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COB-2021-0581 DNS SIMULATION OF COMPRESSIBLE FLOWS OVER GAPS: STRATEGIES FOR REACHING A BASE FLOW UNDER STABLE AND UNSTABLE CONDITIONS

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Abstract. *An accurate Direct Numerical Simulation (DNS) of flows may lead to important insights on the origins of turbulence which, in turn, allows quieter, greener and more efficient aircraft to be designed. Often, it is important to obtain a base flow to be used as initial state for a DNS analysis; such flow is defined as an equilibrium point, in which the time derivatives of all variables in the system are null. For stable flows, it is possible to simply let the simulation run for long enough until all oscillations are damped out; nonetheless, it is often a time-consuming process due to near-critical modes, which may be of physical or numerical nature. Unstable flows, on the other hand, are defined by an unstable equilibrium at the base flow and, therefore, will not converge to it naturally. In this work, two techniques are combined to achieve rapid and consistent convergence to base flows. A time-averaging process is carefully done so that whole periods of the least stable modes are contained in the window, damping such modes by roughly an order of magnitude at each cycle. In the case of unstable flows, the well-documented Selective Frequency Damping (SFD) method artificially stabilizes all modes; however, we only employ this technique in selected portions of the domain, where we can maximize its benefits. Our test scenario is a small gap in a subsonic boundary layer, for which unstable Rossiter modes may appear depending on the Mach number and on the gap geometry. By activating the SFD near the gap's trailing edge, we were able to successfully damp unstable Rossiter modes by breaking their feedback mechanism. The time-averaging routine is used to remove lower frequency phenomena, such as a standing wave we have observed in our domain, which is of numerical origin and spans from the inflow to the outflow. By using both techniques in our DNS code, we can achieve residua at least twelve orders of magnitude lower than the free-flow, which allows us to have accurate measures of Tollmien-Schlichting waves well into the linear regime, as well as use a time-stepping method for obtaining the flow's global stability modes.*

Keywords: *Direct Numerical Simulation, Selective Frequency Damping, Boundary Layer, Open Cavities*

1. INTRODUCTION

The push for greener aircraft has driven researchers and designers towards innovative solutions in several areas of aircraft design. In the aerodynamics front, laminar flow designs are widely regarded as an important step towards reducing drag and, therefore, fuel consumption. Reneaux (2004) estimates that a 1% reduction in drag would be allow an extra 1.6 metric tons of payload to be carried by an Airbus A320 sized aircraft, which corresponds to about 10 additional passengers. However, current drag prediction methods may err with margins higher than 1%. For instance, Tinoco *et al.* (2018) summarize the data from the Sixth AIAA CFD Drag Prediction Workshop, in which eighteen participant groups used state-of-the-art CFD codes to predict drag for a standard transonic aircraft model, their results fall within a 10 Drag Counts margin, corresponding to about 4% of the total drag for this model. A considerable part of the difference comes from skin-friction drag, which is strongly related to the turbulent transition location.

An accurate simulation of the transition phenomenon often requires knowledge of the base flow, which is defined as an equilibrium point, in which the time derivatives of all variables in the system are null. For stable flows, it is possible to simply let the simulation run for long enough until all oscillations are damped out; nonetheless, it is often a time-consuming process due to near-critical modes, which may be of physical or numerical nature. Unstable flows, on the other hand, are defined by an unstable equilibrium at the base flow and, therefore, will not converge to it naturally.

In the paper, we combine two different techniques to reach the base flow in a quick and reliable manner. The Selective Frequency Damping (SFD), by Åkervik *et al.* (2006), which acts as a low-pass filter is able to stabilize the unstable modes of the flow, allowing it to reach a steady state if given enough time. A time averaging scheme is also used, aiming to remove low-frequency oscillations, that might be either of numerical or physical nature.

