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INFLUENCE OF THE FLUID FREE SURFACE ON THE NONLINEAR RESONANCE CURVES OF A FLEXIBLE TANK PARTIALLY FILLED WITH FLUID

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Abstract. *A flexible partially fluid-filled tank is analyzed considering the tank as a simply supported cylindrical shell described by the Donnell's nonlinear shallow-shell theory. The perturbation method is used to choose an appropriate displacement field to discretize the equilibrium equations. An external harmonic load is applied in the transversal direction with the shape of the first natural mode of vibration. The internal fluid, initially considered inviscid and irrotational, can be described by a potential flow and in a linear analysis, the velocity potential can be divided in two parts. The first one is responsible to describe the movement of the fluid body, that follows the velocity of the flexible tank and implies a velocity boundary condition that forces the fluid and the structure to have the same velocity in their interface. The second velocity potential contribution represents the free surface vibration, also known as sloshing. But in this situation, the tank is rigid, and this parcel must be added to the first one. By the Euler's equilibrium equation, a hydrodynamic pressure is derived from the velocity potentials and applied in the tank walls. The same equation is used in the free surface with aim to guarantee the equilibrium in this region, called as sloshing equation, where a damping term is added using the Rayleigh dissipation function. With the coupled system composed by the tank nonlinear equilibrium equation with an applied hydrodynamic pressure and the sloshing equation, the Garlerkin method is used and, at last, the brute force method is employed to obtain the nonlinear resonance curves. Time responses, phase-portraits and Poincaré section are also obtained for different regions of the resonance curves, showing the nonlinear behavior of the coupled fluid-structure problem.*

Keywords: *sloshing, tank, partially fluid-filled, nonlinear dynamics, sloshing damping.*

1. INTRODUCTION

The storage capability is one of the most common and important uses of structures in engineering and it can be seen in so many different areas such as industrial and offshore structures, as a part of a water distribution system or in aerospace engineering, for example. It can also be noticed that the storage capability is often a fluid storage, and because of this type of application, the fluid-structure interaction is a current situation in engineering structures. Geometrically, it is still possible to attach the use of cylindrical tanks to the storage structures, because of their suitable shape, ideal to avoid losses of material and, at same time, being such lightweight structural elements.

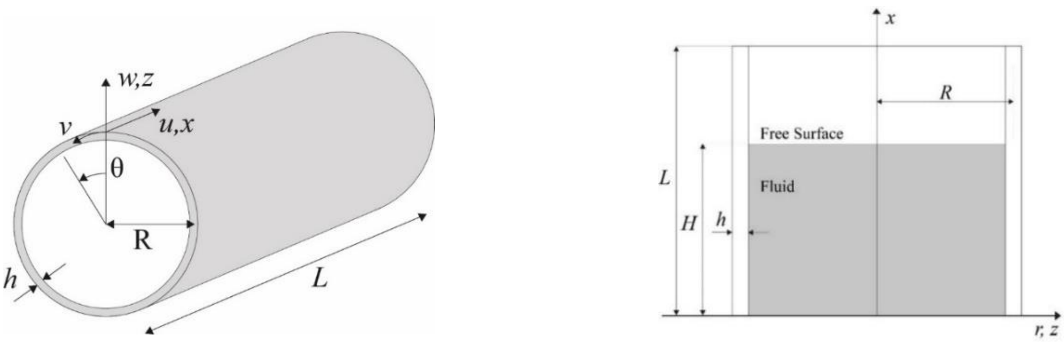
There are many studies in the literature that approach this fluid-structure interaction problem (Amabili and Païdoussis, 2003; Alijani and Amabili, 2014), but the use of low-order systems by some authors is a particularly interesting choice due the reduced number of degrees of freedom, which is important in a study that treats a volume dominium. The Galerkin and Rayleigh-Ritz methods are examples of techniques to obtain reduced order equations and they are employed in Gonçalves and Batista (1987), who studied the natural frequencies of the tank varying the fluid level, using a velocity potential to couple the fluid to the cylindrical tank. Amabili (1997) also used the velocity potential to consider the presence of the fluid partially filling the tank to obtain the equilibrium equations by Rayleigh-Ritz method. The analysis of the nonlinear forced vibrations are also performed using low dimensional models, as can be seen in the work of Pellicano and Amabili (2006), where the stability of a partially fluid filled tank submitted to an axial harmonic load is investigated. Gonçalves, Silva and del Prado (2006) used the Galerkin method to treat the definition of the instability boundaries of a fluid filled tank submitted to harmonic excitations.

There is another phenomenon involving partially filled tanks named as sloshing that is the vibration of the free surface of the liquid. Although, researchers have been developing formulations to take sloshing into account, also using the potential velocity, as can be noted in Kim, Lee and Ko (2004), where both tank and sloshing natural frequencies were obtained using the Rayleigh-Ritz method. In forced dynamic analysis, it is also necessary to consider the structure and sloshing damping. The damping of structures can be estimated by the Rayleigh dissipation function (Amabili, 2008) while the fluid damping has a more complex behavior for its energy dissipation. The reasons of them are the friction between the fluid and the tank walls and bottom, not only the fluid viscosity that has a small influence on the total damping factor. The vibration mode, determined by the tank oscillations, has a strong influence in how the fluid will be damped, as showed by Case and Parkinson (1957), and experimentally confirmed by Mieda et al (1993), where the dependence of the vibration modes is considered.

