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DYNAMIC ANALYSIS AND CONTROL OF A DOUBLE PENDULUM ARM EXCITED BY AN RLC CIRCUIT

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Abstract. *In this paper the dynamic of a double pendulum arm coupled through a magnetic field to a nonlinear RLC based shaker circuit is numerically studied. This kind of electromechanical system are frequently found in robotic systems, and have important applications in Engineering Sciences. The double pendulum is considered as a three degree of freedom system coupled through a magnetic field to an RLC circuit based nonlinear shaker, where the capacitor voltage is a nonlinear function of the instantaneous electric charge. The nonlinear response analysis of the system is done by various techniques, including bifurcation diagrams, phase portraits, power spectral densities (FFT), and Lyapunov exponents. The bifurcation diagram is constructed to explore the qualitative behavior of the system. Numerical simulations show the existence of chaotic and hyperchaotic behavior for some regions in the parameter space and this behavior is characterized by Lyapunov exponents. In order to suppress the chaotic motion, a PID control is proposed and analyzed. Numerical simulations show the effectiveness of the proposed control in suppressing the chaotic motion.*

Keywords: *Electromechanical System, PID control, Chaos Suppression, Hyperchaotic behavior, Chaotic behavior.*

1. INTRODUCTION

The growing number of researches on dynamical systems with pendulum elements has demonstrated the importance of the theme in applications in Engineering Sciences. Dynamics of the parametrically excited pendulum have been investigated (Clifford and Bishop, 1995, 1996; Lu, 2006; Lenci *et al.*, 2008; Lenci and Rega, 2011; Nagamine *et al.*, 2007; Masoud *et al.*, 2012; Bridges and Georgiou, 2001; Rocha *et al.*, 2017; Janzen *et al.*, 2019; Iliuk *et al.*, 2013). The non-ideal pendulum autoparametric system was studied by (Sado and Kot, 2006, 2007; Avanço *et al.*, 2015, 2018, 2019), and the double pendulum autoparametric system with harmonic excitation was studied by (Sado and Gajos 2008; Tusset *et al.*, 2016b). The nonlinear dynamics of a periodically forced triple pendulum is investigated experimentally and numerically in Awrejcewicz *et al.* (2004, 2005, 2007, 2008, 2013).

Dynamic systems with pendulum elements can exhibit different types of behavior, ranging from periodic oscillations to chaotic behavior (Kecik and Warminski, 2012; Stachowiak and Okada, 2006; Xu *et al.*, 2005). In many cases, periodic behavior may be desirable, avoiding chaotic movement (Ge *et al.*, 2001). The suppression of chaotic behavior has received a lot of attention in the past years and various control methods have been proposed to control the chaotic system (Wang and Jing, 2004). Yokoi and Hikihara (2011) analyzed delayed feedback control. Wang and Jing (2004) applied the Lyapunov function method to design the control. Tusset *et al.* (2016a) used the nonlinear SDRE control to suppress the chaotic behavior of a simple pendulum system with parametric excitation. In Tusset *et al.* (2016b), SDRE and Saturation control are considered to suppress the chaotic behavior of a double pendulum system. In Tusset *et al.* (2017), an MR damper was used as a passive control for the suppression of chaotic behavior in a simple pendulum system with autoparametric excitation.

In this paper, the dynamics of a double pendulum arm coupled to a non-linear shaker is numerically studied. The nonlinear response analysis of the system is done by several techniques, including bifurcation diagrams, phase portraits, spectral power densities (FFT) and Lyapunov exponents. In order to eliminate the chaotic behavior, a PID control is used.

2. MATHEMATICAL MODEL

Fig.1 presents the double pendulum of lengths l_1 and l_2 and masses m_1 and m_2 investigated in the paper.

