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AN EFFICIENT UNSTEADY TRANSIENT FRICTION MODELLING FOR LIQUID-FILLED PIPE FLOWS

Douglas Monteiro Andrade
Felipe Bastos de Freitas Rachid
Laboratory of Theoretical and Applied Mechanics – Department of Mechanical Engineering – Universidade Federal Fluminense, Rua Passo da Pátria, 156 – 246210-240 Niterói, RJ – Brazil
dmajnj@gmail.com; rachid@vm.uff.br

Abstract. *This work presents a one-dimensional unsteady friction model to predict the transient responses of a Newtonian fluid under turbulent regimes. Yet the simplicity of the one-dimensional mathematical frame is maintained, the continuum theory of mixtures, which the model is based on, enables the model to access the fluid transient internal structure. In the present approach, the fluid flow is interpreted as a pseudo-mixture formed by constituents of invariant volume-fraction that are geometrically arranged inside the pipe as concentric shells. They all have the same equation of state and their kinematics allow each of them to slide upon their neighbors in the direction of the pipe centerline. The governing equations form a quasi-linear hyperbolic system of partial differential equations whose solution in the context of initial-boundary-value problems are approximated by using the method of characteristics. In the present model, the radius and thickness of each constituent is an arbitrary constitutive choice in such a way that their arrangement appraises how much the internal structure of the flow is accessed. To identify the spatial distribution of the constituents inside the mixture capable of promoting the best cost-effective solution, with just a small number of them well-placed and few spatial grid nodes, a numerical study is carried out taking as reference 2D numerical solutions and experimental data available in the literature. The ideal grid allows the one-dimensional proposed model to present responses quite similar to those found in 2D models ($k-\omega$, five-region) in terms of head and rate of energy dissipation with CPU times that beholds its use when real-time applications are sought.*

Keywords: *unsteady friction, turbulent regime, theory of mixtures, rate of energy dissipation.*

1. INTRODUCTION

Turbulent transient flows in pipes occur in a diverse range of industries and are originated under ordinary as well as abnormal operational circumstances. The pressure surges that arise during this phenomenon are a real concern to the hydraulic analysis field, ranging from pipeline integrity and equipment design to uprising fault detection and hydraulic calibration techniques.

Historically, the very first attempt to model turbulent transient friction in pipe flows has appealed to a quasi-steady friction model, by assuming: the same equation that governs hydrodynamical fully-developed flows. However, such an approach is not capable to properly reproduce all-relevant flow features since it disregards the local and bi-dimensional velocity inversion process which takes place in transient regimes (Riasi et al. 2009). Such a shortcoming turns the wall shear stress to be in phase with the mean velocity of the stream, which does not reflect the physical reality. Therefore, in order to obtain realistic simulations, proper modelling of the friction associated with those distinguished velocity fields must be addressed.

In pursuit of more accurate one-dimensional models to describe pressure and flow fluctuations in turbulent transient regimes, several distinct attempts were proposed in the literature in the past decades (Duan et al., 2020). These models can be cast into two categories regarding the number of space coordinates used: one or two-dimensional. One-dimensional (1D) models do not have access to the instantaneous flow velocity profile inasmuch as the velocity is assumed to be uniform at any pipe cross-section. As a result, it requires a great deal of physical insight to accurately describe the friction phenomenon. Within this category, the concept of unsteady wall shear stress is introduced by splitting the wall shear stress as an additive decomposition of two parcels: one referring to the steady and the other to the unsteady flow contributions. As the steady parcel takes on the usual representation of the fully-developed steady pipe flows, the unsteady parcel is modeled by invoking the acceleration of the fluid, according to two main distinct approaches, so-called: instantaneous acceleration-based models IAB (Brunone et al, 1995) and weighting function-based models WFB (Vardy and Brown, 2007). In contrast to the one-dimensional models, the 2D models assume axisymmetric fluid flows and keep track of the radial variation of the axial component of the flow velocity, allowing the association of the velocity gradient with the shear stresses. A comprehensive review of those models can be found in Shamloo and Mousavifard (2014).