From literature review, there are some works that consider the coupling between an internal fluid and the tank structure, investigating the free and forced vibrations. On the other hand, when the sloshing contributions are considered place, few works detail the dynamical behavior of fluid-srtucture interaction in the framework of reduced order models and sloshing damping considerations. Therefore, in this work, an isotropic tank partially filled with fluid will be studied, while the sloshing phenomenon is also considered. The tank will be analyzed as a cylindrical shell described by the Donnell's nonlinear shallow shell theory and the fluid will be formulated with linear velocity potentials composed by two terms: one describing the fluid body movement following the tank walls without sloshing and the other describing the sloshing while the tank remains rigid. The damping factors will be calculated with the formulation presented by Case and Parkinson (1957), that will appear in the sloshing equilibrium equation. The Galerkin method is applied in the tank equations and in the sloshing equations, obtaining a discrete ordinary differential equations system that describes the fluid-structure interaction problem. Natural frequencies will be obtained for the validation of the present formulation, and, after that, a nonlinear dynamic analysis takes place, where the frequency-amplitude relations, resonance curves and phase-portraits will be developed, showing the complex behavior of the tank when the sloshing is considered.

2. PROBLEM FORMULATION

A thin-walled cylindrical shell is used as the structural element that represents the tank, made by a linear elastic material with Young's modulus E, Poisson's ratio v and density ρ. The geometry, showed in Figure 1 (a), is defined by a radius R, length L and thickness h for the tank partially fluid-filled with a liquid level H, that is also the free surface position, at Figure 1 (b). The coordinate system is the cylindric coordinates composed by the x, θ and z axes with respective displacements u, v and w.



(a) (b)

Figure 1. (a) Cylindrical shell and geometric parameters and (b) partially fluid-filled tank

The mechanical behavior of the cylindrical shell is described by the Donnell's nonlinear shallow shell theory (Donnell and Ohio, 1934), where the transversal equilibrium equation and the compatibility and continuity equation are given by:

$$\rho h\,\ddot{w}+D\nabla^4 w+\beta_1\dot{w}+R\,f_{,xx}-\left(f_{,\theta\theta}w_{,xx}-2f_{,x\theta}w_{,x\theta}+f_{,xx}w_{,\theta\theta}\right)-p_H+p_L=0\,, \tag{1}$$

$$\nabla^4 f=\frac{Eh}{R^4}\left(w_{,x\theta}{}^2-w_{,xx}w_{,\theta\theta}+R\,w_{,xx}\right) \tag{2}$$

where w is the transversal displacement field, f is Airy's stress function, β_1 is the linear viscous damping coefficient, p_H is the hydrodynamic pressure applied by the internal fluid in the structure walls, p_L is an external time-dependent pressure and D (=Eh^3/[12(1- v^2)]) is the flexural rigidity. To discretize Eq. (1), the Airy's stress function, Eq. (2), is solved analytically for a chosen modal solution of transversal field displacement, w. Afterward, the Galerkin method must be applied in the partial differential equation of tank, Eq. (1), to obtain a set of second order ordinary differential equations in time.

The internal fluid is inviscid, incompressible and irrotational, so its behavior can be described by a potential flow, which is analyzed based on velocity potentials noted by φ in way that it is composed by two different contributions. The first one, φ_1, represents the fluid body participation while the tank is flexible and the second one, φ_2, is related only with the sloshing while the tank remains completely rigid, and as a result $\varphi = \varphi_1 + \varphi_2$. Furthermore, the fluid flow has always small amplitudes, thus the problem is studied based on linear equations. The first of them is the Bernoulli's equation, from where the hydrodynamic pressure is dependent of the fluid density, ρ_f, as:

$$p_H = -\rho_f \left[\dot{\varphi}\right]\Big|_{r=R} . \tag{3}$$

The sloshing equilibrium equation must also be determined, and it is done with the Euler equation applied in the position of free surface, $x = H$, that can be written in terms of the velocity potential,

$$\rho_f \, \ddot{\varphi}_2 + \rho_f \, g\left(\varphi_1 + \varphi_2\right)_{,x} - \beta_3 \, \dot{\varphi} = 0 , \tag{4}$$

where it was also added the participation of the dissipative forces representing the friction between the fluid and the tank, produced by a layered shear flow as well as a contribution of dissipation that occurs due the fluid viscosity. It was made using the same process applied in the cylindrical shell equation, by adapting the Rayleigh dissipation function with β_3 representing the fluid damping coefficient.

