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INTERPOLATION CURVE IMPACT IN A FREEFORM WING OPTIMIZATION

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Abstract. *This work presents a wing optimization model that uses three control sections to describe the wing shape. The sections profile and attack angle are defined in a preliminary stage and are not investigated in this paper. Each control section has three variables: offset, chord length and wingspan position. The last variable considered is the chord length close to the fuselage. The optimization objective is to maximize the aircraft's payload. However, a limitation on the length of lane required to takeoff and the wingspan limit the feasible design space. The offset parameters are automatically calculated and swept wings are not allowed. For the analysis, the Vortex Lattice Method was used with AVL (Athena Vortex Lattice) application. The optimization was performed with in-house Genetic Algorithm code that uses a random DOE. The main original contribution of this paper is the comparison of different interpolation models used to define the wing shape. Four different interpolation models are used to define the sections among the three control sections. The first one defines the form using linear interpolation of each two control sections. The second model uses a single linear interpolation to adjust all control sections and it defines a straight wing shape. The other models use second and third-degree equations, that result in organic shapes. The results showed that a high order interpolation curve (cubic model) resulted in 0.3% better performance than linear model but with 13 times more computational cost. However, in the analysed scenario, the linear model has the best general performance and will provide a good initial solution for a high order CFD optimization process.*

Keywords: *Optimization model, Shape optimization, Wing, Interpolation models, AVL.*

1. INTRODUCTION

Wing optimization is a topic extensively discussed in the literature. There are two traditional approaches: the parametric and the freeform optimization. As high-precision benchmarking is a time-consuming process, a freeform optimization using RANS/DNS CFD code is difficult. Fot this reason, a good initial solution is an important requirement.

In traditional approach, wings are represented by coordinate points. Although this method can represent a variety of forms and allow local changes, it is difficult to be used in optimization processes, because it involves enormous numbers of parameters with a high computation cost to explore the design space.

Sadrehaghighi (2021) showed that computational simulation are extremely useful to explore the space of feasible solutions, but also have a high computational cost due to the huge number of possibilities to be analyzed. Furthermore the author showed that the shape parameterization methods have a large impact on an aerodynamic wing optimization.

Masters (2016) analysed six parameterization models: Bezier Curves, B-spline surfaces, Class/Shape function transformation method (CST), Hicks-Henne bump functions, Radial basis function domain element (RBF-DE) deformation, camber-line-thickness parameterization and Singular Value Decomposition (SVD) method. His research showed that smoothness had a significant impact on the robustness and rate of convergence of the optimizations. Some methods that result on un-smooth shapes have negative impact on the optimized result.

It was presented by George R. Anderson (2015) a study of adaptive shape parameterization for discrete aerodynamic geometries, in order to obtain computational cost reductions in the optimization cycles. That work showed that with a progressive parameterization, the optimization curve is smoother.

In Manohara Selvan (2015), four parameterization techniques are analyzed. Among them, the Class-Shape Transformation and Hicks-Henne Bump functions were able to increase the airfoil lift-drag ratio by 30%. This result shows that the selection of a suitable parameterization technique is one of the most significant factors affecting the quality of the solution found during the optimization process.

In Akram and Kim (2021), that parameterized and optimized two aerodynamic profiles, it is shown that the significant reduction of a large number of design parameters allows the optimization algorithm to be more efficient. The results are significant and show gains of more than 10% in some cases.

In Bortolete (2017) it is shown a wing optimization model using modeFRONTIER as optimization tool and AVL as analysis tool. The work presents a force model to describe the takeoff as a single equation. The problem is solved using Genetic Algorithm and an optimal solution is reached after 700 generations.

Finally, about the different parameterization models of a wing (or profile), Samareh (2001) points out that such models should have a compact number of design variables and a high flexibility to cover the design space. They should also be able to represent the existing geometries with high accuracy and be able to produce smooth and realistic shapes.

In this work, we use a Genetic Algorithm, which, although slower, offers interesting metrics to evaluate the proposed parameterization models. The proposed approach is similar to the one used by Akram and Kim (2021). However, this work focuses on evaluating the impact of interpolation model on the optimization efficiency and on the quality of the solution.

The main original contribution is an enlighten comparison using four different interpolation models to define the sections among the three control sections. The first one, is the default option used by AVL, that defines the form using linear interpolation of each two control sections. The second model uses a single linear interpolation to adjust all control sections (straight wing shape). The other models use second and third-degree equations, that result in organic shapes.

For modeling, this work uses the mesh points technique, which as demonstrated by Castonguay and Nadarajah (2007) and Mousavi and Nadarajah (2007) provided good results in all test cases. Only the coordinates of control sections are used. The coordinates of interpolated sections are calculated using each of the model studied.

