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ON FIXED-STRESS AND FIXED-STRAIN SOLUTION SCHEMES FOR LARGE STRAIN POROELASTICITY APPLIED TO SOFT BIOLOGICAL TISSUES

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Abstract. *Biological fibrous tissues such as tendon, ligaments and intervertebral discs are highly hydrated structures. Since these connective tissues are, in the most cases, poorly vascularized, diffusive mechanisms have been hypothesized to be the primary form of mass transport of nutrients into cells within the tissue. The interstitial fluid flow through those tissues gives rise to dissipation micromechanisms that can be investigated using poroelastic theories. The interactions between the fluid flow and the solid mechanics have been modelled using different types of coupling schemes to solve both mechanical equilibrium and mass balance equations. In the fully-implicit scheme, the governing equations are solved simultaneously at every time step. Iteratively-coupled methods consists of an inner iterative loop at each time step, solving the mechanical and the fluid flow problem separately until the solution reaches an acceptable convergence criterion. This work proposes to investigate of the fixed-stress and the fixed-strain iteratively-coupled approaches to solve the biphasic problem at finite strains. The main goal of this research is to apply these schemes on the investigation of the interstitial fluid flow within the tendon's microstructure, which may shad light about diffusion of nutrients and their relations with the mechanobiology of tendons.*

Keywords: *poroelasticity, solution schemes, finite element method, finite strains, soft biological tissues*

1. INTRODUCTION

The first developments of poroelastic theories were motivated by the study of geomechanical problems (Terzaghi, 1923; Biot, 1941). In addition to their use in soil mechanics, the biphasic theories are becoming increasingly important in the context of biomechanics of soft biological tissues (Mow *et al.*, 1980). These approaches model the interstitial fluid and its interactions with the surrounding solid matrices (solid skeleton) in a homogenised way, without an explicit distinction between each phase. Due to the kinematics amongst which soft biological tissues are commonly subjected, the biphasic models applied to this materials are generally based on the assumption of finite strains (Suh *et al.*, 1991; Almeida and Spilker, 1997a,b).

The classical poroelastic model leads to a system of equations accounting for the mechanical balance and the mass conservation of the biphasic media. Considering a poroelastic theory subjected to finite strains, the solid skeleton constitutive behaviour is generally ruled by a hyperelastic model, and the fluid transport is modelled by the Darcy's law. One of the main parameters of Darcy-type laws, namely, permeability, plays a major role in the fluid flow, and it is strongly related to the level of biphasic coupling. In soft biological tissues, such as tendons, cartilage, and intervertebral discs, the experimental assessment of the permeability parameter is an arduous task that is not yet well established in literature. Moreover, it has been suggested that this biphasic parameter may be strain dependent (Holmes and Mow, 1990; Yin and Elliott, 2004), porosity dependent (Lai *et al.*, 1981; Holmes and Mow, 1990) and may present different levels of anisotropy

(Ateshian and Weiss, 2010; Federico and Grillo, 2012).

The solution of finite strains biphasic problems can be obtained by two main strategies: the fully-implicit (monolithic) approach and iteratively-coupled (staggered) approaches (Kim *et al.*, 2011a; Yi and Bean, 2017; Duran *et al.*, 2020). In one hand, the fully-implicit scheme solves the governing equations simultaneously at each time-step, generally using an implicit Newton-Raphson algorithm. This approach is unconditionally stable and convergent, but can suffer with ill-conditioning problems and non-symmetric systems of equations, becoming computationally costly (Hirabayashi and Iwamoto, 2018). On the other hand, iteratively-coupled schemes can be used to solve these problems. In this case, the solution method splits the biphasic problem into two sub-problems: one related to the mechanical equilibrium and the other regarded to the conservation of mass. For each time-step, the sub-problems are iteratively solved until a given criterion on the solution variables is fulfilled. Such schemes result in two smaller symmetric system of equations, enabling the use of different discretization methods for each sub-problem. However, the iteratively-coupled methods may be limited due to the lack of stability and convergence issues (Kim *et al.*, 2011a,b; Yi and Bean, 2017).

