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PERFORMANCE AND ROBUSTNESS COMPARISON OF TYPE-2 FUZZY AND OPTIMAL CONTROL IN ROBOTIC JOINT POSITIONING

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Abstract. *Fuzzy systems were introduced in 1965 and have been applied in many areas, such as control systems among others. Generally, fuzzy systems are based on a type-1 fuzzy set where membership functions are precise and the real value concerning specific membership function corresponds to only one membership grade. In 1975 fuzzy type-2 systems were introduced and this new theory has been also applied in many applications. Type-2 fuzzy sets possess an additional imprecision with relation to type-1 fuzzy sets, where a real value corresponds to an interval of membership degrees. It means there is uncertainty with relation to membership degree where a real value corresponds to many membership grades concerning the linguistic variable. There is an expectation that type-2 fuzzy sets can generate fuzzy controllers more efficiently in control systems applications that are linear or even nonlinear. In this work, it was designed a fuzzy controller for a position of motorized rotary joint utilizing type-2 fuzzy sets. The objective of this research is to investigate whether type-2 fuzzy controllers can present performance and robustness comparable to optimal controllers for this application which is of great interest in the robotics area. Initially, the comparison was performed numerically by simulation. Performance was evaluated employing reference inputs and disturbances of a wide-range frequency spectrum. Its robustness was investigated from the parametric and non-parametric point of view to represent parametric uncertainties but also the unmodeled dynamics, normally present in the modeling stage. The system was also evaluated experimentally in real-time for lab-scale equipment. The results observed suggest that type-2 fuzzy controllers can be used in applications like this with similar performance when compared to optimal controllers and having a more natural form of design.*

Keywords: *type-2 fuzzy system, optimal control, fuzzy logic*

1. INTRODUCTION

The robust control problem is related to the fact that the plant model used for design purposes is uncertain. The origin of model uncertainty is the lack of completeness of the model and the errors associated with its parameters. When the errors are due to unmodeled dynamics, the uncertainties are said to be non-parametric and, when the errors are associated with the values of the model parameters, the uncertainties are said to be parametric (Skogestad, 2005).

Control theory research has addressed the issue of parametric and non-parametric robustness to incorporate information about uncertainties in the design phase (Da Cruz, 1996). However, the design method is complex and usually, only systems engineers can carry out a design with these methodologies.

Our motivation for this paper is precisely to evaluate whether the fuzzy control technique, especially the type-2 fuzzy can lead to satisfactory results in applications that are more common in mechanical engineering, such as in controlling the positioning of a robotic joint, a common task of the area. The intention is not to formally prove the robustness of a type-2 fuzzy controller (Biglarbegian, 2010), but to convince the audience through an example that there is inherent design performance and robustness for the application being considered. A lot of applications of the type-2 fuzzy controller have been utilized as can be seen in (Ren, et al., 2020, Huang, 2018), among others.

For comparison purposes, we investigated the performance obtained from a type-2 fuzzy controller with the performance of an optimal controller type linear quadratic regulator, whose robustness is high when state observers are not used (Doyle and Stein, 1979). As is well known, fuzzy controllers are a mature application for handle with a wide variety of uncertainty and do not require the mathematical model of the plant, so if the results are relevant for this application, the fuzzy controller may be an alternative more intuitive and simpler. The comparison of the two techniques was made through real experiments in laboratory-scale equipment. The modeling used for the optimal controller design was intentionally simplistic so that the model contains uncertainties that will be inherently compensated for by the linear quadratic control law. The criterion used for the adequacy of the type-2 fuzzy controller is the comparison of responses in the sense that, if the fuzzy control law is similar to the optimal control, it is assumed that the control system has good performance and robustness.

2. TYPE-2 FUZZY SYSTEM

Fuzzy systems were introduced by Zadeh in 1965 (Zadeh, 1965) and have been applied in many areas, such as control systems (Ariff, et al, 2014), among others. Generally, fuzzy systems are based on a type-1 fuzzy set where membership functions are precise and the real value concerning specific membership function corresponds to only one membership grade. A fuzzy set of a universe of objects X is defined as function μ that maps X into $[0;1]$. When X is a finite set $x_1, \dots \dots, x_2$ a fuzzy set F on X is defined as $F = (x, \mu_F(x) | x \in X)$, where $\mu_F(x)$ is denominated membership grade of x in F that is obtained through a membership function. Figure 1 shows an example of a type1- fuzzy set.