It is still necessary to estimate the value of the fluid damping coefficient β_3, and it was made analytically by Case and Parkinson (1957), from a φ modal approximation chosen for a simple supported tank, that was applied in a dissipation function defined by Lamb (1945). Therefore, the viscosity contribution from the free surface α_s, the contributions from the dissipation produced by the shear flow between the fluid and the tank bottom α_B and the lateral shear flow contribution α_L are presented as following:

$$\alpha_s = 2\eta\lambda_{mn}^2 , \; \alpha_B = \frac{1}{2R}\sqrt{\frac{\nu\omega_{mn}}{2}}\,\frac{2\lambda_{mn}R}{\sinh(2\lambda_{mn}h)} , \; \alpha_L = \frac{1}{2R}\sqrt{\frac{\nu\omega_{mn}}{2}}\left[\frac{1+\left(n/\lambda_{mn}R\right)^2}{1-\left(n/\lambda_{mn}R\right)^2} - \frac{2\lambda_{mn}h}{\sinh(2\lambda_{mn}h)}\right] , \tag{5}$$

where η is the fluid kinematic viscosity, ω_{mn} is the sloshing natural frequency given by:

$$\omega_{mn}^2 = g\lambda_{mn}\tanh(\lambda_{mn}h) , \tag{6}$$

and λ_{mn} are the roots of the Eq. (7), where m and n represent the longitudinal half-wave number and the circumferential wave number, respectively.

$$J_n'\left(\lambda_{mn}R\right) = 0 . \tag{7}$$

Finally, the sloshing damping coefficient β_3 can be defined as:

$$\beta_3 = \alpha_B + \alpha_L + \alpha_s . \tag{8}$$

Lastly, each part of the velocity potential must be chosen according to their proper boundary conditions. The part associated to the flexible tank and the fluid body, φ_1, must be zero at the free surface ($x = H$) and the fluid velocity must be zero at the bottom ($x = 0$). The shell velocity needs to be the same as the fluid velocity at the interface ($r = R$). These boundary conditions can be expressed as:

$$\varphi_1\big|_{x=H} = 0 , \; \left[\varphi_{1,x}\right]_{x=0} = 0 , \; \left[\varphi_{1,r}\right]_{r=R} = \dot{w} . \tag{9}$$

For the velocity potential that describes the sloshing, φ_2, the tank bottom and the tank walls must be rigid that implies:

$$\left[\varphi_{2,x}\right]_{x=0} = 0 , \; \left[\varphi_{2,r}\right]_{r=R} = 0 . \tag{10}$$

From these previous criterions, the approximation chosen for φ_1 is presented in Eq. (11) (Kim *et al*., 2004).

$$\varphi_1 = \sum_{i=1}^{M}\sum_{j=1}^{\bar{M}} A_{ij}(\mathrm{t})\cos\left(\frac{(2j-1)\pi x}{2H}\right) I_n\left(\frac{(2j-1)\pi r}{2H}\right) f_i(\theta) , \tag{11}$$

where A_{ij} is the velocity potential amplitude, determined by the boundary conditions, I_n is the modified Bessel function of first class and order n and $f_i(\theta)$ is the harmonic function of θ present on the mode i of the modal solution for transversal displacement field. The discretization is chosen based on the number M of modes used to describe w and in the proper φ_1 discretization $\bar{M}$.

The velocity potential adopted for φ_2 is given by in Eq. (12) (Kim *et al*., 2004):

$$\varphi_2 = \sum_{i=1}^{M\theta}\sum_{k=1}^{K} B_{ik}(\mathrm{t})\,\omega h \cosh\left(\frac{\varepsilon_{ik}x}{R}\right) J_n\left(\frac{\varepsilon_{ik}r}{R}\right) f_i(\theta),\;\; \varphi_{20} = \sum_{k=1}^{K} B_{0k}(\mathrm{t})\,\omega h \cosh\left(\frac{\varepsilon_{0k}x}{R}\right) J_0\left(\frac{\varepsilon_{0k}r}{R}\right), \quad (12)$$

with φ_2 representing the parcels for asymmetric modes ($n > 0$), present in the modal solution of the transversal field displacement, while φ_{20} the parcels for the axisymmetric modes ($n = 0$) of the modal solution of w. J_n is the Bessel function of the first kind and order n, $M\theta$ is the number of different functions $f_i(\theta)$ used in w. The arguments of the Bessel function, ε_{ik}, are determined using the boundary condition of the rigid tank, also applying the Galerkin method with the Bessel function being the weight function.

At last, with the velocity potentials well defined, the Galerkin method must be applied in the sloshing equation, Eq. (4), using the respective Bessel function, according to the φ_2, as weight function to transform the partial differential equations in ordinary differential equations in time, that will be solved coupled with the tank equations containing the hydrodynamic pressure.