When practical applications become the main concern, the 2D model category is still a minority. The reason for that comes from the fact that this approach requires harder implementation in practice and greater CPU computational times when compared to their 1D competitors (Duan et al, 2020). Though this fact seems to discourage attempts to come up with 2D modeling, the importance of such analysis is increasing due to its significant impact on the fundamental research about fluid transients. Its ability to more accurately describe the shear stresses and overall dissipation in the fluid flow in addition to provide a glance into the flow turbulence structure with snapshots of the velocity profiles has had a discernible importance for the understanding of the transient mechanisms of momentum diffusion and dissipation of energy. Because of all those capabilities, 2D modeling can be not only applied to validate one-dimensional models but also be employed to provide insights into the development of 1D unsteady friction models (Pezzinga, 2000). Besides, 2D modeling is also more reliable when precise water quality management comes into play since they strongly depend on the knowledge of the flow radial velocity distribution (Ghidaoui et al., 2005). Uprising inverse transient-based techniques used to calibrate pipe visco-elastic parameters and identify pipeline defects also depend on precise and complete transient responses to provide realistic estimations (Tijsseling et al., 2020)

Aiming to gather within the same context the distinct qualities and functionalities that exist in each of the two categories of models, a versatile unsteady model is proposed herein. Such an approach is unique since it retains the simplicity of the 1D nature while inherits a fair approximation of some capabilities of the 2D models.

The model is developed within the framework of the continuum theory of mixtures by assuming that flow is viewed as a structured virtual mixture of constituents. The constituents are assumed to be spatially distributed across the pipe cross-section as cylindrical shells. Each of them has its own kinematics and can interact only with its adjacent neighbors. The resulting motion of the pseudo-mixture is governed by the mass and linear momentum conservation principles, forming a system of hyperbolic partial differential equations.

As being a novel and unusual kind of modeling, its capabilities and potentialities were not fully explored yet. To bridge this gap, we carry out a study to assess the ideal arrangement and number of constituents of the pseudo-mixture. The purpose is to set forth the main parameters to make the model a useful tool when small computational times are sought. In the sequel, we validate the model by comparing their predictions with an 2D model of the literature found in the work of Riasi *et al.* (2013) which applies the well-known $k - \omega$ two-equation turbulence model. With this initial step, we show that the proposed model is capable to predict quite similar responses of the transient in terms of head and rate of energy dissipation. Finally, the performance evaluation of the model is completed by addressing the computational time of a numerical run made by Duan *et al.* (2009). The proposed model is found to be quick enough to be applied when low computational times are required.

2. PROPOSED MODEL

The mechanical model to be explored is developed within the framework of the continuum theory of mixtures (Atkin and Craine, 1976). As a well-established rational theory, it has been widely employed to describe the thermomechanical behavior of mixtures made of different constituents, all of them treated as continuum media. The mixture is interpreted as being made of superimposed interacting constituents, each of them having its own motion, for which the principles of the mechanics of a single continuum are consistently generalized.

In contrast to the classical mixture theory, an unconventional approach is used herein by assuming that the fluid flow is formed by n spatially distributed constituents having the same physical properties. More specifically, they form cylindrical shells arranged concentrically in such a way that one layer can slide over the other with velocity v_j of the $j-th$ constituent. Each constituent has a fixed volumetric fraction α_j and travel independently along the x-direction of the pipe centerline as shown in Fig.1.

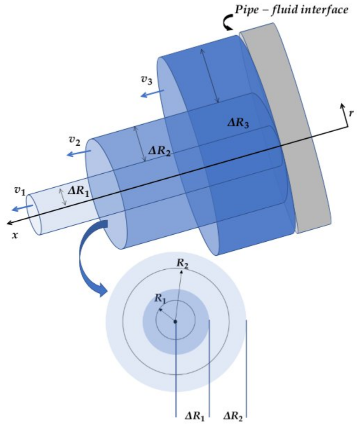

Figure 1 – Virtual structure of the mixture for $n = 3$.