Although this technique is not new, numerical metrics about their importance are presented, which can be applied to a large number of designs that involve shape optimization.

2. The design problem

Four interpolation model are investigated in order to assess the impact of the interpolation model on the optimization process. Since the objective is to evaluate this influence, a small wing and an aerodynamic analysis tool based on vortex theory are used to reduce the computational cost, allowing a greater number of analyzes and comparisons.

The model presented considered only aerodynamic performance once the fabrication and structural aspects are not investigated in this paper.

2.1 Decision variables

The wing geometry is defined by 3 control section. Each one has a single chord and span value. The 7^{th} parameter refers to the section chord close to the aircraft fuselage (Figure 1).

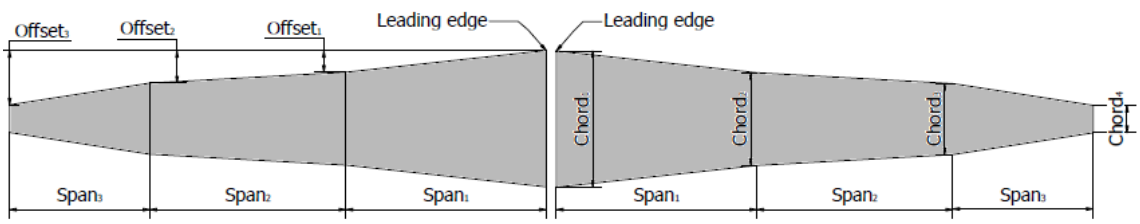


Figure 1. Wing surface parameters

Finally, it is well-known that the wing with swept is more common in supersonic aircraft design. Thus the sections offsets are defined by Equation (1) in order to prevent swept wings.

$$Offset(s) = \frac{Chord_1 - Chord(s)}{2} \quad (1)$$

Where s represent a generic longitudinal position along the wing.

It is important to highlight that the profile used in wing modeling is fixed, and it is always the profile whose characteristics are shown in Tab. 1.

Table 1. Wing profile characteristics

Features	Values	Shape
Thickness	12.60 [% of chord]	
Camber	9.06 [% of chord]	
Maximum thickness position	23.23 [% of chord]	
Maximum camber position	40.40 [% of chord]	
Stall angle	13.5°	

As already mentioned, the geometry is defined by 7 variables, which are called x_i and grouped in a vector $\bar{X}$:

$$[\bar{X}] = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] \tag{2}$$

The limits of each variable must be carefully defined to minimize the generation of unfeasible solutions. This definition was made using data from the literature and is shown in Tab. 2.

Table 2. The design space

Geometric parameter	Design variable	Min. value[m]	Max. value[m]
$Span_1$	x_1	0.30	2.00
$Span_2$	x_2	0.60	2.20
$Span_3$	x_3	0.90	2.40
$Chord_1$	x_4	0.10	1.00
$Chord_2$	x_5	0.05	1.00
$Chord_3$	x_6	0.05	1.00
$Chord_4$	x_7	0.05	1.00

2.2 Objective Function

The objective function (Equation 3) aims to maximize the aircraft payload, which is calculated from the aerodynamic performance and an empirical model capable of estimating the aircraft weight. Where $\bar{X}$ refers to the design variables vector and $W_{aircraft}$ refers to the empty aircraft mass, which is estimated based on the empirical model calculated using the area S referring to the total surface of the wings measured in m^2 (Equation 4). And $W_{takeoff}$ refers to the total mass that the aircraft is able to take off and it is calculated by Equation (5).

$$F(\bar{X}) = Payload = W_{takeoff} - W_{aircraft} \tag{3}$$

$$W_{aircraft} = 1.539331 * S^2 + 1.341043 * S \tag{4}$$

$$W_{takeoff} = \frac{L_{takeoff}}{g * F_s} \tag{5}$$

Where g refers to the acceleration of gravity (9.81 m/s^2) and F_s refers to a correction factor for the inclusion of the dynamic effects present in the takeoff, being arbitrated the value of 1.09. The takeoff speed used was 1.2 times greater than the stall speed, as recommended by Regulations (1989). The lift and drag were calculated with 0.7 of this speed value, as recommended by Anderson Jr (2015).

Finally, using Equation (4) and Equation (5) in Equation (3) it is possible to define the objective function (Equation 6).

$$F(\bar{X}) = \frac{L_{takeoff}}{g * F_s} - (1.539331 * S^2 + 1.341043 * S) \tag{6}$$

Where $L_{takeoff}$ refers to the total lift generated by the wings at takeoff, which is determined based on the Algorithm 1, where at each iteration the aircraft mass increases 0.25 kg. The L_{lane} equation was developed in Bortolete (2017).