3. NUMERIC RESULTS

To discretize the partial differential equilibrium equation of the tank, Eq. (1), firstly, it is necessary to consider an accurate modal solution for transversal displacement field, w, that it can consider all the modal coupling present in the nonlinear quadratic and cubic terms in the terms of Eq. (1). From the perturbation method (Gonçalves, 1987; Silva *et al.* 2008), w is defined as:

$$\begin{aligned}
w &= W_1(t)h\sin\left(\frac{m\pi x}{L}\right)\cos(n\theta) + W_1^c(t)h\sin\left(\frac{m\pi x}{L}\right)\sin(n\theta) \\
&+ W_2(t)h\sin\left(\frac{m\pi x}{L}\right)\cos(3n\theta) + W_2^c h\sin\left(\frac{m\pi x}{L}\right)\sin(3n\theta) \\
&+ W_3(t)h\sin\left(\frac{3m\pi x}{L}\right)\cos(n\theta) + W_3^c h\sin\left(\frac{3m\pi x}{L}\right)\sin(n\theta) \\
&+ W_4(t)h\sin\left(\frac{3m\pi x}{L}\right)\cos(3n\theta) + W_4^c h\sin\left(\frac{3m\pi x}{L}\right)\sin(3n\theta) \\
&+ W_5(t)h\left[\frac{3}{4} - \cos\left(\frac{2m\pi x}{L}\right) + \frac{1}{4}\cos\left(\frac{4m\pi x}{L}\right)\right] \\
&+ W_6(t)h\left[\frac{3}{4} - \cos\left(\frac{2m\pi x}{L}\right) + \frac{1}{4}\cos\left(\frac{4m\pi x}{L}\right)\right]\cos(2n\theta) + W_6^c h\left[\frac{3}{4} - \cos\left(\frac{2m\pi x}{L}\right) + \frac{1}{4}\cos\left(\frac{4m\pi x}{L}\right)\right]\sin(2n\theta)
\end{aligned}, \quad (13)$$

where $W_i(t)$ and $W_i^C(t)$ (i=1...6) are, respectively, the modal amplitudes of driven and companion modes. This modal solution can describe the vibration of cylindrical shell up to twice shell thickness (Gonçalves *et al.*, 2008).

Table 1. Validation of the natural frequencies for bulging and sloshing modes (rad/s) (n = 4).

Mode	ω_1			ω_2		
m	**Kim *et al.* (2004)**	**Present**	**Dif. (%)**	**Kim *et al.* (2004)**	**Present**	**Dif. (%)**
1	14.054	14.396	2.43	1.4427	1.4398	0.20
2	34.672	36.620	5.62	1.9085	1.9081	0.02
3	49.629	48.906	1.46	2.2308	2.2306	0.01
4	61.556	59.224	3.79	2.5029	2.5002	0.11
5	71.476	69.360	2.96	2.7445	2.7613	0.61

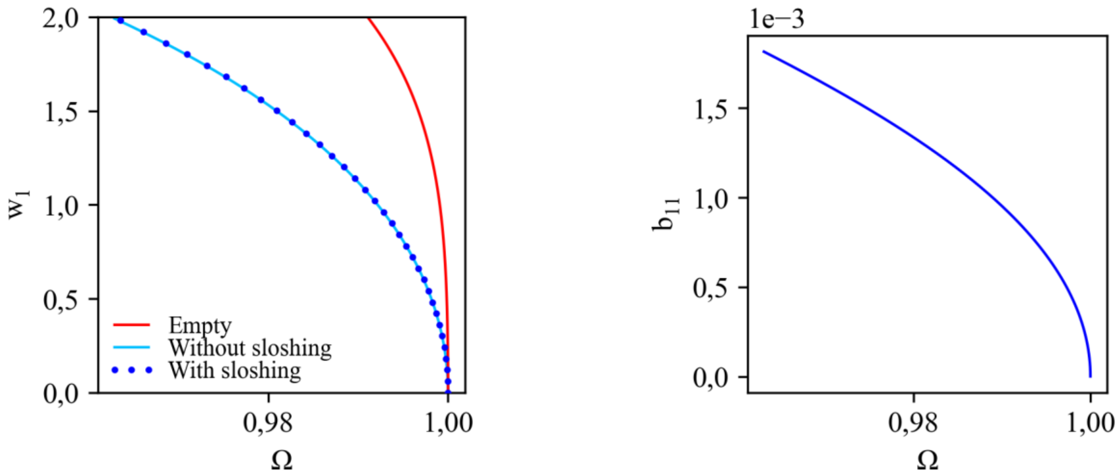
The natural frequencies of the tank and the sloshing are compared with the values presented by Kim *et al.* (2004), considering the simply supported boundary condition. To obtain these results, only fundamental vibration mode, $W_1(t)$ ($=w_1\cos(\omega t)$), of modal solution, Eq. (13), is considered, where ω is the natural frequency. The problem is defined by a tank with L= 30 m, R = 25 m, h = 0.03 m, with a linear isotropic material E = 206 GPa, ν = 0.3 and ρ = 7850 kg/m^3. The internal fluid has an internal level H= 21.6 m and ρ_f = 1000 kg/m^3 and its discretization considers $\bar{M}$ = 30 and K = 1. Then, the frequency values are presented in Table 1 where ω_1 is the frequency of the tank while ω_2 is frequency of the sloshing. The largest difference observed is 5.62% in the bulging mode (natural frequency of tank) results and 0.61% in the sloshing results. Thus, it is possible to consider that the present formulation is consistent.