Considering the advantages and the potential of the iterative schemes, different methods can be proposed to solve poromechanics problems. Among the various methods reported in literature, two common methods are presented herein: the *fixed-strain* and the *fixed-stress* (Kim *et al.*, 2011b; Yi and Bean, 2017). Although in geomechanics applications the iterative methods are widely investigated, in the biomechanics context, where the problems widely differs in scale and constitutive behaviour, iteratively-coupled schemes have been poorly studied.

Motivated by these observations, the main goal of this paper consist of investigating fixed-stress and fixed-strain schemes to solve biphasic problems applied to soft biological tissues. The time-continuum variables were discretized by the backward Euler integration and a mixed finite element formulation was used in the approximation of the primary variables (displacement and pore pressure). To investigate these methods, a confined compression test is performed, and the results are compared with the fully-implicit (monolithic) solver. The geometry and the constitutive characteristics of the numerical samples were estimated considering the range of properties addressed in literature of soft biological tissues. The solid skeleton constitutive behaviour is modelled by a neo-Hookean hyperelastic model, and the permeability model is considered exponentially strain-dependent. This paper also aims to contribute investigating how the exponential constant *M* of the employed permeability model can affect the coupling level of biphasic problems.

2. THEORETICAL FRAMEWORK

2.1 General poroelasticity theory

The poroelastic boundary value problem (BVP) consists in the solution of the conservation of linear momentum (together with the conservation of angular momentum) and the conservation of mass, either concurrently or sequentially, on a coupled manner. In mechanics of soft biological tissues, these equations are usually based on a fundamental set of hypothesis. In the microstructural level, both fluid and solid particles are assumed incompressible. It is important to note that this assumption does not imply in an incompressible behaviour of the macro continuum. The present poroelastic model accounts for the presence of two constituents (biphasic model), resulting in a fully saturated mixture of the fluid and the solid skeleton phases. Following the effective stress principle, the total Cauchy stress is equal to the stress induced by the solid phase deformation (effective Cauchy stress tensor) plus the purely volumetric stress acting on the solid phase caused by the pore pressure.

Suppressing the body and the inertial forces from the governing equations, the poroelastic problem in a spatial framework can be stated as the solution of the BVP:

$$
\left|\begin{array}{lr}
\text{Conservation of linear momentum} & \operatorname{div}\boldsymbol{\sigma} = \mathbf{0} \\
\text{Conservation of mass} & \operatorname{div}\,(\mathbf{v}^{\mathrm{s}} + \mathbf{w}) = 0 \\
\text{Conservation of angular momentum} & \boldsymbol{\sigma} = \boldsymbol{\sigma}^T \\
\text{Cauchy stress decomposition} & \boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathrm{s}} - p\boldsymbol{I} \\
\text{Relative velocity} & \mathbf{w} = (1-\phi_0)\left(\mathbf{v}^{\mathrm{f}} - \mathbf{v}^{\mathrm{s}}\right) \\
\text{Dirichlet boundary condition on } \Gamma^{\mathrm{s}}_{\mathbf{u}} & \mathbf{u} = \bar{\mathbf{u}} \\
\text{Neumann boundary condition on } \Gamma^{\mathrm{s}}_{\mathbf{t}} & \mathbf{t} = \boldsymbol{\sigma}\mathbf{n} = \bar{\mathbf{t}} \\
\text{Dirichlet boundary condition on } \Gamma^{\mathrm{f}}_{p} & p = \bar{p} \\
\text{Neumann boundary condition on } \Gamma^{\mathrm{f}}_{q} & q = \mathbf{w}\cdot\mathbf{n} = \bar{q},
\end{array}\right. \tag{1}
$$

together with the constitutive behaviour of the solid skeleton and the transport law of the fluid. In this set of equations, $\boldsymbol{\sigma}^{\mathrm{s}}$ and $\mathbf{v}^{\mathrm{s}}$ are the solid Cauchy stress tensor (effective) and the solid velocity vector, respectively, $\mathbf{w}$ is the relative velocity vector, $\mathbf{v}^f$ is the fluid velocity vector, ϕ_0 is the solidity and $\boldsymbol{I}$ is the identity matrix. Here, the dirichlet boundary conditions are expressed in terms of the primary variables $\mathbf{u}$ (displacement field) and p (pore pressure field), and the

Neumann boundary conditions are given in terms of the tractions $\mathbf{t}$ and flux q along the normal vector $\mathbf{n}$.