2. METHODS

2.1 Flow simulation and global stability analysis

We used an in-house Direct Numerical Solver (DNS), which features structured meshes that are refined in regions of interest. A fourth-order Runge-Kutta scheme is used for time marching and fourth-order compact spectral-like finite differences are used for the spatial derivatives (Lele, 1992). A pencil-slab domain decomposition is used for code parallelization (Li and Laizet, 2010). A tenth-order spatial high-frequency filter is also employed (Gaitonde and Visbal, 1998) to prevent very short wavelength spurious oscillations. Buffer zones are placed around the useful domain to attenuate undesirable open boundary condition effects such as reflections. They employ a combination of grid stretching, lower order spatial derivatives and Selective Frequency Damping (SFD) (Åkervik *et al.*, 2006). The SFD acts as a low pass temporal filter and may also be turned on in the whole domain to allow base flows to be generated faster or at unstable conditions. Further details of these methods and their implementation in our codes are given by Souza *et al.* (2005); Silva *et al.* (2010); Bergamo *et al.* (2015); Mathias and Medeiros (2018).

To access the global stability, we use a bi-global analysis routine, that uses a time-stepping approach, in which the Jacobian matrix of the governing equations is not explicitly needed (Theofilis, 2011; Gómez *et al.*, 2015). The method uses the Arnoldi algorithm (Arnoldi, 1951) which is based on Krylov subspaces. It just requires the ability to compute vector multiplications which, due to the way in which the algorithm is built, corresponds to a call to the flow numerical solver, in our case, the code described in the previous section.

The time-stepping global instability analysis can be regarded as an established procedure and the current implementation closely followed that of Chiba (1998) and Tezuka and Suzuki (2006). In summary, the method iteratively disturbs the base flow and uses the DNS to capture its response. The successive iteration involves disturbances that are orthogonal to all previous ones. The flow response is used to form a corresponding Hessemberg matrix, which is several orders of magnitude smaller than the flow's Jacobian matrix. If the number of iterations is sufficiently large, the leading eigenvalues and eigenvectors computed from this matrix are good representations of the flow modes and provide good estimates of their respective amplification rates and frequency. In our convention, the real part of the eigenvalue represents the growth rate in time, while the imaginary part represents its angular frequency. Further details on the implementation are given by Mathias and Medeiros (2018).

2.2 Selective Frequency Damping

The Selective Frequency Damping method was proposed by Åkervik *et al.* (2006), who searched for an alternative to Newton iteration methods, which are often expensive to run and require considerable reworking of the code base to be implemented into flow solvers. This new method adds a new term to the Navier-Stokes equation, and therefore can be easily implemented into existing code.

Considering that the flow evolution is given by:

$$\frac{dY}{dt} = f(Y), \tag{1}$$

where Y is a vector that contains all variables in the flow and $f(Y)$ represents the Navier-Stokes equations with all boundary conditions. The first-order SFD scheme, which is used in this paper, is described by the following set of equations:

$$\frac{dY}{dt} = f(Y) - \chi(Y - \bar{Y}) \tag{2}$$

$$\frac{d\bar{Y}}{dt} = \frac{Y - \bar{Y}}{\Delta}. \tag{3}$$

χ and Δ are filter parameters which respectively define damping strength and the cut-off frequency (given by $\omega_c = 1/\Delta$). $\bar{Y}$ acts as a time-averaged flow, which ideally does not contain any high-frequency phenomena.

Assuming that χ and Δ are picked such that the flow becomes stable, given enough time, the flow will reach a steady state, causing Y and $\bar{Y}$ to become equal, which would make the second term of the right-hand side of Eq. 2 to vanish, reducing it back to Eq. 1.

The original paper by Åkervik *et al.* (2006) contains some details on how to pick χ and Δ correctly, for efficient stabilization of the flow. Jordi *et al.* (2015) proposes an adaptive algorithm, that automatically selects χ and Δ based on the current solution of the flow.

In this paper, we also consider the possibility of using different values of χ for each part of the domain. Our test scenario is an open cavity, whose only unstable mode is of the Rossiter type, which involves a spatially unstable region in the mixing layer as well as an acoustic feedback mechanism inside the cavity. By setting χ as non-zero only at the cavity's mixing layer and trailing edge, we are able to attenuate the oscillations in these regions, stabilizing the whole global mode. The advantage of using $\chi = 0$ in most of the domain is that the oscillations outside of the unstable region are not slowed down, allowing for faster convergence of the whole flow.