For this considered geometry, the tank natural frequency is ω_1 = 4.960 rad/s and the sloshing frequency ω_2 = 2.229 rad/s. For comparison purposes, in the case of an empty tank (φ_1=φ_2=0), the natural frequency is equal to 14.532 rad/s. In other hand, for the partially fluid filled tank, without the sloshing consideration (φ_2=0), the natural frequency is 4.927 rad/s. About the natural vibration mode of the tank, it always occurs for the mode (m, n) = (1, 11).

The next step of this work is the nonlinear analysis that begins with the analysis of the backbone curves. For this, only driven modes at Eq. (13) (W_i^C(t)=0, i=1...6) were considered with $\bar{M}$ = 30 and K = 5 to discretize the velocity potentials of the fluid. Then, the Galerkin-Urabe method is applied, and, for this, it is necessary the time discretization of modal amplitudes of driven modes. Obeying the order of time function given by the perturbation method (Gonçalves, 1987), the driven modes are discretized in time as:

$$W_1(t) = W_2(t) = w_i \cos(\omega t), \qquad W_3(t) = W_4(t) = w_i \cos^3(\omega t), \qquad W_5(t) = W_6(t) = w_i \cos^2(\omega t), \\ B_{ij} = b_{ij} \sin(\omega t). \tag{14}$$

The backbone curves are obtained considering three situations: empty tank (red curve), partially fluid filled tank without the sloshing consideration (light blue curve) and partially fluid filled tank with the sloshing consideration (blue dots). The results are presented in Figure 2, where Ω is the adimensional frequency parameter referent to the natural frequency of each situation. In Figure 2 (a) the backbone curve of the fundamental mode is presented and it can be seen a softening behavior for all situations that was intensified when the fluid was present. The curves considering, or not considering, the sloshing effects are overlapping. This is not to say that the sloshing mode will not influence the forced response of the cylindrical tank, as will be shown later. The sloshing amplitude, showed in Figure 2 (b), follows the same behavior of the tank amplitude with a softening behavior.



(a) Amplitude of fundamental mode

(b) Amplitude of sloshing mode correlated with fundamental mode

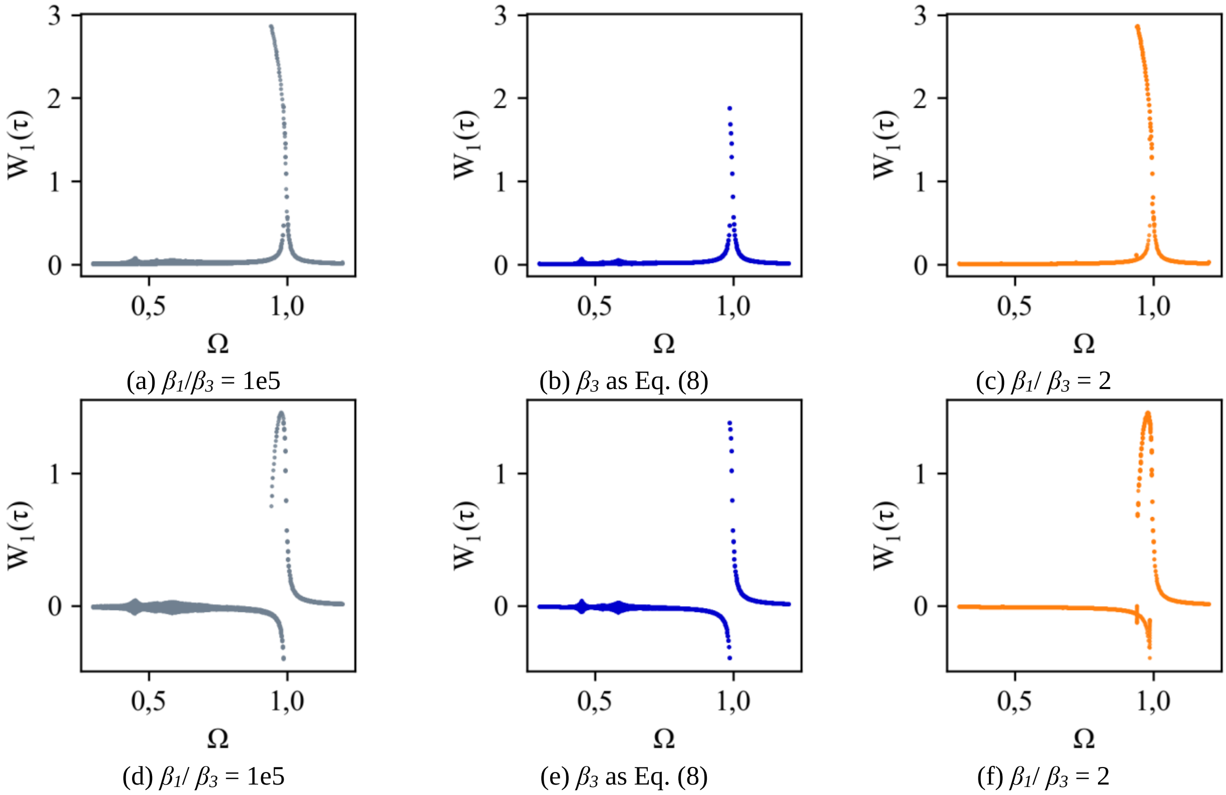
Figure 2. Frequency-Amplitude Relations for the empty and partially fluid-filled tank with and without sloshing.