2.2 Constitutive model

To describe the solid skeleton material, we choose a hyperelastic constitutive model known as *compressible neo-Hookean*, whose stored strain energy function is defined as

$$\Psi = \frac{\mu}{2}(I_1 - 3) - \mu \ln J + \frac{\lambda}{2}(\ln J)^2, \tag{2}$$

where μ and λ are the Lamé's parameters, $I_1 = \operatorname{tr} \boldsymbol{C}$ is the first invariant of the right Cauchy-Green deformation tensor and J is the determinant of the deformation gradient $\boldsymbol{F}$. The second Piola-Kirchhoff stress tensor $\boldsymbol{S}$ is obtained by the relation

$$\boldsymbol{S} = 2\frac{\partial \Psi(\boldsymbol{C})}{\partial \boldsymbol{E}}, \tag{3}$$

where $\boldsymbol{E}$ is the Green's strain tensor. In order to further describe an *updated Lagrangian* formulation, the Cauchy stress tensor is obtained by the *push-forward*:

$$\boldsymbol{\sigma} = J^{-1}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^T. \tag{4}$$

2.3 Fluid transport model

The fluid flow through the porous domain is based on the Darcy's transport law,

$$\mathbf{w} = -\boldsymbol{k} \operatorname{grad} p, \tag{5}$$

where $\boldsymbol{k}$ is the second-order permeability tensor. This linear relation between the relative velocity vector and the pore pressure gradient vector is well-established in the literature of mechanics of porous media (Coussy, 2003). Here, we employ the *exponential isotropic permeability* model, which states:

$$\boldsymbol{k} = k(J)\boldsymbol{I}, \tag{6}$$

where,

$$k(J) = k_0 \exp\left(M\frac{J-1}{J-\phi_0}\right), \tag{7}$$

in which k_0 is the initial spatial permeability constant and M is an exponential constant that controls the nonlinearity level of the model (Ateshian and Weiss, 2010).

3. SOLUTION STRATEGIES

Coupled solution strategies are widely investigated to solve poromechanics problems, mainly in geomechanics applications. However, as aforementioned, the governinig equations of the biphasic problem are based on the incompressibility of the fluid and the solid particles in the microstructural level. This condition often gives rise to convergence and stability problems in the well-known *operator splits* (Kim *et al.*, 2011a,b; Kim, 2018). Other than that, low permeability values hypothesized in soft biological tissues, also known as *hydraulic permeability*, play an important role in the computational effort to solve the coupled system of equations.

In this work, we employ well-established coupling strategies, such as the *fixed-stress* and the *fixed-strain* iteratively-coupled schemes, and compare them with the fully-implicit (monolithic) method.

3.1 Fully-implicit

In the fully-implicit algorithm, the conservation of mass and linear momentum equations are solved simultaneously, *i.e*, the convergence is obtained at the end of each solution step using typically the Newton-Raphson scheme. The fully-implicit solution method can be expressed as,

$$\begin{vmatrix} \mathbf{u}^n \\ p^n \end{vmatrix} \xrightarrow[\operatorname{div}(\mathbf{v}^s + \mathbf{w}) = 0]{\operatorname{div} \boldsymbol{\sigma} = \mathbf{0}} \begin{vmatrix} \mathbf{u}^{n+1} \\ p^{n+1} \end{vmatrix}, \tag{8}$$

where the indices $(\cdot)^n$ and $(\cdot)^{n+1}$ represent the previous and current time stepping iteration. Although this method presents unconditional stability, its resulting system of equations is not symmetrical. Other than that, due to the distinct nature of each partial differential equation (mechanical balance and energy conservation), the different orders of magnitude of the material parameters (permeability and Lamé's constants) generally lead to ill-conditioning problems. This issues are usually surpassed with the use of preconditioners and robust linear solvers.