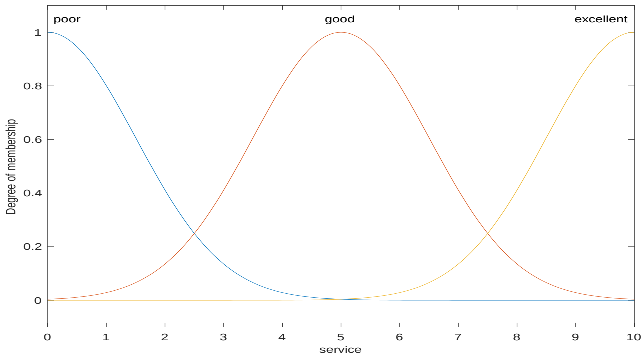


Figure 1: Type 1- Fuzzy Membership Functions

As can been seen, the real value service equal to 3 corresponds to only one membership grade concerning linguistic variable poor and only one membership grade about linguistic variable good. In the same fuzzy set, service equal to 3 corresponds to only one membership degree. The type-1 fuzzy set was known popularly as fuzzy set until 1975 when Zadeh (Zadeh, 1975) introduced the type-2 fuzzy system and this new approach provided additional freedom degrees to a design (Mendel, 2007).

Type-2 fuzzy sets possess an additional imprecision with relation to type-1 fuzzy sets, where a real value corresponds to an interval of membership degrees as shown in figure 2.

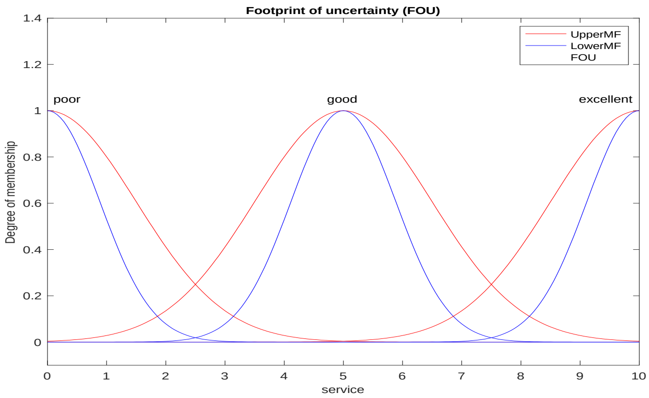


Figure 2: Membership Functions Type 2- Fuzzy

So, a type-2 fuzzy set $\tilde{F}$ is represented by a type-2 membership function $\mu_{\tilde{F}}(x,u)$, where $x \in X, \forall\, u \in J_x \subseteq [0;1], i.e.,$

$$\tilde{F} = \left\{ \left((x,u), \mu_{\tilde{F}}(x,u) \right) \mid \forall\, x \in X, \forall\, u \in J_x \subseteq [0;1] \right\} \quad (1)$$

As can be seen in figure 2, there is uncertainty with relation to membership degree where a real value corresponds to many membership grades related to the linguistic variable. For example, the linguistic variable good possesses an uncertainty denominated FOU (Footprint of Uncertainty) that generates an additional degree of freedom with the principal aim of capture more information about the linguistic variable.

Taking as an example the membership function good, the number 3 related to this function has various degrees of membership, that is, all that intercept the blur part called FOU, and no longer a simple value. So, type 2 fuzzy is useful when there is uncertainty on membership function, uncertainty on the limits of these functions, and uncertainty on membership degrees. Figure 3 shows the basic configuration of a type-2 fuzzy system.