In this context, the volumetric fraction of the $j-th$ mixture component may be expressed as

$$\alpha_j = \frac{2R_j\Delta R_j}{R^2},\ j = 1,..n, \tag{1}$$

in which R, R_j and ΔR_j are the internal radius of the pipe, the radius of the $j-th$ mixture component and its thickness, respectively, so that:

$$R_j = R_{j-1} + \frac{\Delta R_{j-1}}{2} + \frac{\Delta R_j}{2} \text{ for } j = 1,...n, \text{ with } R_0 \equiv 0 \equiv \Delta R_0 \text{ and } R \equiv R_n + \frac{\Delta R_n}{2}. \tag{2}$$

This abstract structure compels the postulation of the momentum balance of each of the constituents in addition to the one-dimensional mass and momentum balances of the flow (mixture as a whole) to turn the problem mathematically well-posed. In addition, as pressure fluctuations induce fluid density changes as well as pipe wall deformation, both phenomena must be taken into account to a more realistic mathematical description of fluid transients (Rachid and Costa Mattos, 1998). Because of that, an equation of state for the liquid is assumed to be the one of slightly compressible fluids and the pipe is assumed to be at the small linear elastic deformation range.

As the flows in this work is low mach number and the mass density, pressure and velocity of the pseudo-mixture as a whole can be defined such that $\rho = \sum_{j=1}^{n} \rho_j$, $p = \sum_{j=1}^{n} p_j$ and $v = \sum_{j=1}^{n} \alpha_j v_j$, respectively, in which $\rho_j = \alpha_j \rho$, v_j, $p_j = \alpha_j p$ stands for these parameters of the $j-th$ constituent of the mixture, the local form of the mechanical balances can be expressed as

$$\frac{1}{a^2}\frac{\partial p}{\partial t} + \rho_0 \frac{\partial v}{\partial x} = 0 \tag{3}$$

$$\rho_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{2}{R}\sum_{j=1}^{n} a_j = 0, \tag{4}$$

$$\alpha_j \rho_0 \frac{\partial v_j}{\partial t} + \alpha_j \frac{\partial p}{\partial x} + m_j + \frac{2}{R} a_j = 0, \quad j = 1,..., n, \tag{5}$$

In those equations, ρ_0 is the undisturbed mass density, a is the wavefront velocity and a_j represents the reactive contact friction force per unit lateral area that acts on the pipe-fluid interface. Further, m_j represents the internal interaction force per unit of cross-sectional area exerted by the other constituents on the $j-th$ constituent.

Based on the kinematics of constituents and the fundamental principle of objectivity, the expression to the interaction force m_j is devised to based on the velocity difference among neighbor constituents that results in

$$m_j = C_{j,j-1}\left(v_j - v_{j-1}\right) + C_{j,j+1}\left(v_j - v_{j+1}\right) \text{ for } j = 1,..., n, \text{ with } C_{1,0} \equiv 0 \text{ and } C_{n,n+1} \equiv 0 \tag{6}$$

in which $C_{j,j-1}$ and $C_{j,j+1}$ are material constants of the model in such a way that $C_{j,j+1} = C_{j+1,j}$,for $j = 1,...,n$.

Invoking the same principles used to develop the constitutive equations of m_j's it is possible to obtain an expression for the friction reactive force as

$$a_j = \begin{cases} 0, & j = 1,...., n-1 \\ Cv_n, & j = n \end{cases}. \tag{7}$$

