This section shows the urban turbine design conditions, through which the optimization procedure is carried out to optimize the wind blade at low Reynolds number (Re). In this case, SG6040 airfoil for Re = {100000, 150000, 200000} are described. A numerical model based on the finite volume method is used to determine lift and drag coefficients.

2.1 The turbine design conditions

To analyze the performance of the urban turbine, the parameters described in Table 1 are considered. The SG6040 airfoil (Fig. 1a) is used to design the rotor blades because it is a good foil for small wind turbines, according to Wood

(2011). Figure 1b shows lift and drag coefficients for low Reynolds numbers of 100000, 150000 and 200000, respectively. It is worth noting that for Re = 100000, the aerodynamic characteristic is indeed unstable, meaning that low Reynolds number can impose a significant impact on design of small urban turbine.

Table 1. Design parameters and conditions of the urban wind turbine.

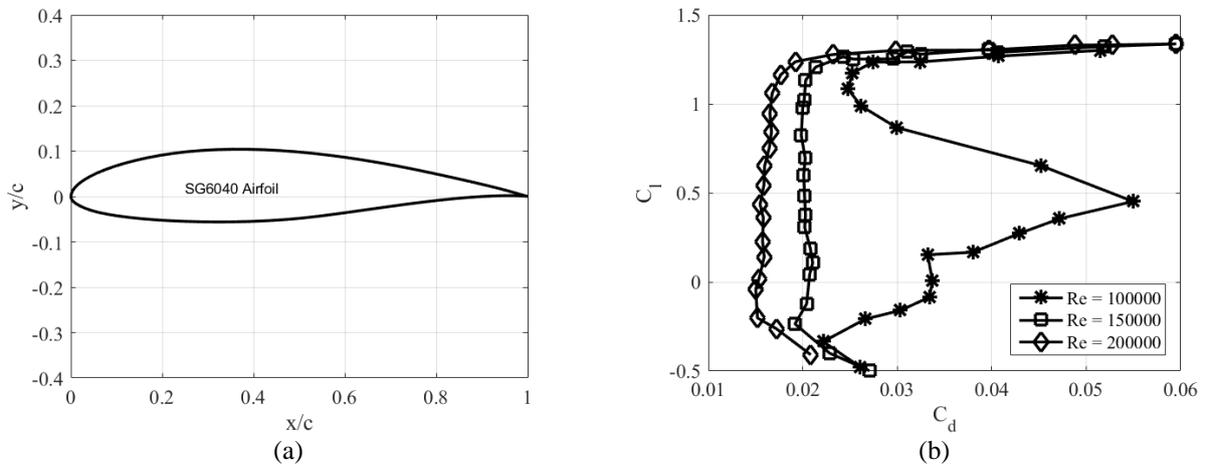| Parameters	Value
Turbine diameter, m	2.0
Hub diameter, m	0.2
Number of blades	4
Freestream velocity, m/s	6.5
Air density, kg/m$^{3(1)}$	1.19
Angular velocity, rad/s	26.18

[1] measured at 25°C

Figure 1. (a) SG6040 airfoil. (b) Lift and drag coefficients for different low Reynolds numbers (Giguère and Selig, 1998).

2.2 The optimization procedure

According to Glauert, the power coefficient of a turbine may be expressed in terms of tangential and axial induction factors, a' and a, respectively, as

$$C_P = \frac{8}{\lambda^2} \int_0^\lambda a'(1-a)Fx^3 dx, \tag{1}$$

where $\lambda = \frac{\Omega R}{V_0}$ and $x = \frac{\Omega r}{V_0}$ are the local-speed ratio and the tip-speed ratio, respectively. The optimum design for a wind blade is obtained through maximization of the power coefficient, Eq. (1), using only the term $a'(1-a)$ because F is not dependent on the induction factors. This yields

$$\frac{d}{da}[a'(1-a)] = \left[(1-a)\frac{da'}{da} - a'\right] = 0, \tag{2}$$

Equation (2) can be simplified as

$$(1-a)\frac{da'}{da} = a', \tag{3}$$

As reported in Vaz and Wood (2016), combining the equations for the flow angle shown in the velocity diagram (Fig. 2), gives