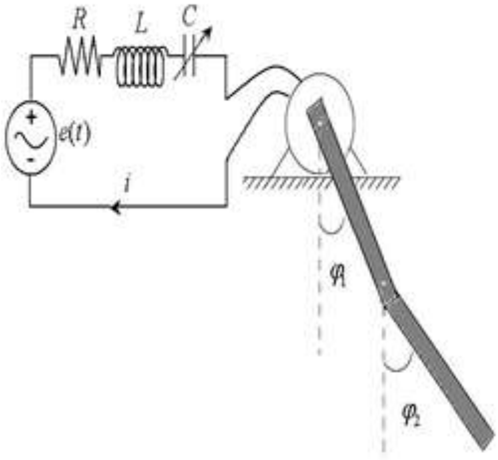


Figure 1. Schematic diagram of the electrical excited pendulum system (Tusset et al., 2016b).

In Fig. 1 it is observed that the electrical part of the system consists of a capacitor *C*, an inductor *L*, a resistor *R* and a source of voltage *e*(*t*), where the pendulum deflection angles φ_1 and φ_2 are measured from the vertical axis.

The dimensionless mathematical model for the system presented by Fig. 1 can be expressed by the following system of equations (Tusset *et al.*, 2016b):

$$\begin{cases} \ddot{x} + \mu\dot{x} + x + kx^3 + B_1\dot{\varphi}_1 = Ecos(\omega\tau) \\ \ddot{\varphi}_1 + (1/(b+1)a)\ddot{\varphi}_2\cos(\varphi_1-\varphi_2) + (1/(b+1)a)\dot{\varphi}_2^2\sin(\varphi_1-\varphi_2) + \omega_1^2\sin(\varphi_1) = \\ -\mu_1\dot{\varphi}_1 - (\mu_2/(b+1)a^2)(\dot{\varphi}_1-\dot{\varphi}_2) + B_2\dot{x} \\ \ddot{\varphi}_2 + a\ddot{\varphi}_1\cos(\varphi_1-\varphi_2) - a\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2) + (\omega_1^2 \mathrm{a})\sin(\varphi_2) = \mu_2(\dot{\varphi}_1-\dot{\varphi}_2) \end{cases} \quad (1)$$

The equation (1) can be rewritten in state space form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\mu\dot{x} - x - kx^3 - B_1\dot{\varphi}_1 + Ecos(\omega\tau) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -(1/(b+1)a)\ddot{\varphi}_2\cos(\varphi_1-\varphi_2) - (1/(b+1)a)\dot{\varphi}_2^2\sin(\varphi_1-\varphi_2) - \omega_1^2\sin(\varphi_1) \\ -\mu_1\dot{\varphi}_1 - (\mu_2/(b+1)a^2)(\dot{\varphi}_1-\dot{\varphi}_2) + B_2\dot{x} \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = -a\ddot{\varphi}_1\cos(\varphi_1-\varphi_2) + a\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2) - (\omega_1^2 \mathrm{a})\sin(\varphi_2) + \mu_2(\dot{\varphi}_1-\dot{\varphi}_2) \end{cases} \quad (2)$$

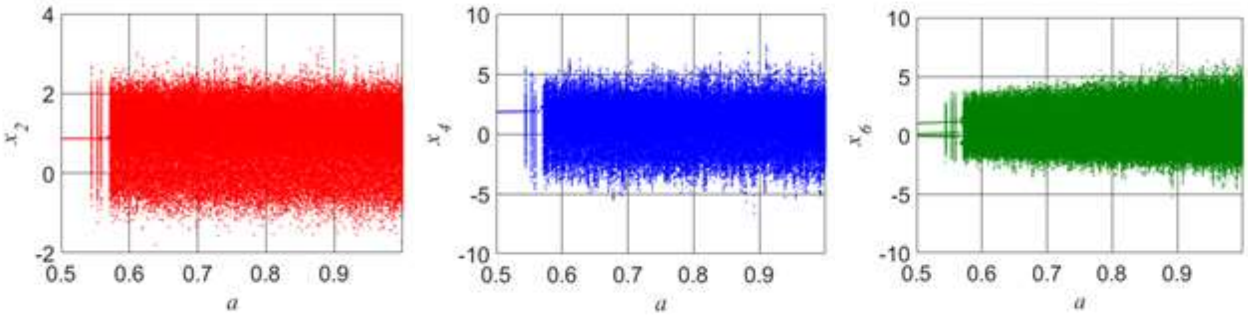
where: $x = q/Q_0$, $a = l_1/l_2$, $b = m_1/m_2$, $\mu = R/L\omega_e$, $k = a_3Q_0^2/L\omega_e^2$, $\omega_e^2 = 1/LC_0$, $\omega_2^2 = g/l_2\omega_e^2 = \omega^2 a$, $\omega_e^2 = \Omega/\omega_e$, $E = \upsilon_0/LQ_0\omega_e$, $\mu_1 = c_1/(m_1+m_2)l_1^2\,\omega_e$, $\mu_2 = c_2/m_2 l_2^2\,\omega_e$, $B_1 = nB\sigma^2 l^2/2LQ_0\omega_e$, $B_2 = nB\sigma^2 l^2 Q_0/2(m_1+m_2)l_1^2\omega_e$.