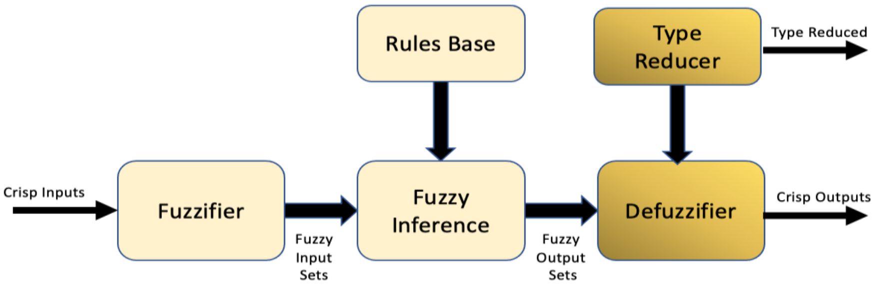


Figure 3: Type-2 Fuzzy System.

The difference between type- 1 fuzzy sets and type- 2 fuzzy sets is only related to the nature of the membership functions, as shown in figure 2. Figure 3 shows a block called Type Reducer that reduces type-2 after defuzzifier block maps fuzzy numbers into a crisp number. The rules continue presenting the same structure, as following:

i) Mamdani inference method: $If\ x_1\ is\ F_1\ and\ x_2\ is\ F_2\ then\ y = B_k$

ii) Takagi-Sugeno inference method: $If\ x_1\ is\ A_1\ and\ x_2\ is\ A_2\ then\ y = g(x_1,\ x_2)$

where, $x_1\ and\ x_2$ are inputs, $F_1\ and\ F_2\ and\ B_k$ are type 2- fuzzy sets.

There is a hope that a type-2 fuzzy system can generate fuzzy controllers more efficiently in practice concerning the performance and robustness of the system. In this work is proposed a type-2 fuzzy controller for a position control utilizing the Takagi-Sugeno inference method.

3. PRELIMINARY RESULTS

In this work was designed a fuzzy controller for position control of a servomechanism utilizing a type-2 fuzzy system with Takagi-Sugeno fuzzy inference. The real process or plant is represented by a servomechanism and the communication between the real plant and the type-2 fuzzy controller is done utilizing a data acquisition board National Instruments 6221- 37 pin installed in the computer and an interface National Instruments as illustrated in figure 4. The figure of the servomechanism is only illustrative and is not intended here to detail its electronics.

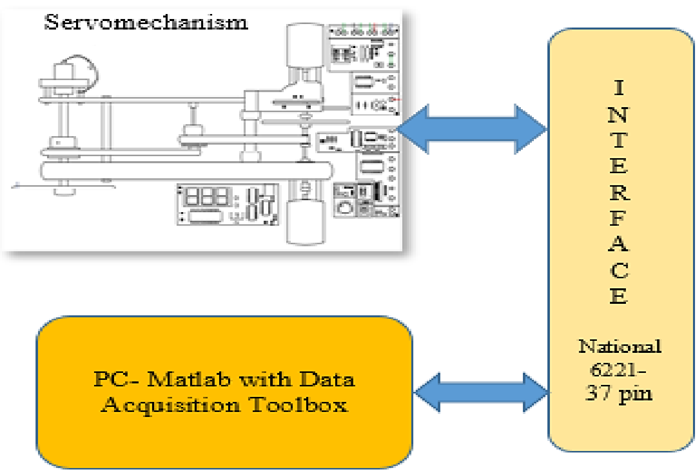


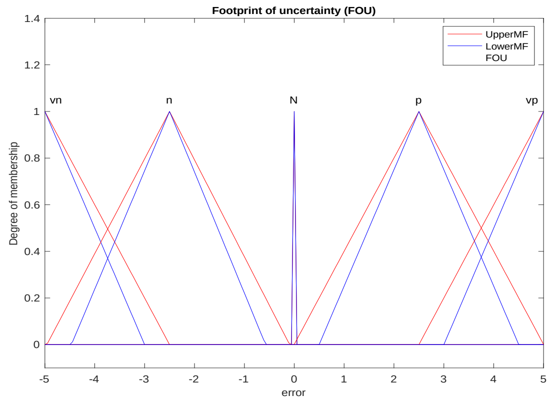
Figure 4: Layout of the Practical Application.