The material constants of model $C, C_{j,j+1}$ can be achieved by stating that the viscosity structure of the unsteady flow remains the same of the one found in the flow at the pre-transient state. In this context, these material constants have same form in both transient and steady flow regimes. Thus, an incompressible fully-developed velocity profile ($v_j\left(R_j\right)$) can be employed into the momentum balances of the $j-th$ constituents to generate a linear system given by

$$C_{12}\left(v_1 - v_2\right) = -\alpha_1 \frac{\partial p}{\partial x}, \tag{8}$$

$$C_{21}\left(v_2-v_1\right)+C_{23}\left(v_2-v_3\right) = -\alpha_2\frac{\partial p}{\partial x},$$

$$C_{32}\left(v_3-v_2\right)+C_{34}\left(v_3-v_4\right) = -\alpha_3\frac{\partial p}{\partial x},$$

$$\vdots \qquad \vdots$$

$$C_{j,j-1}\left(v_j-v_{j-1}\right)+C_{j,j+1}\left(v_j-v_{j+1}\right) = -\alpha_j\frac{\partial p}{\partial x},$$

$$\vdots \qquad \vdots$$

$$C_{n-1,n-2}\left(v_{n-1}-v_{n-2}\right)+C_{n-1,n+1}\left(v_{n-1}-v_{n+1}\right) = -\alpha_{n-1}\frac{\partial p}{\partial x},$$

$$\frac{2}{R}Cv_n + C_{n,n-1}\left(v_n-v_{n-1}\right)+0 = -\alpha_n\frac{\partial p}{\partial x},$$

, which solution are the material constants of the model. In this work, the algebraic turbulent model described in Vardy & Brown (2007) is applied to obtain an expression for the velocity profile. This approach defines an idealized turbulent viscosity distribution with two distinct regions, namely an outer annulus and an inner core. The core region ranges from $r = 0$ to $r = 0.8R$ and has a fixed turbulent kinematic viscosity ν_c while the annulus, with thickness equal $b = 0.2\,R$, the turbulent kinematic viscosity varies linearly from the wall $r = R$, where it assumes the value ν_w, to a maximum value of ν_c.at the interface of these regions. The material constants connected with the interaction forces of the model in the core and annulus regions are found to be (Andrade, 2019)

$$C_{j,\,j+1} = \frac{4\rho_0\nu_c}{R_{j+1}^2-R_j^2}\left(\sum_{i=1}^{j}\alpha_i\right), \tag{9}$$

$$C_{j,j+1} = \frac{2\nu_w\rho_0(1-\sigma_{cw})/b^2}{\left\{\dfrac{R_{j+1}-R_j}{b}+\dfrac{(-4+5\sigma_{cw})}{(1-\sigma_{cw})}\ln\left[\dfrac{\left(\dfrac{1-\sigma_{cw}}{b}\right)R_j-4+5\sigma_{cw}}{\left(\dfrac{1-\sigma_{cw}}{b}\right)R_{j+1}-4+5\sigma_{cw}}\right]\right\}}\left(\sum_{i=1}^{j}\alpha_i\right), \tag{10}$$

respectively. Whereas, the reactive material constant can be expressed as

$$C = \frac{\nu_w\rho_0R\left(1-\sigma_{cw}\right)/b^2}{\left\{\dfrac{R-R_n}{b}+\dfrac{\left(-4+5\sigma_{cw}\right)}{\left(1-\sigma_{cw}\right)}\ln\left[\left(\dfrac{1-\sigma_{cw}}{b}\right)R_n-4+5\sigma_{cw}\right]\right\}}\left(\sum_{i=1}^{n}\alpha_i\right), \tag{11}$$

In order to solve the quasi-linear hyperbolic system given by Eqs. (3-5), the method of characteristics (Whitham, 1974) is applied with the aid of a Crank-Nicholson approximation given by

$$\int_t^{t+\Delta t}\varphi\, dt = \left\{[1/2\varphi]_i^{t+\Delta t}+[(1-1/2)\,\varphi]_i^t\right\}\Delta t. \tag{12}$$

Where φ may stand for a_j or m_j.

3. NUMERICAL RESULTS

To identify the spatial distribution of the constituents inside the mixture able to promote the best cost-effective solution, that is, by employing a small number of them well-placed with few spatial grid nodes, a numerical study is carried out taking as reference 2D numerical solutions and experimental data available in the literature. All of these data is based on a reservoir-pipe-valve installation, in which the valve is located at the pipe downstream end. The transients in this piping system are generated by a sudden valve closure maneuver. In the discretized domain, the reservoir is located at $x = 0$, while the valve is positioned at $x = L$.