Algorithm 1: Takeoff Algorithm

Result: $L_{takeoff}$
$L_{lane} = 0$
$L_{takeoff} = 0$
while $L_{lane} < 90$ **do**
  $V = \sqrt[2]{\frac{2*L_{takeoff}}{\rho_{air}*S*C_L}} * 1.2 * 0.7$
  $T = a(0.7 * V)^2 + b * 0.7 * V + c$
  $D = 0.5 * C_D * S * \rho_{air} * V^2$
  $L = 0.5 * C_L * S * \rho_{air} * V^2$
  $L_{lane} = \frac{1.44*L_{takeoff}^2}{g*C_L*S*\rho_{air}*(T-D-\mu_{lane}*(L_{takeoff}-L))}$
  if $L_{lane} < 90$ **then**
    $L_{takeoff} = L_{takeoff} + 0.25 * g$
  end
end

Where ρ_{air} refers to the specific mass of air (1.225 kg/m^3). μ_{lane} refers to the soil friction coefficient and second Rodrigues (2011), it is 0.025. The parameters [a, b, c] refer to the thrust curve, which were estimated in [-0.0126, -0.5248, 40.0248]. Finally, the coefficients C_D and C_L are obtained in the aerodynamic analysis of the modeled wing.

2.3 Constraints

As constraints of the problem, it was arbitrated that the maximum wingspan allowed is 4.2 m and the aircraft need take off on a lane of 90 m. About the wing geometry, the sections chords can never increase from a section closer to the fuselage to a section closer to the wing tip.

To satisfy the conceptual constraints the optimization model received a set of constraints that are shown in Tab. 3. The position marked with * in Tab. 3 refer to conceptual constraints that is satisfied by the limitation of the design space.

Table 3. The constraints equations

Concept	Equation
$2 * Span_3 < 4.2$	$x_3 < 2.1$
$Span_1 > 0.3$	$*$
$Span_2 - Span_1 > 0.3$	$x_2 - x_1 > 0.3$
$Span_3 - Span_2 > 0.3$	$x_3 - x_2 > 0.3$
$Chord_2 - Chord_1 < 0$	$x_5 - x_4 < 0$
$Chord_3 - Chord_2 < 0$	$x_6 - x_5 < 0$
$Chord_4 - Chord_3 < 0$	$x_7 - x_6 < 0$

To penalize the objective function, a single value of 10000 was adopted, since all constraints have the same magnitude order and there is only one objective function in the model.

3. Interpolation models

As cited in introduction, the main original contribution is a comparison using four different interpolation models to define the sections among the three control sections.

The strategic used is shown in Figure 2. All analyses done used the same optimization model but before evaluate the objective function, a different interpolation model is used to define the wing geometry.

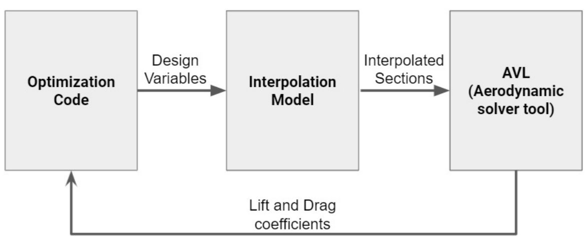


Figure 2. Full process flowchart

The first model is called 'segmented' and it is the default option used by AVL. In this model the form is defined by linear interpolation of each two control sections. The second model, called 'linear', use a single linear interpolation to adjust all control sections and generate a straight wing shape. The other models used second and third-degree equations, that result in organic shapes. The second-degree model is called 'quadratic' model, while the third-degree is called 'cubic' model. The interpolation models are synthesized in Tab. 4.

Table 4. The interpolation models

Model	**Equation**	**Interpolated sections**
Segmented	(no equation)	(same as control sections)
Linear	$ax + b$	(defined from interpolation equation)
Quadratic	$ax^2 + bx + c$	(defined from interpolation equation)
Cubic	$ax^3 + bx^2 + cx + d$	(defined from interpolation equation)

In Tab. 4 the x parameter refer to a generic longitudinal position along the wing. To maintain a compromise between curve smoothness and computational cost, all analyzes used 10 interpolated sections uniformly distributed along the wingspan. So, when fuselage section is included, each wing has 11 sections. In this way, the distance between two interpolated sections is defined by the Equation (7).

$$\Delta x = \frac{Span_3}{10} \tag{7}$$

To assure that the wing has only positive taper, a new constraint was included in the optimization model (Equation 8). Finally a last constraint (Equation 9) was included to avoid that interpolation results in unrealistic shapes with negative chords.

$$min(Chord_n - Chord_{n+1}) \geq 0 \tag{8}$$

$$min(Chord_n) > 0.05 \tag{9}$$

Where n refer to any of interpolated sections used in geometric model.