Table 1 shows the input data, as well as the linguistic values and their respective notation.

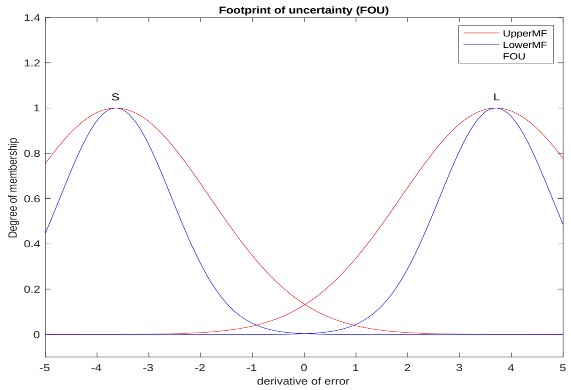
Table 1. Variables and Linguistic Values.

Linguistic variable: error (e)		***Linguistic variable: derivative of error (de)***	
Linguistic Value	***Notation***	***Linguistic Value***	***Notation***
very negative	*vn*	*Small*	*S*
negative	*n*	*Large*	*L*
null	*N*		
positive	*p*		
very positive	*vp*		

After many tests and adjustments, the membership functions of the error and derivative of error inputs for the type-2 fuzzy system, which presented the best results, are shown in figure 5.



a



b

Figure 5: a. Type-2 fuzzy membership functions- input error; b. Type-2 fuzzy membership functions- input derivative of the error

As can be seen in the figure, each real value of the variables error and error derivative are converted into a range of membership degrees, characterizing type- 2 fuzzy that produces an uncertainty in the membership degrees. The Simulink/ Matlab diagram shown in figure 6 was utilized together with the data acquisition board. Applying a set point represented by a square wave of 0.1Hz, the following response was obtained as shown in figure 7.

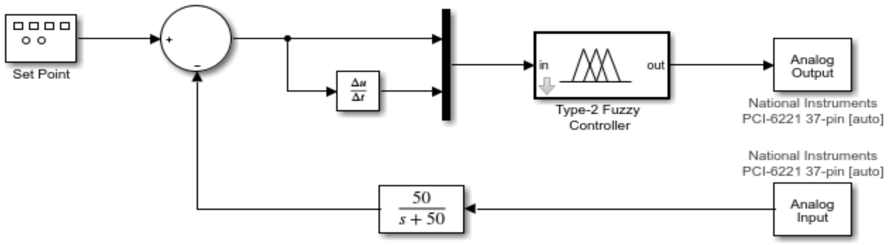
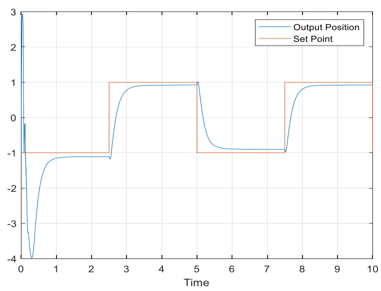


Figure 6: Block diagram of type-2 fuzzy control system



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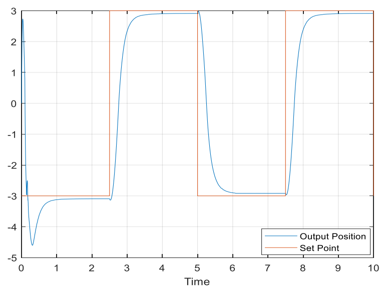

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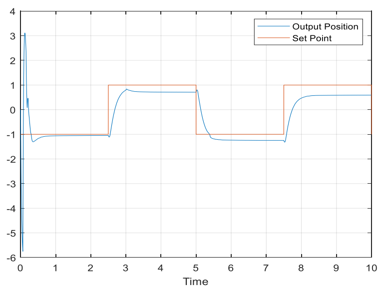


c

Figure 7: a. Response to square wave 1Vpp; b. Response to square wave 3Vpp; c. Response to square wave 1Vpp with disturbance of 1Vpp at t=3s

The performance of the control system was evaluated for the reference tracking and for the load disturbances rejection. The same scenario is considered in section 4 to be able to make a comparison with the objective of having a perception if the design using type-2 fuzzy control is appropriate for this application and has advantages or drawbacks.