Being υ_0 and Ω the amplitude and frequency, respectively, C_0 is the linear value of *C*, a_3 is the nonlinear coefficient depending on the type of the capacitor used, c_1 and c_2 is damping coefficients, *B* is the magnetic field, σ is the permeability coefficient, *l* is the length of the conductor and *n* is the number of turns per unit length.

2.1 Numerical results

The equations of motion in dimensionless form (2) have been solved numerically for: E=1, b=0.5, $\mu = 0.02$, $\mu_1 = 0.01$, $\mu_2 = 0.01$, k=0.95, $\omega_1 = 1$, $\omega = 1$, $B_1=0.2$ and $B_2=0.4$ (Tusset et al., 2016b).

In Fig. 2 one can observe the Bifurcation diagram for the system (2) considering $0.5 \le (a = l_1/l_2) \le 1$.

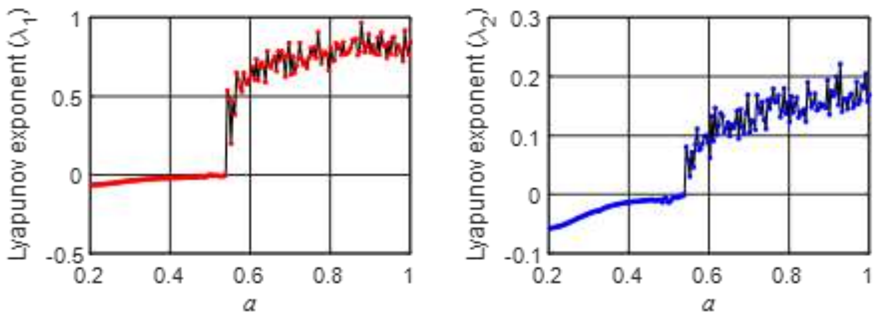


(a) (b) (c)

Figure 2. Bifurcation diagram. (a) Electrical system. (b) First link. (c) Second link.

Analyzing the results obtained in Fig. 2, it can be perceived that the system has a chaotic behavior for a range of values of "$a = l_1/l_2$", demonstrating that the length of the links directly interferes with the system dynamics.

In Fig. 2 it is presented the most significant Lyapunov exponent of the system (2) considering $0.5 \leq a \leq 1$.

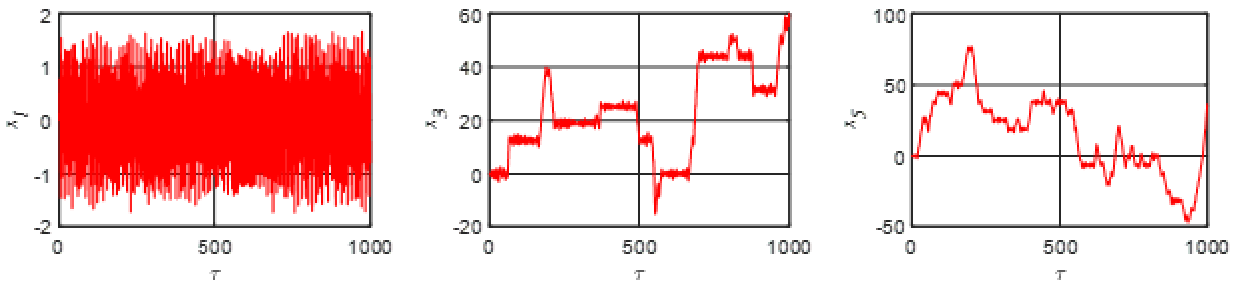


(a) (b)

Figure 3. Most significant Lyapunov exponent. (a) First exponent of Lyapunov. (b) Second exponent of Lyapunov.