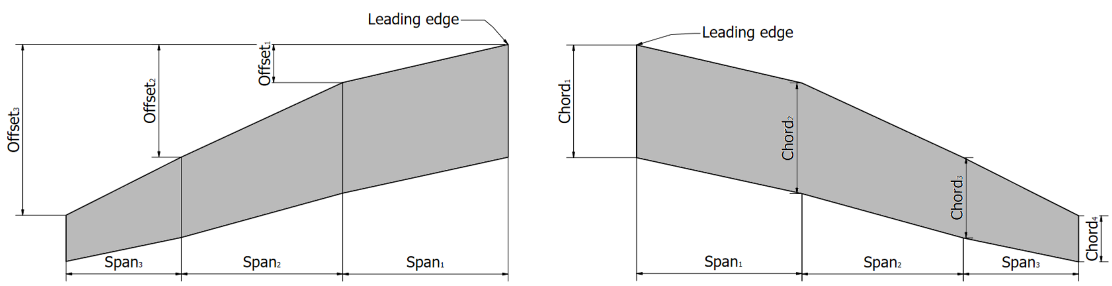
4. Methods and tools

4.1 Analysis

For the analysis, the Vortex Lattice Method was used with AVL (Athena Vortex Lattice) application. In this technique the lift calculation use an inviscid (non-viscosity) vortexs of horseshoe type distributed over panels along the three-dimensional wing surface. Details about the solver are not important for this paper, but they can be seen in Mark Drela (2004).

4.1.1 Verification of the numerical model

Since the tool used in the aerodynamic analysis has a numerical approach, the mesh influence on the response quality must be evaluated. Therefore, a study case was done to identify the appropriate discretization for the studied geometry. The characteristics of the geometry used in mesh convergence study are shown in Figure 3.

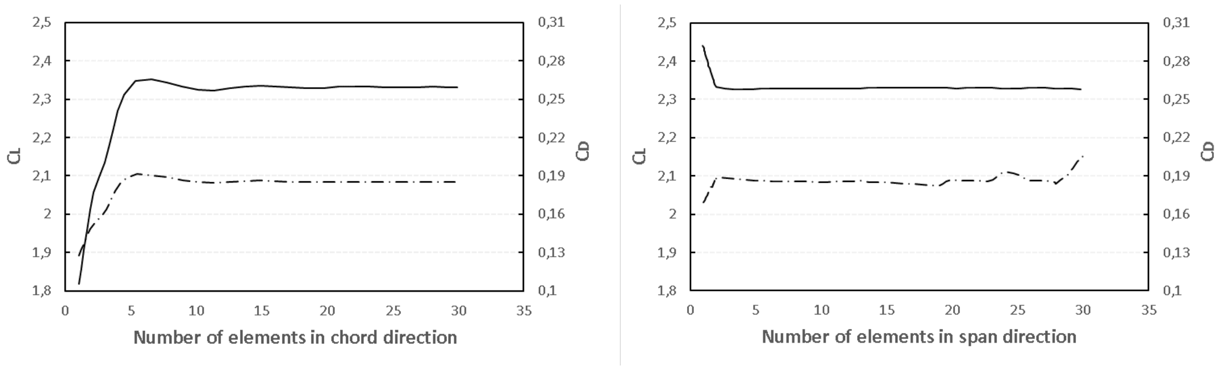


$Span_1$	$Span_2$	$Span_3$	$Chord_1$	$Chord_2$	$Chord_3$	$Chord_4$	$Offset_1$	$Offset_2$	$Offset_3$
$0.50m$	$0.90m$	$1.30m$	$0.35m$	$0.30m$	$0.25m$	$0.15m$	$0.05m$	$0.10m$	$0.20m$

Figure 3. Wing surface parameters

The AVL program uses a discretization based on a number of elements in the chord direction and a number of elements in the wing span direction. Both parameters should be specified for each wing's segment defined between control sections. Initially, a parametric analysis was done, varying the number of elements in chord direction from 1 to 30, keeping the number of elements in span direction equal to 10 (Figure 4a).

As it is possible to observe, the analysis seems to reach convergence with 22 elements in the chord direction. Then, a parametric analysis was done, varying the number of elements in the span direction from 1 to 30, keeping the number of elements in the chord direction equal to 22 (Figure 4b).



[a] [b]

Figure 4. Mesh convergence study

As can be seen, there is a region with numerical stability around 10 elements. However, it is important to note that the sections positions are also design variables of the problem. Once the segment's length can change significantly, the best approach is to define a typical element size and not a typical number of elements. Since in the studied geometry the segments length are between 0.4 m and 0.5 m (Figure 3), it was defined a typical element size of 0.05 m.