3.2 Fixed-stress

The fixed-stress schemes are amongst the so-called *staggered-type* coupling algorithms. The infinitesimal version of this scheme is traditional in reservoir geomechanics (Kim *et al.*, 2011b; Almani *et al.*, 2016; Yi and Bean, 2017). In contrast, its appliance to large strains kinematics is yet to be well-elucidated in the literature. In Kim (2018), it is proposed the *fixed second Piola-Kirchhoff stress* scheme, in which the conservation of linear momentum is solved in a *total Lagrangian* finite element formulation, and the mass balance is solved by the finite volume method. In this work, we extend the aforementioned proposal, employing the fixed second Piola-Kirchhoff stress scheme in an updated Lagrangian finite element formulation for both fluid flow and mechanics. The fixed-stress scheme solves sequentially the conservation of mass equation at constant second Piola-Kirchhoff stress $\delta\dot{\boldsymbol{S}} = \mathbf{0}$, followed by the solution of the conservation of linear momentum fixing the pore pressure field $\delta\dot{p} = 0$.

The fixed-stress iterative procedure can be expressed as,

$$\begin{vmatrix} {}^{k}\mathbf{u}^{n+1} \\ {}^{k}p^{n+1} \end{vmatrix} \xrightarrow[\delta\dot{\boldsymbol{S}}=\mathbf{0}]{\operatorname{div}(\mathbf{v}^{\mathrm{s}}+\mathbf{w})=0} \begin{vmatrix} {}^{k}\mathbf{u}^{n+1} \\ {}^{k+1}p^{n+1} \end{vmatrix} \xrightarrow[\delta\dot{p}=0]{\operatorname{div}\boldsymbol{\sigma}=\mathbf{0}} \begin{vmatrix} {}^{k+1}\mathbf{u}^{n+1} \\ {}^{k+1}p^{n+1} \end{vmatrix}, \tag{9}$$

where the additional indices ${}^{k}(\cdot)$ and ${}^{k+1}(\cdot)$ represent the previous and current staggered iterative procedure. This scheme is maintained until the primal variables ${}^{k+1}\mathbf{u}^{n+1}$ and ${}^{k+1}p^{n+1}$ converge to ${}^{k}\mathbf{u}^{n+1}$ and ${}^{k}p^{n+1}$, respectively, for a given tolerance.

3.3 Fixed-strain

The fixed-strain scheme is also amongst the class of *staggered-type* coupling algorithms, but it presents common convergence issues, as approached in the literature of infinitesimal geomechanics (Kim *et al.*, 2011b). This scheme consists of keeping the Euler-Almansi strain field fixed through the staggered iterations $\delta\dot{\boldsymbol{e}} = \mathbf{0}$ while solving the mass balance equation. Later, we keep the pore pressure field fixed $\delta\dot{p} = 0$ while solving the conservation of linear momentum equation.

The fixed-strain iterative procedure can be expressed as,

$$\begin{vmatrix} {}^{k}\mathbf{u}^{n+1} \\ {}^{k}p^{n+1} \end{vmatrix} \xrightarrow[\delta\dot{\boldsymbol{e}}=\mathbf{0}]{\operatorname{div}(\mathbf{v}^{\mathrm{s}}+\mathbf{w})=0} \begin{vmatrix} {}^{k}\mathbf{u}^{n+1} \\ {}^{k+1}p^{n+1} \end{vmatrix} \xrightarrow[\delta\dot{p}=0]{\operatorname{div}\boldsymbol{\sigma}=\mathbf{0}} \begin{vmatrix} {}^{k+1}\mathbf{u}^{n+1} \\ {}^{k+1}p^{n+1} \end{vmatrix} \tag{10}$$

where the additional indices ${}^{k}(\cdot)$ and ${}^{k+1}(\cdot)$ represent the previous and current staggered iterative procedure. This scheme is maintained until the primal variables ${}^{k+1}\mathbf{u}^{n+1}$ and ${}^{k+1}p^{n+1}$ converge to ${}^{k}\mathbf{u}^{n+1}$ and ${}^{k}p^{n+1}$, respectively, for a given tolerance.

4. FINITE ELEMENT MODEL

In order to gain insight into the nonlinear volumetric behaviour of a representative soft biological tissue sample and assess the numerical results of the developed methods, we propose a numerical experiment enforcing a confined kinematical condition (see Fig. 1 for the applied boundary conditions). To achieve the large deformation regime, Fig. 1 shows that a displacement controlled compression of 4 mm/s is imposed until 0.4 mm, resulting in 20% axial compression (X_3 − direction). The applied displacement is kept fixed until t = 200 seconds. The numerical simulations employ the fully-implicit, fixed-stress and fixed-strain schemes to solve the poroelastic problem.