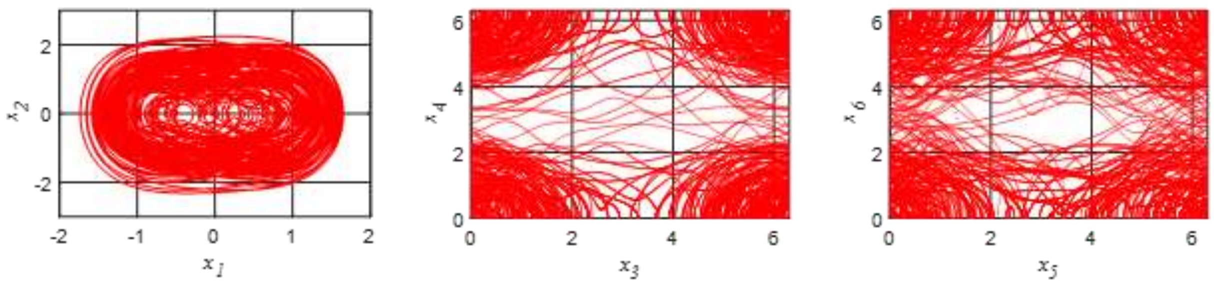
Analyzing the results presented in Fig. 3, it can be seen that the system shows chaotic behavior for a>0.55. As the system has two positive Lyapunov exponents for a>0.57, it can be said that the system have hyperchaotic behavior (Wu *et al.*, 2018).

In Fig. 4, 5 and 6 one can observe the behavior of the system (2) for a=0.91.



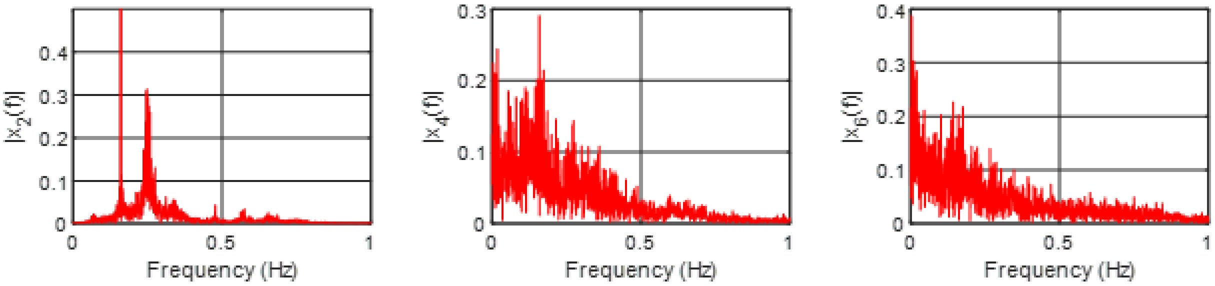
(a) (b) (c)

Figure 4. Time history of the states. (a) Electrical system. (b) First link. (c) Second link.



(a) (b) (c)

Figure 5. Phase portrait. (a) Phase diagram to x_1 versus x_2 . (b) Phase diagram to x_3 (from $x_3 \in[0,2\pi]$) versus x_4 (from $x_4 \in[0,2\pi]$). (c) Phase diagram to x_5 (from $x_5 \in[0,2\pi]$) versus x_6 (from $x_6 \in[0,2\pi]$).



(a) (b) (c)

Figure 6. Power Spectral Density (FFT). (a) Electrical system. (b) First link. (c) Second link.