However, although the chords also can change, to keep a structured mesh it is a good approach to keep the number of elements in chord direction constant.

4.2 Optimization

The optimization process was conducted using a genetic algorithm implemented in Python in-house code. The algorithm uses a random initial population with 50% viability. That is, random individuals are generated until a number of feasible solutions is equal to half the population size.

The evolution process uses classical mutation and crossover operators described in Deb (2002). Finally, an elitism operator is applied, keeping the population size constant and keeping the best individuals in each generation.

All analyzes done used a population of 20 individuals, a mutation rate of 4% and it were limited to 300 generations. All crossovers are done with a random percentage of genome cross.

The utilization of maximum generation number as the only criteria to finish the optimization, although inefficient, facilitates data processing and ensures more uniform metrics to compare the results.

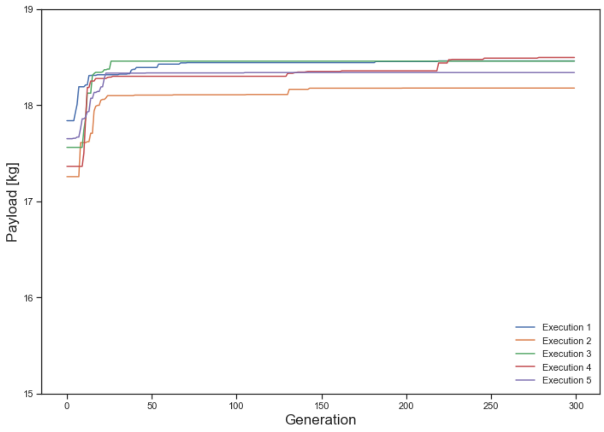
5. Results

The optimization results are presented below. To ensure a minimum of statistical reliability, each interpolation model was solved 5 times, always with a randomly initial populations. Figure 5 shows the evolution of the highest payload obtained in each generation for each of the models executions (only feasible individuals are considered).

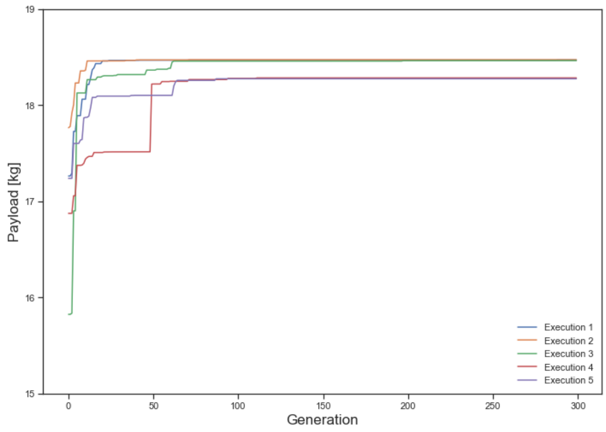
With this results, it is visible that the linear model has a faster convergence, requiring a lower number of generations to reach the optimal solution. Furthermore, it is visible that the final solution has lower dispersion between executions than other models.

To complete the payload analyse, the Figure 9 shows the wingspan and the wing surface area evolution across the optimization process. These results agree with the previous analyse that the linear model has the best performance, with the faster convergence and the less dispersion between executions.

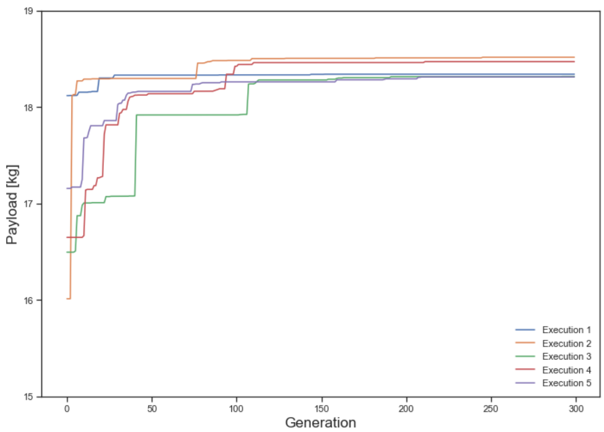
To better assess the impact of the interpolation model on the efficiency of the optimization process, Figure 6 shows the number of evaluations of single solutions (feasible and non-feasible) along the optimization processes with each interpolation model. As can be seen (Figure 6a), the probability of a mutation or crossover produce a feasible solution using linear interpolation model is at least 3.6x bigger than all others model investigated and it is about 20 times greater then the cubic model.