The reference sign contains steps that are detailed by the red curves in figures 7a and 7b. The disturbance sign was also applied in the form of steps and detailed by the red curve in figures 7b. The disturbance was added to the input of the plant but was not shown in the block diagram.

As can be seen in figure 7, in all cases the type-2 fuzzy control system produced a response without overshoot and settling time minor than 1 second, besides having an error extremely low.

4. OPTIMAL CONTROL

Given a dynamic system $\dot{x} = f(x,u,t)$, its performance can be defined by means of a cost function of type $J = \int_0^{t_f} g(x,u,t)dt$. The state-space linear optimal control problem usually refers to making J minimal via state feedback $u = -Kx$, with $K \in R^{mxn}$, applied to the plant $\dot{x} = Ax + Bu$, resulting in $\dot{x} = \mathbb{F}x$, where $\mathbb{F} = A - BK$.

Considering the problem of minimizing $J = \int_0^{\infty}(x^T x)dt$, to guarantee that the states tend to zero with time, a dynamic constraint linked to the plant dynamics is created, which guarantees that the energy of the function $V(x) > 0$, a function of the states, is always decreasing. This is achieved by imposing that $\dot{V} < 0$ for $V > 0$.

This is equivalent to doing $\dot{V} = e$, where $e < 0$. One possible function is $V = x^T P x$, which is a metric related to the energy of states, where V must be a positive definite function $x^T P x > 0$, that is, with all eigenvalues of P > 0. A reasonable value for the right side is $e = -x^T x$, resulting

$$\frac{d(x^T P x)}{dt} = -(x^T x) = \dot{x}^T P x + x^T P \dot{x} \tag{2}$$

Since $\dot{x} = \mathbb{F}x$,

$$x^T(-I)x = (\mathbb{F}x)^T P x + x^T P(\mathbb{F}x) = x^T(\mathbb{F}^T P + P\mathbb{F})x \tag{3}$$

Thus, the matrix P must agree with the constraint given by the algebraic equation given by $\mathbb{F}^T P + P\mathbb{F} = -I$. The problem including an input penalty $J = \int_0^{\infty}(x^T I x) + \lambda(u^T u)dt$, where the scalar constant $\lambda \geq 0$ makes the balance between the energy of the state and the control, it can be solved by adapting the problem without control penalty.

$$J = \int_0^{\infty}(x^T I x) + \lambda(u^T u)dt = \int_0^{\infty} x^T(I - \lambda K^T K)x dt = \int_0^{\infty} x^T S x dt \tag{4}$$

By analogy to the problem with no control penalty, the matrix P must then agree the restriction given by the algebraic equation given by

$$\mathbb{F}^T P + P\mathbb{F} = -(I - \lambda K^T K) \tag{5}$$

For the value of K of the control law $u = -Kx$ that corresponds an extreme point, the gradient of the cost function to all k_i de K, must satisfy

$$\frac{\partial J}{\partial k_i} = 0 \tag{6}$$

and, for it to be a minimum point, the Hessian of the cost function must be defined positively. The problem presented can be put in a more general way with $u = -Kx$, to determine the matrix K that minimizes the cost function

$$J = \int_0^{\infty}(x^T Q x + u^T R u)dt, \tag{7}$$

called linear quadratic regulator problem, where Q is the state penalty matrix and R is the control penalty matrix, such that Q and R are positive definite and symmetric matrices, that is, $Q = Q^T$, $R = R^T$, $det(Q) = \lambda_1 \cdots \lambda_n > 0$, $det(R) =$

$\gamma_1 \cdots \gamma_r > 0$. It can be shown that, determining the minimum value of the gradient of the cost function J is equivalent to solving the algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (8)$$