The analysis carried out ahead assumes that the reservoir is a constant pressure source and that the valve closure induces a linear reduction in the mean fluid velocity along the time until it is completely closed. Such conditions are mathematically expressed as Dirichlet boundary conditions in which p and v are prescribed as:

$$p(x=0,t) = p_R, \tag{13}$$

$$v(x=L,t) = \begin{cases} v_0\left(1-\frac{t}{t_c}\right) & \text{if } 0 \le t < t_c \\ 0 & \text{if } \quad t \ge t_c \end{cases} \tag{14}$$

where t_c is the valve closure time, p_R is the reservoir pressure and v_0 stands for the average velocity in steady state. In what follows, the transient regimes generated in the reservoir-pipe-valve system are used to numerically investigate the following issues associated with the proposed model: a) the constituents' distribution in the mixture; b) the overall model performance. To achieve this goal, 2D numerical solutions as well as experimental data available in the literature are taken as references.

3.1 Analysis of the constituent's codistribution

One of the goals of this article is to identify conditions to optimize the computational time when applying the proposed transient friction model. One should expect that the model may access enough internal information about the behavior of the transient flow meanwhile preserving fair computational cost.

The issue to be analyzed is the best arrangement of the concentrically shell-shaped constituents of the mixture. In a comprehensive analysis, this structure is responsible for the precision of the model since they are related to the dispersive and dissipative effects of the flow. Herein, structure is meant not just the number of components but also their distribution. To do so, an extensive numerical study was carried out by combining different distributions and number of constituents. The results for two of them, hereafter referred to Mesh I and II are presented and compared. Mesh I is uniform with all constituents having the same thickness. Mesh II is non-uniform, with finer constituents located in the annulus. To identify the ideal mesh to be used in numerical simulations, a qualitative comparison between the results obtained from computed responses with Mesh I and II types is carried out.

The radius of each constituent is defined generically in section 2, but specific definitions for the two kinds of meshes are needed. The first and simpler distribution is created by assuming the thickness of each constituent is constant. The second assumed distribution (Mesh II) is built focused on the annular constituents ($0.8R < r < R_n$). In such a distribution the thickness of each constituent is maintained constant in the core region, while the thicknesses of the constituents of the annular region diminish following a progressive ratio. This arrangement is designed to capture the severe velocity gradients near the pipe-wall which arise in the transient regimes. Thence, Mesh II is expected to attain better results with a small number of constituents than that with the even distribution. Therefore, as the number of constituents is reduced, fewer compatibility equations are solved.

By denoting the total number of constituents as $n = n_a + n_c$, with n_a and n_c standing for the number of constituents in the annulus and core, respectively, the thickness of each constituent in the core region for either Mesh I or Mesh II is given by

$$\Delta R_j = \Delta R^c = \frac{R_M}{n_c}, \text{ for } j = 1, \ldots, n_c. \tag{16}$$

On the other hand, the thickness of each constituent in the annulus for Mesh I is simply

$$\Delta R_j = \Delta R_j^a = \frac{0.2R}{n_a}, \quad for\, j = 1, \ldots, n_a, \tag{17}$$

whereas for Mesh II assumes the values

$$\Delta R_j = \Delta R_j^a = \xi \Delta R^c z^{j-1}, \quad \text{for } j = 1, \ldots, n_a, \tag{18}$$

in which $\xi \in (0,1)$ is a constant, which will be set as 1/2 to enforce a drastic reduction of the thickness of the first constituent in the annular region, while z is the geometric ratio given by the root of the following equation

$$\sum_{j=1}^{n_a} \Delta R_j^a = \frac{\xi \Delta R_c \left(z^{n_a} - 1\right)}{z-1} = 0.2R. \tag{19}$$

The comparison between the responses obtained with Meshes I and II will be based on the flow features of the experimental facility used by Adamkowski and Lewandowski (2006). The features that will be analyzed will be the responses with Meshes I and II in terms of the normalized histories of the head, mean velocity and velocity profiles at distinct times.