$$x^2 a'(1+a') = a(1-a), \tag{4}$$

Eq. (4) differentiated with respect to a yields

$$x^2(1 + 2a')\frac{da'}{da} = 1 - 2a, \tag{5}$$

If Eqs. (3) and (5) are combined with Eq. (4), the optimum relationship between a and a' becomes

$$a' = \frac{1-3a}{4a-1}, \tag{6}$$

Eq. (6) was obtained by Glauert (1963). The optimum relationship between x and a can be obtained by substituting Eq. (6) in Eq. (4) resulting in

$$16a^3 - 24a^2 + (9 - 3x^2)a + x^2 - 1 = 0, \tag{7}$$

Because of Eq. (7), the blade optimization procedure can be expressed as a function of the induction factors once the blade element lift and drag are available. Therefore, the optimum chord and the twist angle at each blade section are calculated through the following expressions:

$$c = \frac{8\pi r F \sin\phi \cos\phi}{B C_n} \frac{a}{1+a}, \tag{7}$$
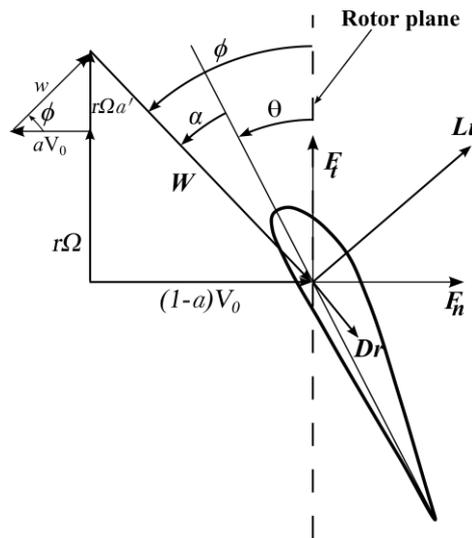
and

$$\theta = \phi - \alpha, \tag{8}$$

Figure 2. Schematic diagram of the velocities at the blade.

3. NUMERICAL VALIDATION

In order to validate the Blade Element Momentum (BEM) model used in the present approach, the experimental data of the MOD-0 100 kW wind turbine is used. The measurements of MOD-0 were collected by NASA Lewis Research Center, Phase VI and the UAE, with experiments carried out by NREL in the wind tunnel at NASA-ASME. The turbine MOD 0 corresponds to a 2-blade rotor, rated power at 100 kW. The used profile is the NACA series 23000, constant rotation of 27 rpm, pitch angle of 0°. The chord distribution is 1.96 m for r/R = 0.234, varying linearly up to 0.67 m for r/R = 1, the rotor diameter is 38m and the hub diameter is 7.84 m (Viterna & Corrigan, 1981; Glasgow, 1978). BEM is a widely used approach for the analysis and design of turbines. Combining the basic principles from both blade element and momentum theories, it is inherently steady, two dimensional, stems from the equivalence between the circulation and momentum theories of lift, and allows the estimation of the inflow distribution along the blade. Fig. 3 displays the comparison between the experimental data and BEM model employed in this work, which yields good agreement.
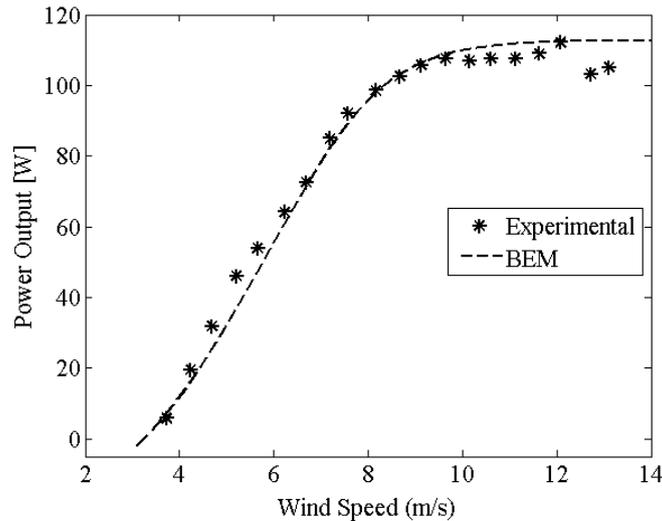
Figure 3. Power output as a function of the wind speed.