If the solution is a positive definite matrix P, the optimal control law results

$$u(t) = -(R^{-1}B^T P)x(t) \quad (9)$$

The linearized model in the state space of the lab-scale equipment is given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -3{,}3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 33{,}3 \end{bmatrix} u \quad (10)$$

$$y = [\ 1 \quad 0\ ]x + [\ 0\ ]u \quad (11)$$

Let the following penalties

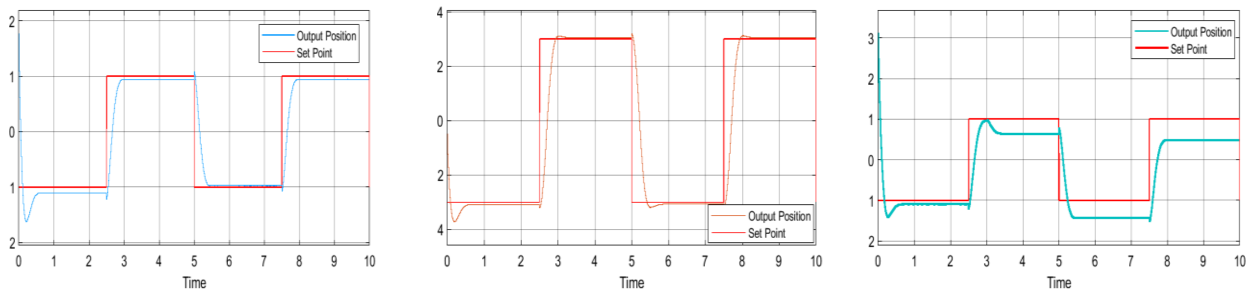
$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \quad (12)$$

$$R = 1 \quad (13)$$

This means the same emphasis on the position error when compared to the error of the velocity. The value of R was determined iteratively preventing the control signal from saturation. Solving the optimal control problem, the feedback law results

$$u = -[\ 2{,}2 \quad 2{,}2]x \quad (14)$$

Utilizing this control law and applying a set point represented by a square wave of 0.1Hz, the following response was obtained as shown in figure 8.



a *b* *c*

Figure 8: a. Response to square wave 1Vpp; b. Response to square wave 3Vpp; c. Response to square wave 1Vpp with disturbance of 1Vpp at t=3s

As can be seen in figure 8a, when the set-point is changed a small overshoot can be observed and when the system is submitted to a disturbance occur an increase in the error, as can be seen in figure 8c. This indicates that the system is most sensitive to variations of set-point and disturbance which can, eventually, affect the system response.

The performance of the fuzzy type-2 control system (see figure 7) in contrast to the performance of the optimal control system (see figure 8) was made based on the closed loop constant time, overshoot and steady state error. The robustness of the controllers was verified since the experiments was done with the real plant.

It is noted that the tracking of the reference signal has very similar characteristics, but the rejection of load disturbances is much more effective with the fuzzy type-2 controller that require a clearly minor steady state error.

5. CONCLUSIONS

In this work, it was proposed two controllers: a type-2 fuzzy controller and an optimal control type linear quadratic regulator. Both controllers were designed and applied for position control of a motorized rotary joint. All tests of performance were done in real-time for lab-scale equipment with and without disturbance applied. Such tests, for being done in a real plant, involve nonlinearities present in the system, backlash effect in the gears, and uncertainty of the plant model, too.

The results obtained showed that type 2- fuzzy controller produced a performance very similar to optimal controller type linear quadratic regulator concerning the response time, overshoot, and sensibility to the disturbances. However, some interesting facts can be observed when the responses utilizing the two controllers are compared based on the obtained results:

i) System, when controlled by a type-2 fuzzy controller, presents a response without overshoot.

ii) The control through type-2 fuzzy controller produces a response less sensitive to external disturbances. So, the system presents a sensibility reduced.

iii) Type-2 fuzzy controller, for being based in the fuzzy logic, does not need the mathematical model of the plant, that is the most difficult phase in the design of a control system and presents a similar performance and sensibly better than optimal controller type linear quadratic regulator.

As future studies could be proposed a hybrid control involving optimal and type-2 fuzzy controllers in the application. Proposed here.

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