The numerical runs for Mesh I (completely uniform mesh) are made with the number of constituents n equal to 5, 10, 40, 80. Meanwhile, as a matter of a huge number of combinations, the results presented for Mesh II will be limited to only one case, which has 5 core and 5 annular constituents, in such a way that the $n-th$ constituent has approximately the same radius of the Mesh I with 80 constituents, i.e. $R_n^{Mesh\ I-80}/R_n^{Mesh\ II} \cong 1$.

Based on the reported experience with the MOC in the work of Andrade (2019), the simulations with Meshes I and II were carried out with the number of spatial nodes to be $N_s = 17$. According to the author, this number of spatial nodes is enough to accurately describe several responses of the transient such as pressure, mean velocity and local rate of energy dissipation.

Figure 2 displays the numerical results we have obtained for the head variation in the pipe mid-length normalized by the Joukowsky head $H^* \equiv av_0/g$ as a function of the normalized time ($t^* \equiv L/a$). As it can be seen, the increase of the constituents number of the Mesh I type leads to a clearer visualization of the characteristic smooth shaping of the pressure responses in addition to more attenuated peaks. The same can be said with respect to the average velocity responses of the model as shown in Fig. 3. Moreover, Mesh II can provide quite similar responses to the one computed by the most refined even distribution (Mesh I with $n = 80$), as it can be seen in Figs. 2 and 3. Thus, it becomes clear that the annular, and specially the vicinity of the wall, is crucial to achieve precise transient responses.

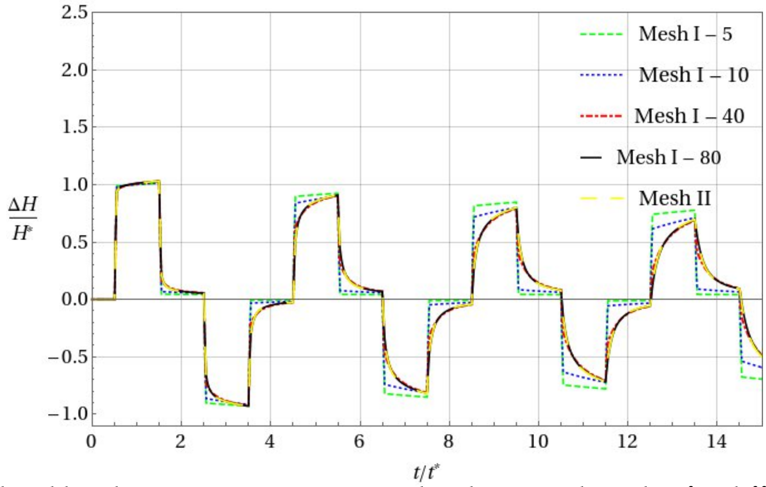

Figure 2. Normalized head responses against normalized time at the valve for different constituent distributions for Mesh I with $n = 5$, 10, 40 and 80 and Mesh II.

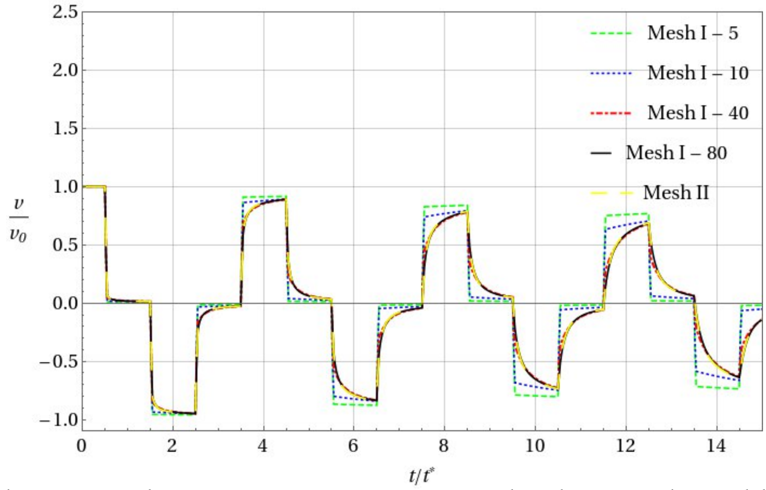

Figure 3. Normalized average velocity responses against normalized time at the mid-length of the pipe for different constituent distributions for Mesh I with $n = 5$, 10, 40 and 80 and Mesh II.