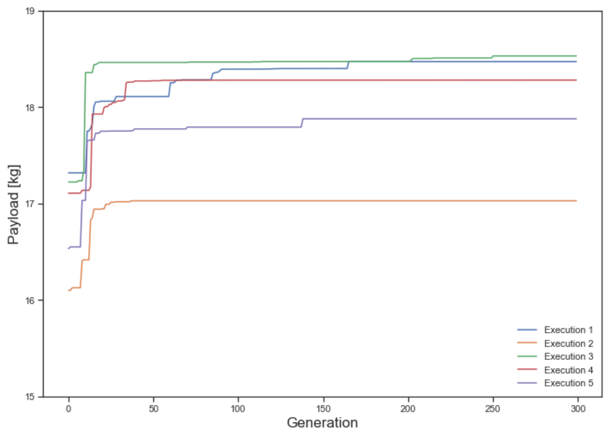
Segmented



Linear

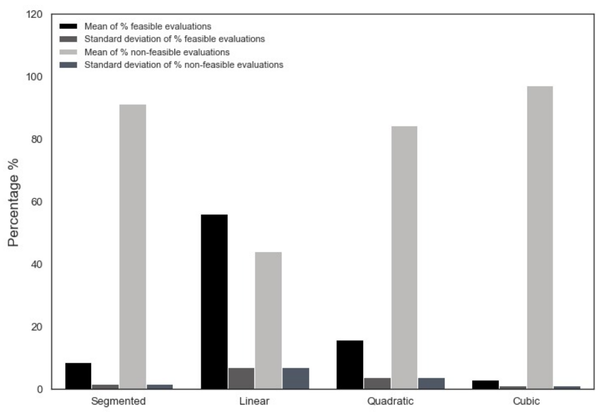


Quadratic

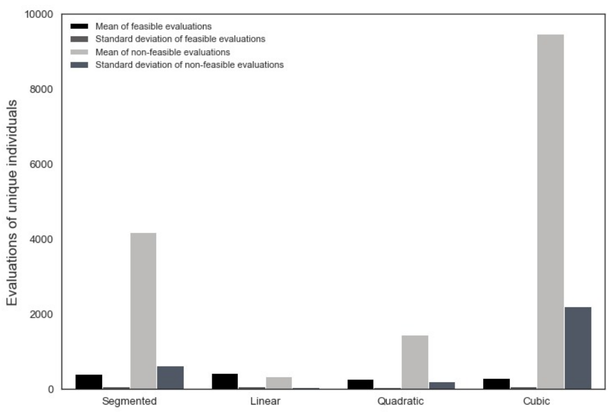


Cubic

Figure 5. Payload evolution across the generations



[a]



[b]

Figure 6. Performance of interpolation models

Analysing the number of single solutions evaluation (Figure 6b), the average number of solutions evaluated during optimization using the linear model is 6.2x smaller than segmented model and 13.1x smaller then cubic model. Furthermore the dispersion of total evaluations with linear model was 1.5 times smaller then cubic model.

Finally, in Figure 7 are systematized the data about the quality of the interpolation model. In theory the design space of segmented model should include the linear model. Likewise the cubic model should include the quadratic and linear model. So, using a infinite time and a infinite number of executions, the cubic model should got the best result, follow by quadratic model, follow by linear model. The segmented model has another nature, so then only affirmation possible is that it should be better then linear model.

However, in the scenario studied, with limitation of generations and limitation of executions; the results shows that the cubic model got the best result, but only 0.3% better than linear model with a huge computational cost and a highest dispersion of the results between all models investigated. On the other hand, the segmented model got a solution 0.1% better then linear model but with 6.2x more computational cost.

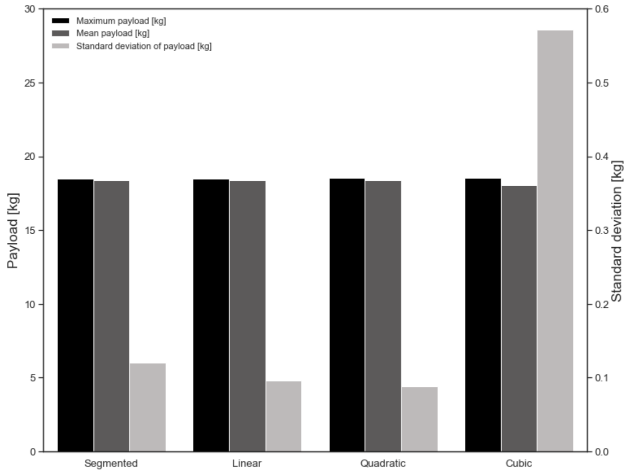
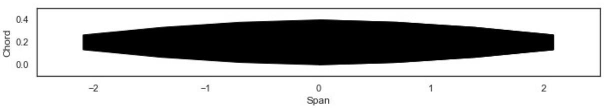
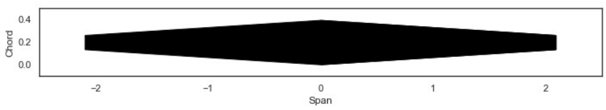