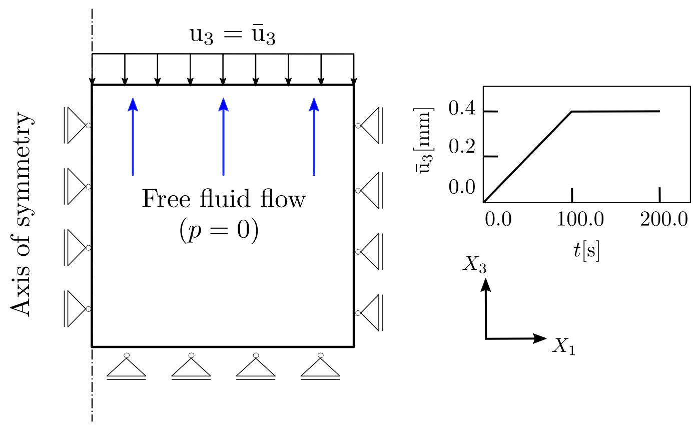

Figure 1. Applied boundary conditions for both displacement and pore pressure.

We employ a three-dimensional mixed finite element model considering second-order Lagrange polynomials to interpolate the displacement field, and first-order ones to interpolate the pore pressure field within the hexahedral element. In this study, a neo-Hookean hyperelastic material is used to model the constitutive behaviour of the solid skeleton (Eq. 2). The fluid transport through the porous domain is based on the Darcy's law using the exponential isotropic permeability model (Eq. 7). In Tab. 1, we introduce the poroelastic material parameters, the geometric characteristics and the time step considered in the analysis. A finite element mesh refinement is performed using five types of regular meshes: mesh 1 (480 elements), mesh 2 (960 elements), mesh 3 (1440), mesh 4 (2400) and mesh 5 (3360). We choose the mesh 4, as the maximum relative errors for the von-Mises stress is 0.1879%.

Table 1. Analysis parameters.

Parameter	Unit	Adopted value
First Lamé's, λ	MPa	15.0
Shear modulus, μ	MPa	10.0
Hydraulic permeability, k_0	$mm^4/N \cdot s$	0.005
Exponential constant, M	-	0.5 - 4.0
Solidity, ϕ_0	-	0.3
Height, H	mm	2.0
Radius, R	mm	2.0
Time step, Δt	s	5.0

5. RESULTS AND DISCUSSIONS

This study employs three different methods to solve the biphasic coupled problem. While the fully-implicit scheme solves the system of nonlinear equations simultaneously, the iteratively-coupled ones solve the equations sequentially, within an iterative process. Despite the distinct way to solve the coupled problem, all simulations reported in this paper are expected to converge to the fully-implicit (monolithic) results. Figure 2 presents the pore pressure and the volumetric Jacobian results using these strategies. We verify that the iterative methods are able to achieve the same results as those obtained by the fully-implicit one.