As can be seen in the results presented in Fig. 4, 5 and 6, the system shows a chaotic behavior with variations in the motor voltage and movements of the links.

3. PROPOSED PID CONTROL

The designed control systems aim to eliminate the chaotic behavior of the system (2) for a=0.91, by introducing a control signal (*U*) to the system (2), equation (3) shows the proposed method.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x_2} = -\mu\dot{x} - x - kx^3 - B_1\dot{\varphi}_1 + E\cos(\omega\tau) + U_1 \\ \dot{x_3} = x_4 \\ \dot{x}_4 = -(1/(b+1)a)\ddot{\varphi}_2\cos(\varphi_1-\varphi_2) - (1/(b+1)a)\dot{\varphi}_2^2\sin(\varphi_1-\varphi_2) - \omega_1^2\sin(\varphi_1) \\ \quad -\mu_1\dot{\varphi}_1 - (\mu_2/(b+1)a^2)(\dot{\varphi}_1-\dot{\varphi}_2) + B_2\dot{x} + U_2 \\ \dot{x_5} = x_6 \\ \dot{x_6} = -a\ddot{\varphi}_1\cos(\varphi_1-\varphi_2) + a\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2) - (\omega_1^2 a)\sin(\varphi_2) + \mu_2(\dot{\varphi}_1-\dot{\varphi}_2) + U_3 \end{cases} \tag{3}$$

The PID controller operates according to the following equations:

$$\begin{cases} U_1 = k_{p_1}e_1 + k_{d_1}\dfrac{de_1}{d\tau} + k_{i_1}\displaystyle\int_0^\tau e_1 d\tau \\ U_2 = k_{p_2}e_2 + k_{d_2}\dfrac{de_2}{d\tau} + k_{i_1}\displaystyle\int_0^\tau e_2 d\tau \\ U_3 = k_{p_3}e_3 + k_{d_3}\dfrac{de_3}{d\tau} + k_{i_1}\displaystyle\int_0^\tau e_3 d\tau \end{cases} \tag{4}$$

where: $e_1 = (1.5\sin(\tau) - x_1)$, $e_2 = (\sin(\tau) - x_3)$, $e_3 = \left(\sin\left(\tau + \frac{\pi}{2}\right) - x_5\right)$, and $k_{p\_}$ is the proportional gain, $k_{i\_}$ is the corresponding integral gain and $k_{d\_}$ is the derivative gain of the control loop, respectively.

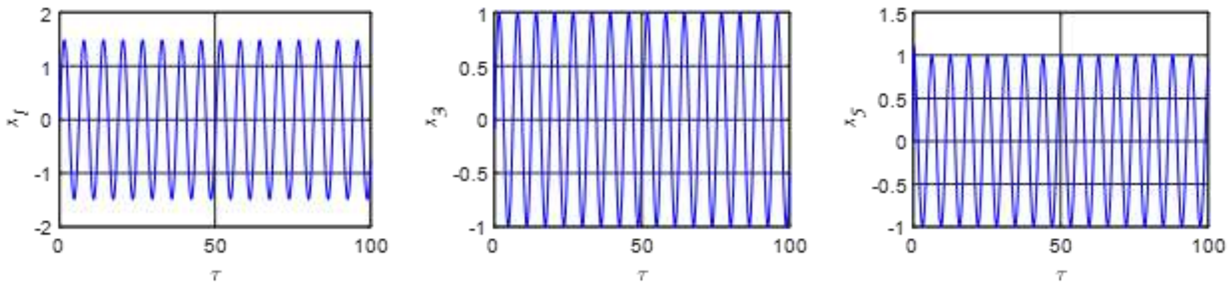
3.1 Case I: Control of the electrical system. First link torque control. Second link torque control