For nonlinear forced vibration, the companion modes have influence on the tank behavior (Amabili, 2008), so the modal solution for transversal field displacement, w, Eq. (13), is considered. The damping used for the tank equation, Eq. (1), is given by $\beta_1 = 2\eta_1\rho h\omega_0$ where η_1 = 0.005 and ω_0 is as the lowest tank natural frequency. The external lateral pressure $p_L(t)$ has the shape of the fundamental mode and it is given by:

$$p_L(t) = P_L \sin\left(\frac{m\pi x}{L}\right)\cos(n\theta)\cos(\omega_f\, t), \tag{15}$$

where ω_f is the load excitation frequency, P_L is the force amplitude, considered as 10 N/m^2.

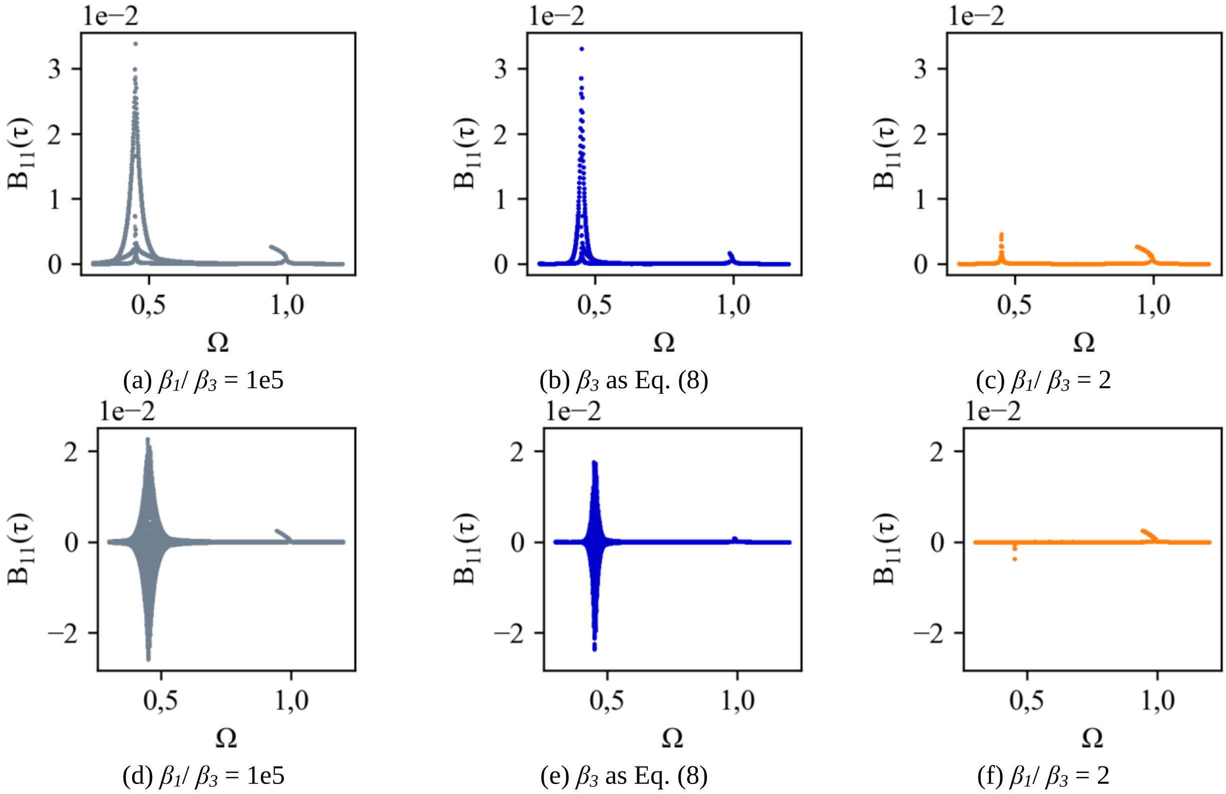
The sloshing damping remains yet to be defined previously in Eq. (8). Three different cases of damping are considered to analyses their effects on the dynamic behavior of the coupled system especially on the resonance curves and phase-portraits.