2.3 Time averaging

The Selective Frequency Damping is very effective for stabilizing high-frequency phenomena in the flow; however, using it to stabilize low-frequency oscillations would require using high values for Δ, which considerably increases the simulation time needed to reach a steady state, as the temporal derivatives of $\bar{Y}$ are inversely proportional to Δ, as per Eq. 3.

For those long-period oscillations, we repeatedly compute a time-averaged flow with a span that is carefully chosen to contain a whole number of oscillation periods.

3. RESULTS

The scenario used in this study is a small rectangular cavity in a subsonic compressible flow. Its parameters are as follows: $Re_{\delta_0^*} = 600$, $Ma = 0.5$, $L = 15\delta_0^*$, $D = 5\delta_0^*$, where L and D are the length and depth of the cavity, respectively. δ_0^* is the displacement thickness of the boundary layer at the cavity's leading edge, which is also used as norm for all lengths in the domain. All velocities are normalized by the free-flow velocity. The coordinates origin is placed at the leading edge of the flat plate. The cavity position is calculated with the Blasius equation so that $\delta^* = 1$ at the leading edge. The physical domain spans $0 \leq X \leq 418\delta_0^*$ and $-5\delta_0^* \leq Y \leq 21\delta^*$. Including the buffer zones, the domain spans $-517\delta_0^* \leq X \leq 1012\delta_0^*$ and $-5\delta_0^* \leq Y \leq 272\delta^*$. Figure 1 shows the base flow with contours of stream-wise velocity, each color represents 20% of the free-flow velocity.

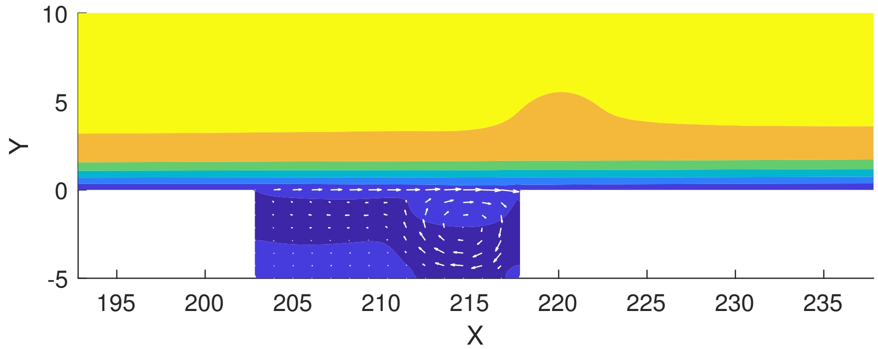

Figure 1. Base flow of the reference case, each contour represents 20% of the free-flow velocity.

A global stability analysis of this base flow reveals that there is one unstable mode, namely the second Rossiter mode, with a corresponding eigenvalue of $\sigma = 0.0037 \pm 0.27i$. In our signal convention, a positive real part of the eigenvalue indicates an unstable mode, and the imaginary part corresponds to its angular frequency. The first Rossiter mode is on the verge of becoming unstable as well, at $\sigma = -0.0007 \pm 0.17i$. Figure 2 (Left) shows the eigenvalues retrieved by the global stability analysis, with the unstable mode highlighted. Figure 2 (Right) shows the corresponding eigenfunctions for stream-wise velocity and density.

The presence of, at least, one unstable mode causes the flow as a whole to become unstable. In a linearized framework, this mode would grow infinitely with time; however, when the amplitude becomes large enough, a limit cycle is reached because the non-linear terms become large enough to stop the growth of the unstable mode. Figure 3 illustrates this with a snapshot taken at an arbitrary time after the limit cycle was reached.