4. RESULTS AND DISCUSSION

The optimization procedure used is the model proposed by Vaz and Wood (2016) and extended by do Rio Vaz et al. (2018) for the case of hydrokinetic turbines. In the present work, Cl/Cd = 56.31 for the optimum angle of attack α = 9.33o, and SG6040 for the Reynolds number of 150000 are employed to optimize the wind blade used for all simulations. It is worth noting that Vaz and Wood (2016) developed an approach for optimizing horizontal axis wind blades under the diffuser effect, but the diffuser is not considered in this investigation. This optimization is based on an extension of the well-known blade element momentum theory in which it is assumed that the same conditions for the axial velocity in the wake of an ordinary wind turbine can be applied on the flow far downwind of a turbine with diffuser. The blade optimization procedure can be expressed through power coefficient maximization once the blade element lift and drag are available as described in subsection 2.2. The iterative procedure for the calculation of optimum chord and twist angle starts at its most external section and uses the following as inputs: r, Cl, Cd, and V0 for a given λ as further detailed in Vaz and Wood, (2016). Figure 4a shows the chord and twist angle distributions for optimized blade use in the simulations as well as the 3D visualization of the wind blade in Fig. 4b.
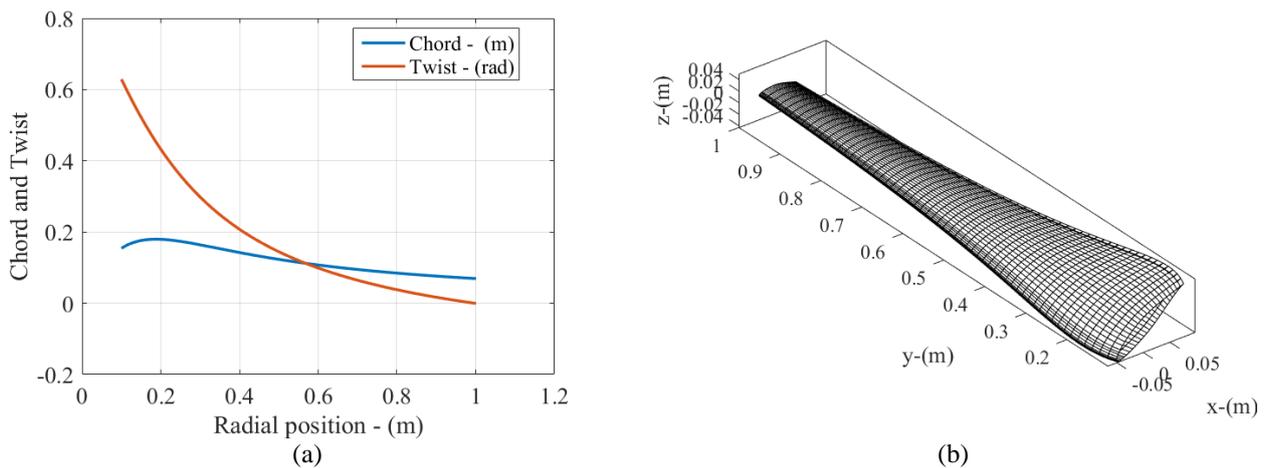

Figure 4. (a) Chord and twist angle at each blade section. (b) Rotor blade in 3D.

Figure 5 presents the C_l/C_d ratio for different low Reynolds numbers. As reported by do Rio Vaz *et al.* (2013), to aerodynamically optimize wind blades, the optimum angle of attack needs to be obtained at the maximum value of C_l/C_d ratio. Thus, Tab. 2 shows the optimum angle of attack, drag and lift coefficients for Re = {100000, 150000, 200000}. Note that a low Reynolds number increases the optimum angle of attack. This aerodynamic aspect makes the design of small urban wind blades difficult as the angle of attack approaches the stall angle at low wind velocity (V_0 = 6.5 m/s), demonstrating the importance of further studies on aerodynamics at low wind velocity.
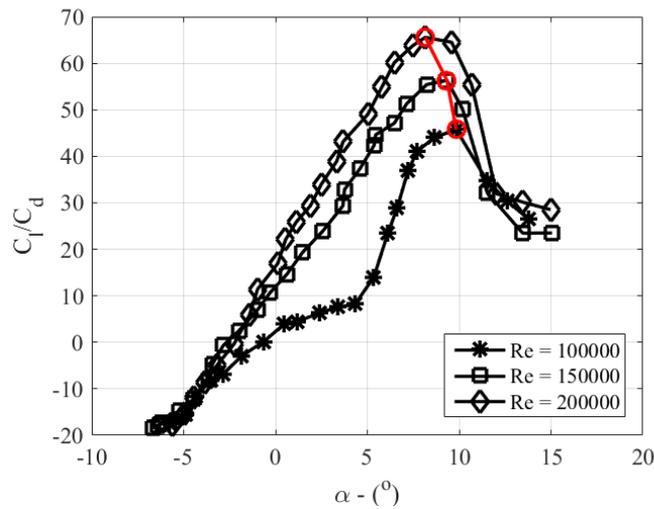
Figure 5. C_l/C_d ratio for different low Reynolds numbers. Experimental data from Giguère and Selig (1998).