(a) β_1/β_3 = 1e5 (b) β_3 as Eq. (8) (c) $\beta_1/\ \beta_3$ = 2

(d) $\beta_1/\ \beta_3$ = 1e5 (e) β_3 as Eq. (8) (f) $\beta_1/\ \beta_3$ = 2

Figure 3. Resonance curves (a, b, c) and bifurcation diagrams (d, e, f) for $W_1(\tau)$ for each case of sloshing damping factor.



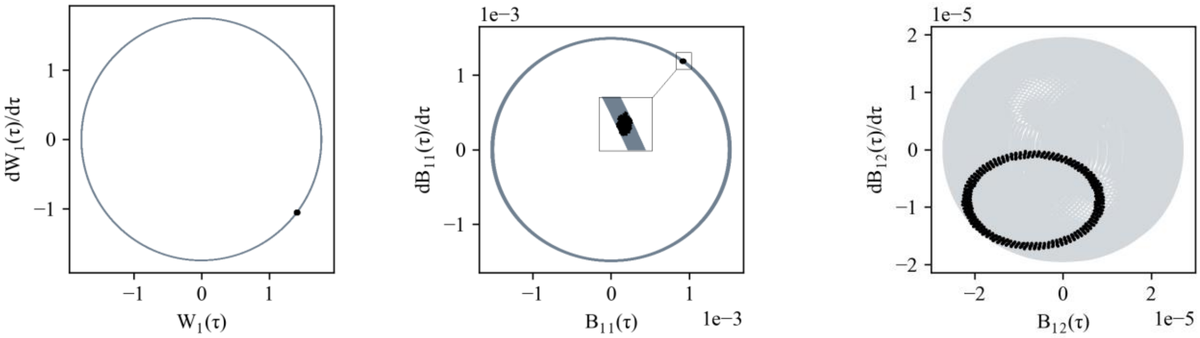
(a) $\beta_1/\ \beta_3$ = 1e5 (b) β_3 as Eq. (8) (c) $\beta_1/\ \beta_3$ = 2

(d) $\beta_1/\ \beta_3$ = 1e5 (e) β_3 as Eq. (8) (f) $\beta_1/\ \beta_3$ = 2

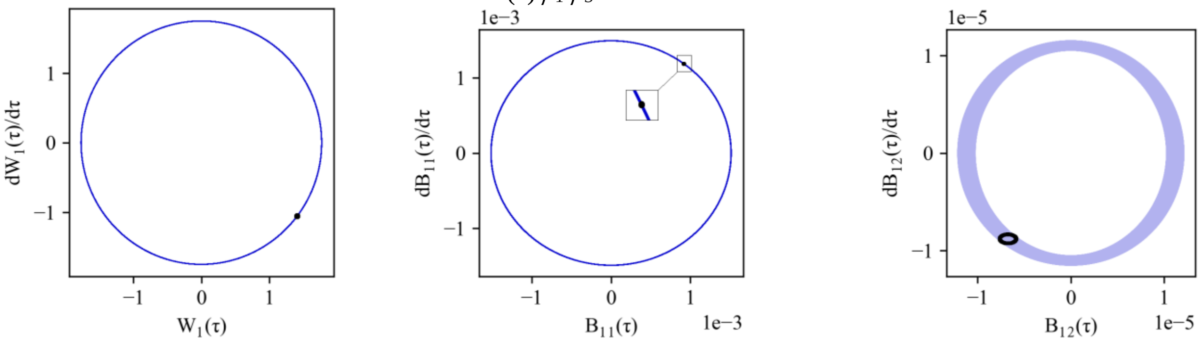
Figure 4. Bifurcation Diagram for $B_{11}(\tau)$ for each case of sloshing damping factor

Then, the first case comes from the application of the formulation previously presented, with the sloshing damping defined as β_3 in Eq. (8) and represented by blue curves in the resonance and bifurcation diagrams in Figures 3 and 4. It is important to notice that, in this case, each sloshing mode will have a different damping value depending on the vibration mode (m, n). In the second case, the fluid damping model has a single damping factor β_3 for all sloshing modes with a value smaller than all the coefficients used at the first case, characterized by the relation β_1/β_3 = 1e5, being represented in the resonance and bifurcation diagrams by gray curves in Figures 3 and 4. At last, a sloshing damping factor closer to the tank damping coefficient β_1 is applied, defined as β_1/β_3 = 2, and noted in the resonance and bifurcation diagrams by orange curves in Figures 3 and 4.