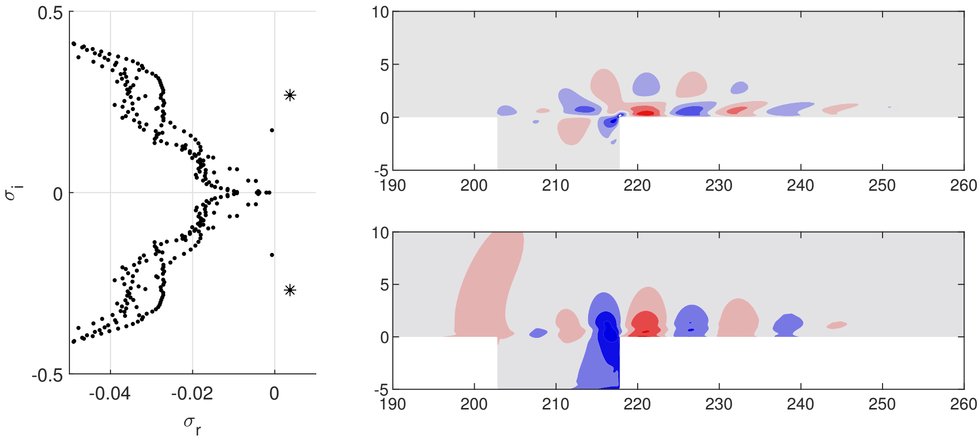

Figure 2. (Left) Eigenvalues of the global modes of the reference case. (Right) Stream-wise velocity and density fluctuations of the first Rossiter mode.

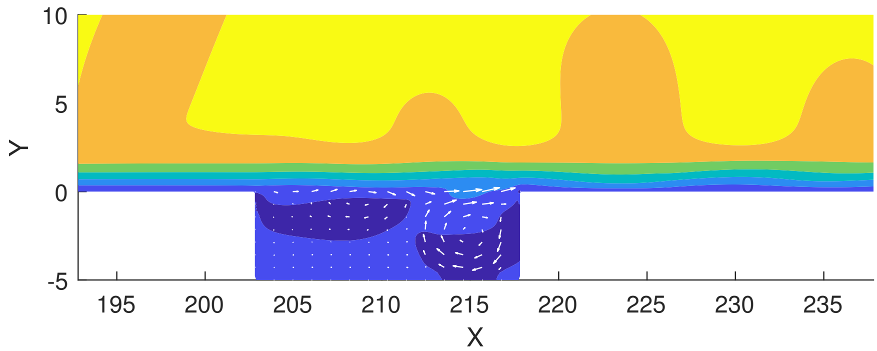

Figure 3. Snapshot of the limit cycle, each contour represents 20% of the free-flow velocity.

3.1 Effect of the SFD on the time series

To compare the effect of different strategies for the Spectral Frequency Damping, we have started the flow simulation with a Blasius boundary layer profile over the flat plate and a stagnated flow in the cavity. Figure 4 shows the time series of stream-wise velocity for three different probes in the flow. The blue line represents the physically accurate case, when SFD is turned off. The red line depicts the case in which SFD was turned on only close to the trailing edge of the cavity. The yellow line is the time series with SFD turned on in the whole domain.

The physically accurate case lets the unstable Rossiter mode to grow until it reaches a limit cycle, as predicted by the global stability analysis. In the probes just before the cavity ($X = 200$) and downstream from it ($X = 300$) there is also a visible shift in the mean flow.

Both "Trailing edge" and "Whole domain" cases were able to successfully damp the unstable mode. All three cases present a long-term oscillation, with a period just above 5000 time units or longer for the "Whole domain" case. This oscillation was identified to be a standing wave between the inlet and the outlet of the domain, i.e. a numerical phenomenon. Due to its very long period, SFD parameter Δ would need to be set as a very high value to damp this mode, causing the whole simulation to evolve very slowly.