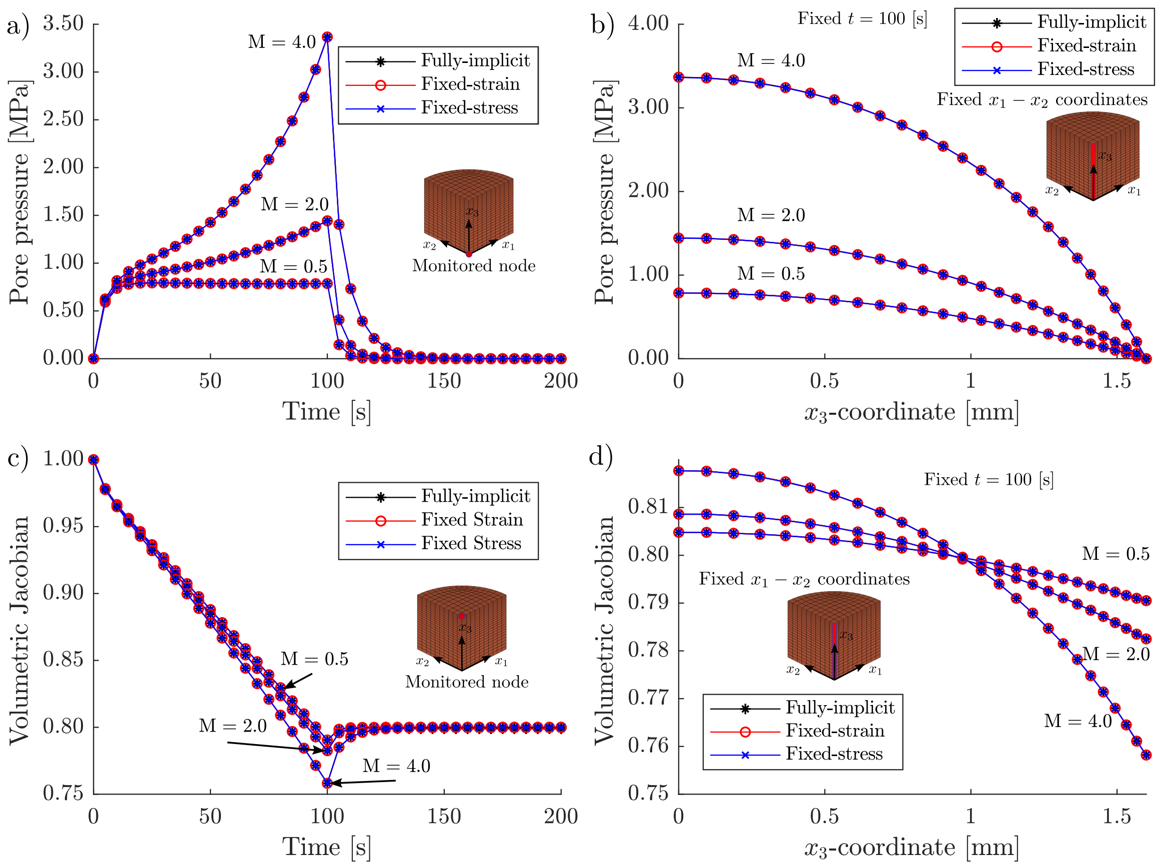

Figure 2. Pore pressure and volumetric Jacobian values as functions of time and spatial x_3-coordinate using three representative values of exponential parameter M.

Aiming to investigate the influence of the poroelastic results with a variation in the permeability parameter, this work employs a permeability model with a volumetric strain dependence. Accordingly to the model relation (7), the higher the parameter M, the more the permeability decreases with increasing compressive strain. This observation is in agreement with experimental studies of soft biological tissues (Mow and Mansour, 1977; Boschetti *et al.*, 2004). This phenomenon can be associated with the closure of the interstitial spaces while compressive deformation occurs.

Figure 2a-b presents the pore pressure results for three different exponential parameters M. We verify that, for higher M values, the decrease in the permeability during the compression process is intensified. This results in a higher resistance of the fluid flow, which can also be verified due to higher pressures and pressures gradients. Fig. 2c-d also presents the results obtained for the volumetric Jacobian. One can see that higher values of the parameter M increase the time dissipation within the deformation process. Regarding the solution process, it is observed that the exponential parameter M has a high influence on the level of coupling of the analysis. In finite strain regime, this higher level can lead to convergence problems in the solution process.

6. CONCLUDING REMARKS

This paper presents a study of the fixed-stress and fixed-strain solution schemes to solve finite strain poroelasticity problems applied to soft biological tissues. Connective tissues such as tendons, intervertebral disks, articular cartilages, and others, have been considered as poorly permeable structures. In poroelastic analysis, this type of structure is modeled as a biphasic body with low hydraulic permeability values. Numerically, this parameter turns out to play an important role on the stability and convergence of the solution scheme. In this work, we present a summary of the procedures involved in the formulation of these so-called iteratively coupled methods.

In order to assess the importance of the aforementioned parameter in the numerical results, we employ the finite element method to solve a three-dimensional biphasic problem with confined kinematical restrictions. A strain-dependent permeability model is also considered in the analysis, so the permeability value changes progressively with the enforcement of the compression forces. Based on the obtained results, the fixed-stress and fixed-strain iterative schemes show similar results to those obtained by the fully-implicit (monolithic) method. The exponential parameter M seems to have a high influence on the level of coupling of the analysis, which may lead to numerical difficulties. For the employed range of parameters, and considering the compressible neo-Hookean hyperelastic material for the solid phase, we conclude that the three investigated methods are able to solve the benchmark problem.

7. ACKNOWLEDGEMENTS

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