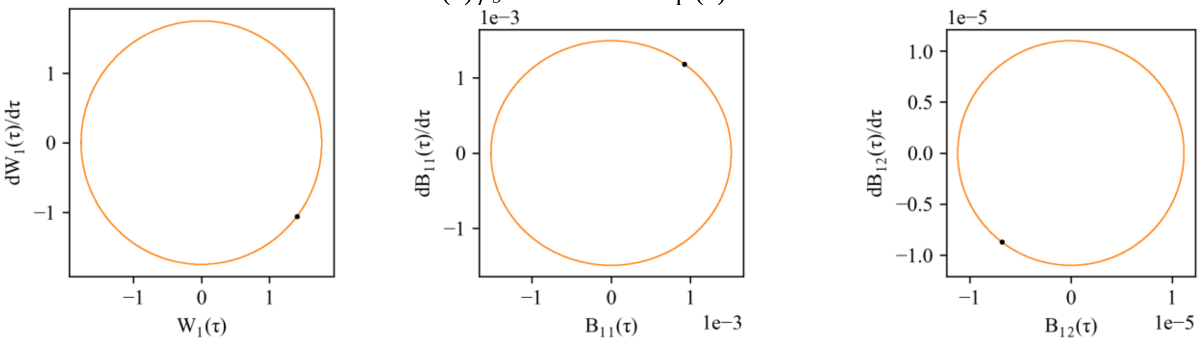
The tank resonance curves for the fundamental vibration mode are presented in Figures 3 (a) - (c) and the bifurcation diagrams for the same modal amplitude are showed in Figures 3 (d) - (f). For the sloshing amplitude $B_{11}(\tau)$, where it is correlated with the fundamental vibration mode of w, the resonance curves are presented in Figures 4 (a) - (c) and its bifurcation diagrams are in Figures 4 (d) - (f). From Figure 3, the region near to the sloshing natural frequency, $\Omega = 0.449$, there is a slightly peak in the resonance curves and bifurcation diagrams, which is more intensive for the damping sloshing cases with β_1/β_3 = 1e5 or β_3 calculated as given by Eq. (8). At this same region, the sloshing resonance curves, in Figure 4, show a significant grow of the sloshing amplitudes and multiple responses appearing in the bifurcation diagrams for the cases of Figures 4 (a) and (b). In the third sloshing damping case, β_1/β_3=2, does not present a new peak near to the sloshing natural frequency in the tank resonance curve, as noted in Figures 3 (c) and (f).



(a) β_1/β_3 = 1e5



(b) β_3 calculated as Eq. (8)



(c) β_1/β_3 = 2

Figure 5. Phase-portraits and Poincaré sections for Ω = 0.9 and analytical β_1/β_3 = 2

Resonances curves and bifurcation diagrams shows the relevance of the damping factor of sloshing on the dynamic behavior of the tank. When smaller values of β_3 were considered, as shown in cases (a), (b), (d) and (e) of Figures 3 and 4, the sloshing amplitudes resonance cause a perturbation in resonance curve and bifurcation diagram of the fundamental vibration mode, while a higher value of β_3, as considered in case (c) and (f) of Figures 3 and 4, provoke practically the total damping of the sloshing amplitudes in the permanent response of the problem, eliminating the sloshing influence on the tank behavior. Comparing the cases 1 (β_1/β_3 = 1e5) and 2 (β_3 calculated as given by Eq. (8)), there is no significant changes in the characteristics of the tank and sloshing behavior, meaning that the use a different damping factor for each sloshing mode is not more important than use the correct order of magnitude of the damping coefficient.

Figure 5 presents the phase-portraits and Poincaré sections for the fundamental vibration mode of *w* and two sloshing modes, correlated with fundamental vibration mode, to the adimensional frequency Ω = 0.9. Figure 5 (a) shows the results for the case β_1/β_3 = 1e5 where it can be seen a periodic behavior for the tank amplitude while the sloshing modes, B_{11} and B_{12}, have a chaotic behavior as can be noted by Poincaré's section. For the second case, Figure 5 (b), referent to β_3 obtained from Eq. (8), the fundamental vibration mode has the same behavior of Figure 5 (a) with a periodic phase-portrait, but the sloshing mode B_{11} has a significant change in its behavior, presenting now a periodic behavior. In other hand, B_{12} becomes quasi-periodic in Figure 5 (b). The last case, β_1/β_3 = 2, Figure 5 (c), shows a periodic behavior for all considered amplitudes, showing that the damping choice is especially important to the behavior of the results because the consideration of a high damping can make the sloshing effects disappear or make them show a behavior that is not the real one, also influencing the coupling that exists between tank and fluid.

4. CONCLUSION

This work treated the fluid-structure interaction in a tank partially filled with fluid, considering the vibration of the free surface to analyze the influence of the sloshing damping value in the dynamic response of the structure and the fluid. The resonance curves, and its relative bifurcation diagrams, phase-portraits and Poincanré sections were obtained for the following three damping cases: the fluid damping very smaller than the tank damping; the analytical estimative of the sloshing damping from a modal formulation; and the sloshing damping values with a close proportion of the tank damping. The sloshing damping has a strong influence in the coupling between the tank structure and the sloshing oscillation. It happens because a quasi-periodic vibration in the sloshing amplitude can provoke changes in the phase-portraits of the tank amplitudes and, at the same time, cause perturbations in the resonance curves and bifurcation diagrams. It was also possible to note that not all the sloshing modes will have the same damping coefficient and the correct attribution for each mode can also have qualitative influences in the sloshing amplitude dynamic response.

5. ACKNOWLEDGEMENTS

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