Considering the case in which it is possible to control the motor voltage and the torque of the two links, the proposed control (equation (4)) can be defined as follows (Tusset et *al.*, 2014):

$$\begin{cases} U_1 = 172.07e_1 + 30.35\dfrac{de_1}{d\tau} + 243.35\displaystyle\int_0^\tau e_1 d\tau \\ U_2 = 3698.05e_2 + 75.46\dfrac{de_2}{d\tau} + 44792.34\displaystyle\int_0^\tau e_2 d\tau \\ U_3 = 74.99e_3 + 17.00\dfrac{de_3}{d\tau} + 66.14\displaystyle\int_0^\tau e_3 d\tau \end{cases} \tag{5}$$

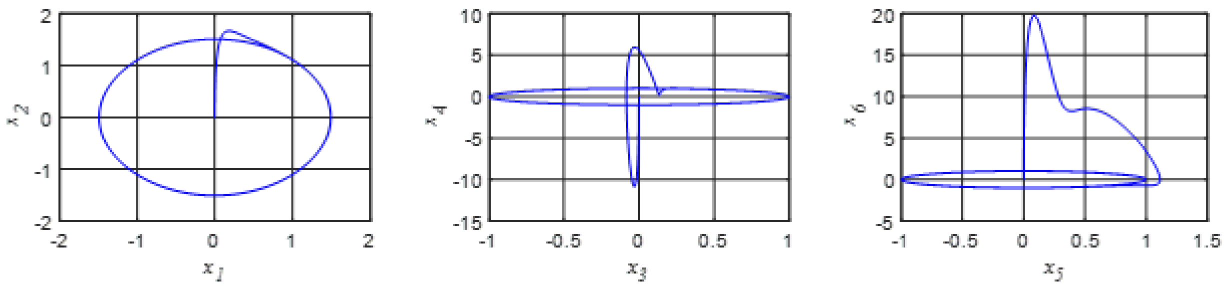
where gains are adjusted using Ziegler-Nichols method.

In Fig. 7 and 8 one can observe the behavior of the system (3) with the proposed control (equation (5)).



(a) (b) (c)

Figure 7. Time history of the states. (a) Electrical system. (b) First link. (c) Second link.



(a) (b) (c)

Figure 8. Phase portrait. (a) Phase diagram to x_1 versus x_2 . (b) Phase diagram to x_3 versus x_4. (c) Phase diagram to x_5 versus x_6 .

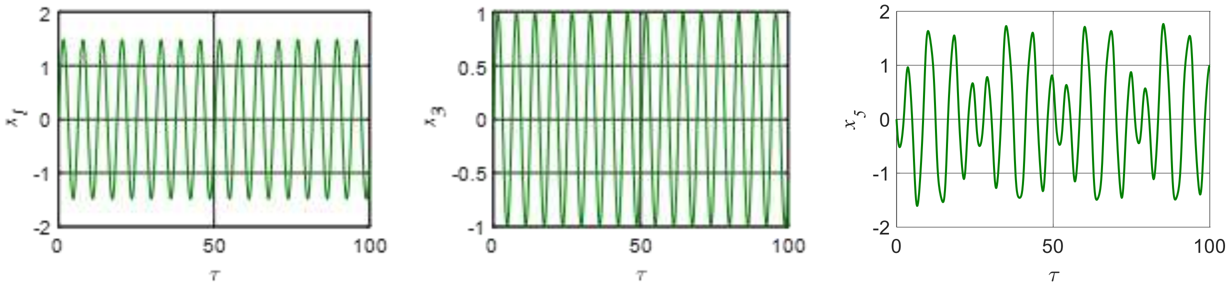
As can be seen from the results shown in Fig, 6 and 7, the PID proposed can efficiently control the movements of the system, considering the situation of acting in the electric motor and the two links.