Figure 7. Objective function quality with each interpolation model

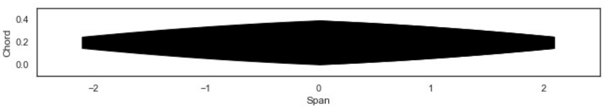
Just to allow a better visualization of the impact of interpolation model in the wing geometry, the Figure 8 shows the best solutions of each model studied.



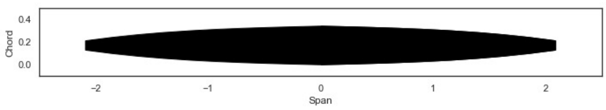
Segmented



Linear



Quadratic



Cubic

Model	**Payload** [kg]	**Wingspan** [m]	**Wing area** [m^2]	$\mathbf{C}_L$	$\mathbf{C}_D$	$\mathbf{W}_{aircraft}[kg]$
Segmented	18.50	4.18	1.18	2.50	0.13	3.72
Linear	18.47	4.19	1.14	2.51	0.13	3.54
Quadratic	18.52	4.19	1.15	2.51	0.13	3.60
Cubic	18.53	4.18	1.09	2.54	0.13	3.28

Figure 8. Characteristics of best solution with each interpolation model

6. Conclusions

This paper presented a comparison of four interpolation models for an aircraft wing design using the same design space and the same optimization algorithm. The results considered only aerodynamic performance once the fabrication and structural aspects are not investigated in this paper.

The more important results are the good performance of linear model. Since the maximum efficiency gain expected during an optimization process is around 5%, the variations got between interpolation models are significant and should be better studied.

How commented before, in theory, the design space of segmented model should include the linear model. Likewise the cubic model should include the quadratic and linear model. So, using an infinite time and an infinite number of executions, the cubic model should got the best result, follow by quadratic model, follow by linear model.

Although the cubic model got the best results, it also was the most time-consumed model. However, engineering is not done with infinite time, so sometimes the simplest solutions are the best solutions.

7. ACKNOWLEDGEMENTS

To CNPq to support this work.
In memory of the 500 thousand lost souls in Brazil until June 2021, mown down by Covid 19.

8. REFERENCES

Akram, M.T. and Kim, M.H., 2021. “Cfd analysis and shape optimization of airfoils using class shape transformation and genetic algorithm—part i”. *Applied Sciences*, Vol. 11, No. 9, pp. 2049–2075. doi:10.3390/app11093791.

Anderson Jr, J.D., 2015. *Fundamentos de engenharia aeronáutica*. AMGH Editora.

Bortolete, F.A., 2017. “Algoritmo de otimização aerodinâmica de asas voltadas a uma aeronave de baixo numero de reynolds "aerodynamic optimization algorithm of low reynolds number aircraft"”.

Castonguay, P. and Nadarajah, S., 2007. “Effect of shape parameterization on aerodynamic shape optimization”. *AIAA-2007-59*.

Deb, K.; Pratap, A.A.S.M.T., 2002. “A fast and elitist multiobjective genetic algorithm: Nsga-ii. ieee transactions on evolutionary computation”. *SIAM journal on optimization*, Vol. 6, No. 2.

George R. Anderson, M.J.A., 2015. “Adaptive shape parameterization for aerodynamic design”. *NAS Technical Report: NAS-2015-02*.

Manohara Selvan, K., 2015. “On the effect of shape parameterization on aerofoil shape optimization”. *International Journal of Research in Engineering and Technology.* doi:04. 123-133. 10.15623.

Mark Drela, H.Y., 2004. “Avl guide”. `https://web.mit.edu/drela/Public/web/avl/`.

Masters, Dominic A., P.D.T.N.R.T..A.C., 2016. “Impact of shape parameterisation on aerodynamic optimisation of benchmark problem.” *54th AIAA Aerospace Sciences Meeting [AIAA2016-1544] American Institute of Aeronautics and Astronautics Inc. (AIAA)*. doi:https://doi.org/10.2514/6.2016-1544.

Mousavi, A., C.P. and Nadarajah, S.K., 2007. “Survey of shape parameterization techniques and its effect on three-dimensional aerodynamic shape optimization”. *AIAA Computational Fluid Dynamics Conference*. doi:2007-3837.