Table 2. Optimum drag and lift coefficients for different low Reynolds numbers.

Optimum parameters	Re = 100000	Re = 150000	Re = 200000
Angle of attack, (º)	9.84	9.33	8.15
Drag coefficient, (-)	0.0263	0.0211	0.0175
Lift coefficient, (-)	1.2026	1.1898	1.1511

Figure 6 shows the impact of low Reynolds numbers on the performance of small urban wind turbines. In this case, the nomenclature Designs 1, 2 and 3 are the turbines simulated considering Re = {100000, 150000, 200000}, respectively. Note that, for the Design 1, the power coefficient demonstrates a significant instability (Fig. 6a), leading the turbine efficiency to incorrectly exceed Betz limit (59.3%). The same behavior is also depicted in Fig. 6b, which shows the power output in relation to the freestream wind velocity. The instability occurs at small wind velocities, demonstrating how difficult the design of small urban wind turbines can be.
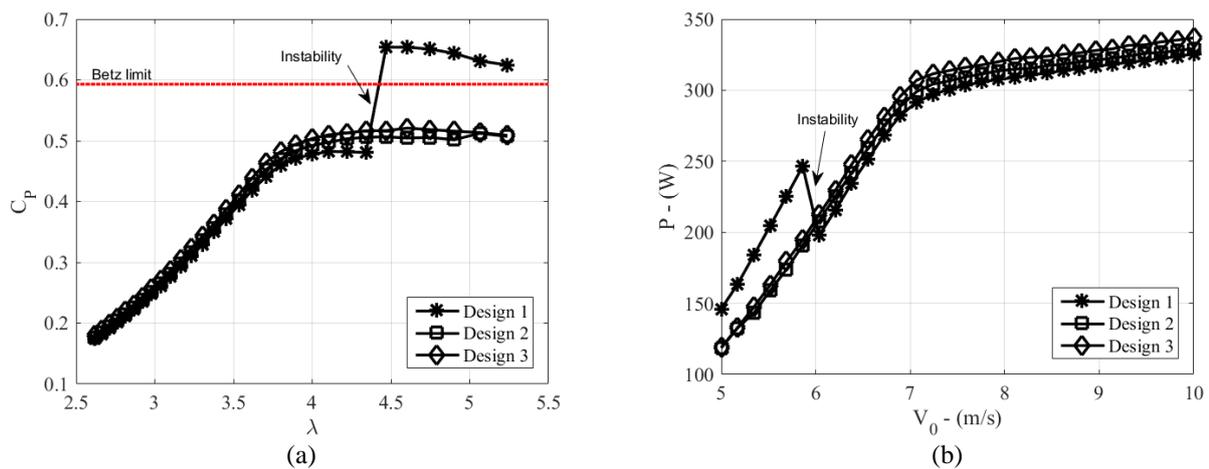

(a)

(b)

Figure 6. (a) Power coefficient as a function o tip speed ratio. (b) Power output as a function of the freestream wind velocity.

5. CONCLUSIONS

This work presents an investigation of lift and drag coefficients for the SG6040 airfoil applied to small urban wind turbines considering different low Reynolds numbers (100000, 150000 and 200000). These aerodynamic aspects of designing small wind turbine blades are important as it can impose instability on the performance of the wind rotor,

mainly at low wind velocity. As previously stated, the power coefficient significantly demonstrates instability (Fig. 6a), leading the turbine efficiency to incorrectly exceed Betz limit (59.3%), i.e., a behavior not physically consistent. Therefore, to design small turbines, the Engineer shall consider the aerodynamics characteristics of the airfoil chosen.

6. ACKNOWLEDGEMENTS

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