3.2 Case II: Control of the electrical system. First link torque control. Second link without control

Considering now the case that it is only possible to directly control only the electric motor and the first link, the proposed control (equation (4)) can be changed to the following form:

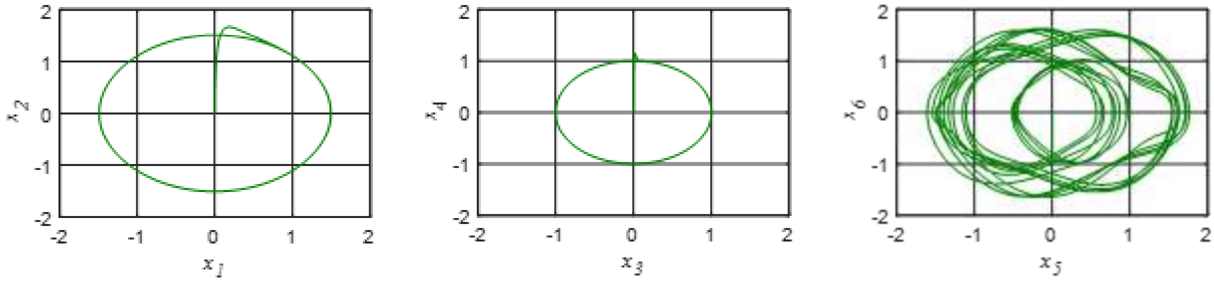
$$\begin{cases} U_1 = 172.07e_1 + 30.35\dfrac{de_1}{d\tau} + 243.35\displaystyle\int_0^\tau e_1 d\tau \\ U_2 = 3698.05e_2 + 75.46\dfrac{de_2}{d\tau} + 44792.34\displaystyle\int_0^\tau e_2 d\tau \\ U_3 = 0 \end{cases} \tag{6}$$

In Fig. 9 and 10 one can observe the behavior of the system (3) with the proposed control (equation (6)).



(a) (b) (c)

Figure 9. Time history of the states. (a) Electrical system. (b) First link. (c) Second link.



(a) (b) (c)

Figure 10. Phase portrait. (a) Phase diagram to x_1 versus x_2 . (b) Phase diagram to x_3 versus x_4. (c) Phase diagram to x_5 versus x_6 .

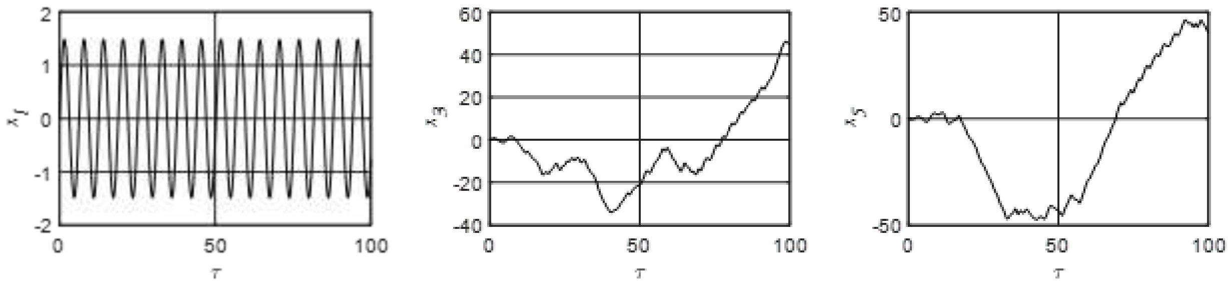
As can be seen in the results presented in Fig. 9 and 10, the control remains efficient in controlling the system in the situation of directly acting only in the electric motor and the first link. However, the fact that the second link is no directly controlled has influenced the final behavior of the first link.

3.3 Case III: Control of the electrical system. First link without control. Second link without control

Considering the case that it is only possible to include control in the electric motor, the proposed control (equation (4)) can be show in following form:

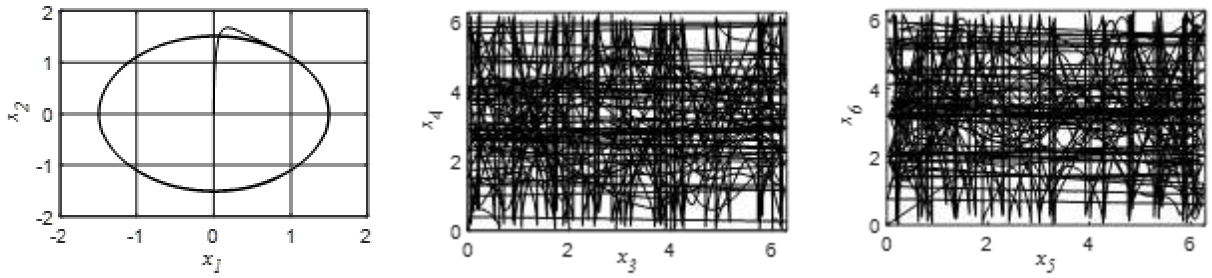
$$\begin{cases} U_1 = 172.07e_1 + 30.35\dfrac{de_1}{d\tau} + 243.35\displaystyle\int_0^\tau e_1 d\tau \\ U_2 = 0 \\ U_3 = 0 \end{cases} \tag{7}$$