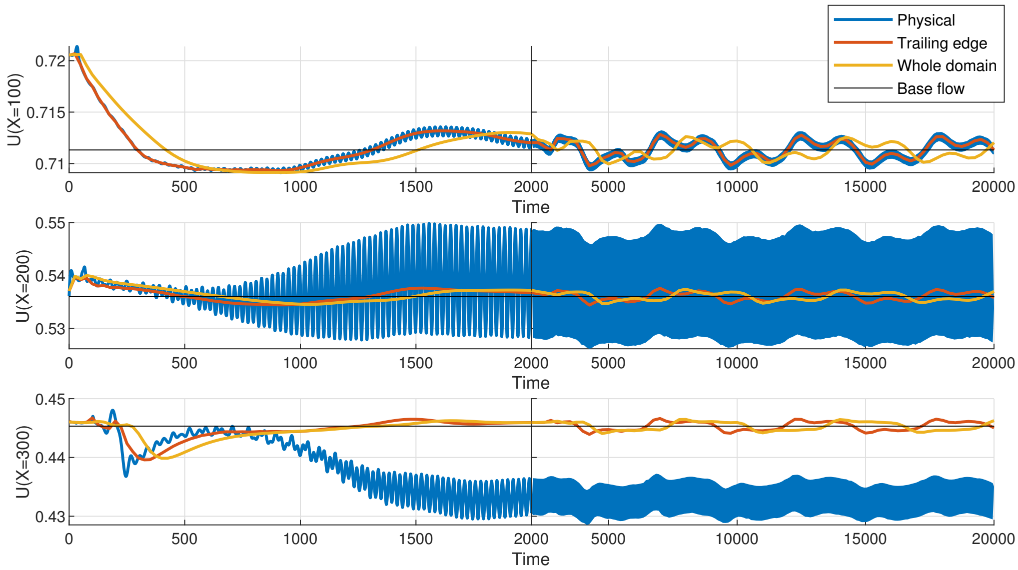

Figure 4. Time series of stream-wise velocities for three different strategies of Selective Frequency Damping.

3.2 Effect of the time averages

The long-term oscillation seen in Fig. 4 would eventually disappear; however, it would take an exceedingly long time, at a great computational cost. By using time averages, we could greatly speed up this process. It is especially important to include a whole number of periods in the averaging window. The oscillation period can be either measured from the time series or estimated, based on its nature. In this case, as the time series is already available, we have measured the period as 5500 time units. In this particular case, a snapshot of the flow is stored every 100 time units, therefore, this period corresponds to 55 snapshots. The time average was computed every 6500 time units, this way, there are 1000 time units for the flow to dissipate any non-physical phenomenon that may arise from the averaging process. Figure 5 shows the maximum residual of each variable as the simulation runs. The averaging process was repeated 5 times, each reducing the residuals by over an order of magnitude. The routine was stopped after the residuals have reached the noise floor of the DNS, which is more than 12 orders of magnitude smaller than the free flow.

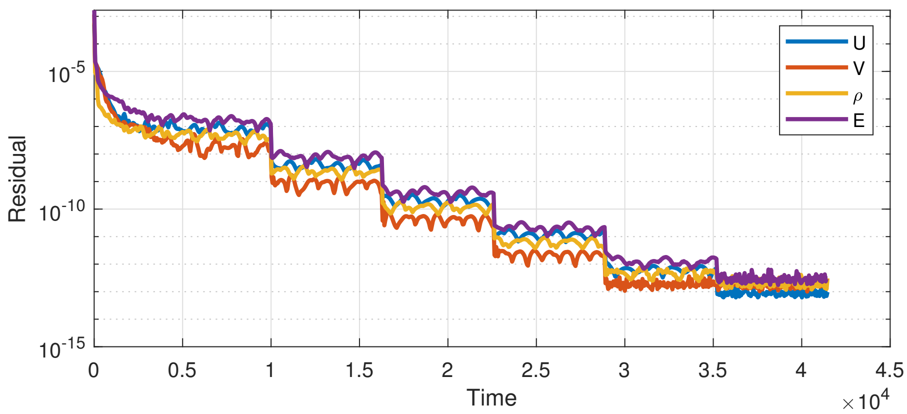

Figure 5. Maximum residual of each variable during the simulation, each drop represents the computation of a time-averaged flow.

4. CONCLUSION

The combination of both techniques has allowed us to base flows quickly and reliably, even for unstable scenarios, such as the one depicted here. By restricting the SFD acting region to only specific parts of the domain, we could successfully remove the instabilities with little effect in the rest of the flow. We have also observed that this strategy is more robust in terms of the χ and Δ parameters that are chosen, allowing a greater range of values, without slowing down the convergence of the rest of the flow, which can be of great importance, especially in scenarios where we have little knowledge of the flow and, thus, cannot make great estimates for the optimal SFD parameters.

The time averaging process was able to reduce the solution residual by over an order of magnitude in each iteration, which has allowed us to reach the base flow in a much shorter time than would be needed for it to be reached naturally. Using the SFD to remove such low-frequency oscillations would require the value of Δ to be set very high, which would slow down the convergence as a whole.

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