Regulations, F.A., 1989. “Part 23 airworthiness standards: Normal, utility, acrobatic and commuter category airplanes”. *Washington DC, US Government Printing Office, Revised January*.

Rodrigues, L.E.M.J., 2011. “Fundamentos da engenharia aeronáutica aplicações ao projeto sae-aerodesign”. *NAS Technical Report: NAS-2015-02*, Vol. 1.

Sadrehaghighi, J.A., I., 2021. “Aircraft wing shape optimization”. *Applied Sciences*. doi: https://www.researchgate.net/publication/318562320_Aircraft_Wing_Shape_Optimization.

Samareh, J., 2001. “Survey of shape parameterization techniques for high-fidelity multidisciplinary shape optimization”. *AIAA Journal*.

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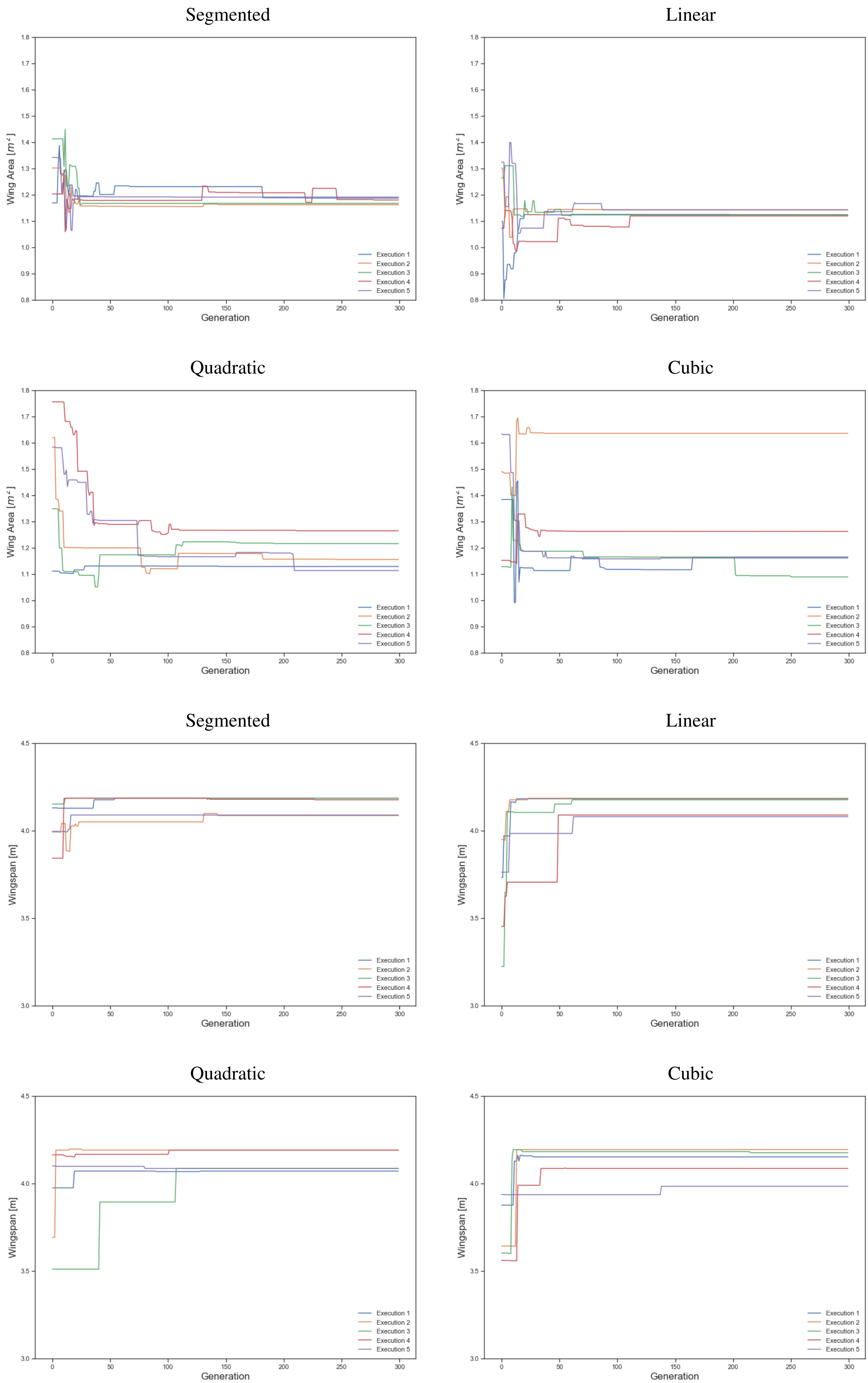

Figure 9. Wingspan and wing surface area evolution across the generations