In Fig. 11 and 12 one can observe the behavior of the system (3) with the proposed control (equation (7)).



(a) (b) (c)

Figure 11. Time history of the states. (a) Electrical system. (b) First link. (c) Second link.



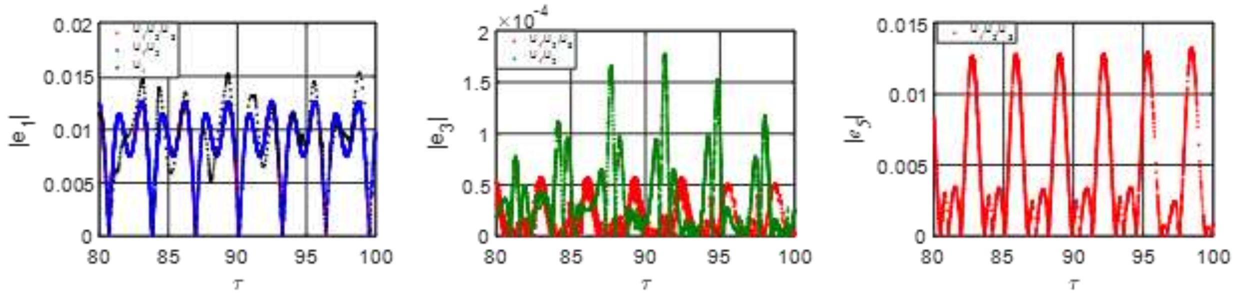
(a) (b) (c)

Figure 12. Phase portrait. (a) Phase diagram to x_1 versus x_2 . (b) Phase diagram to x_3 (from x_3 ∈[0,2π]) versus x_4 (from x_4 ∈[0,2π]). (c) Phase diagram to x_5 (from x_5 ∈[0,2π]) versus x_6 (from x_6 ∈[0,2π]).

As can be seen in the results presented in Fig. 11 and 12, the control remains efficient in controlling the electric motor. However, the fact that the two links are moving freely turned the behavior of the system in a chaotic one, even if the electrical system has periodic behavior.

3.4 Analysis of proposed control errors

In Fig. 13, the absolute errors for the three cases considered are presented.



(a) (b) (c)

Figure 13. Phase portrait. (a) Electrical system. (b) First link. (c) Second link.

In Fig. 13a, the error for the electrical system (x_1) is presented. As can be seen, the error increases when the system is connected to the motor and does not have the control signal (U_2), and it remains practically the same when the system directly controls both links (x_3 and x_5), or only has control in the first link (x_3). This behavior clearly demonstrates that the system is of a non-ideal type.

Analyzing Fig. 13b, it is perceived that the movement of the second link (x_5) interferes with the movement of the first link (x_3). If the second link is in free movement, the error of the first link increases in relation to the desired one ($\sin(\tau)$).

4. CONCLUSIONS

This paper presented the dynamic interactions of a double pendulum arm and an electromechanical shaker by means of phase portraits, bifurcation diagrams, the power spectrum (FFT) and Lyapunov exponents, showing the existence of chaotic and hyperchaotic behavior for variations in the length of the links ($a = l_1/l_2$).

In applications where it is desirable to eliminate the chaotic behavior of the system, we proposed a PID control. The effectiveness of the proposed control is demonstrated through numerical simulations in order to eliminate the chaotic behavior of the system.

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