As one expects, the greater the number of constituents in Mesh I is, the better the picture of the instantaneous velocity profiles becomes, as it can be seen in Fig. 4 for different normalized time instants $t/t^* = 0, 1, 2$ and 3. Thus, the internal structure of the transient flow is better captured that results in more realistic transient responses at the light of the proposed model. The rather surprising fact is that the incredibly small number of constituents used in Mesh II can capture the whole picture of the velocity profiles, including the high-velocity gradients near the wall during different stages of the transient. The same cannot be said about the Mesh I type for of 5 and 10 constituents, which shows poor results for this variable since they are unable to capture the velocities near the wall nor the pipe centerline (see Fig. 4(d)). This fact explains the poor quality of the results obtained with theses meshes for mean-velocity and head histories observed in Figs. 2 and 3.

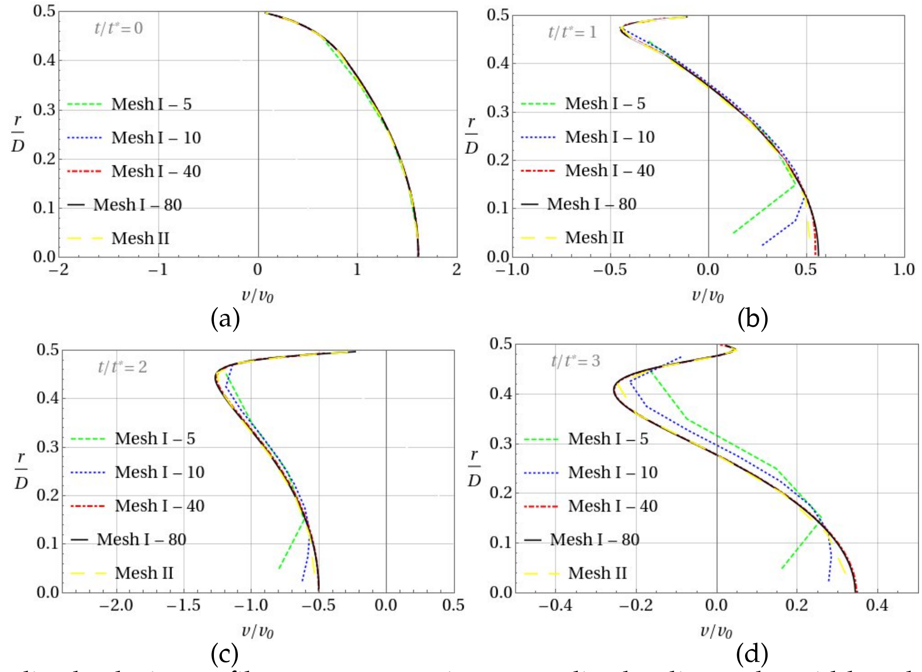

Figure 4. Normalized velocity profiles responses against normalized radius at the mid-length of the pipe for different constituent distributions for Mesh I with n = 5, 10, 40 and 80 and Mesh II at different normalized times t/t^* = 0, 1, 2 and 3.

5.4 Analysis of the overall model performance

The past subsection was devoted to identify the adequate structure of the constituents to produce precise results with low computational costs. This task was important to make the present approach attractive from the computational viewpoint, so that it can be used in a variety of practical situations, such as those in which real-time application is the main concern. To sum up, we concluded that a non-uniform mesh like Mesh II with 17 spatial nodes is enough to get precise and quite reasonable transient responses. Now, we focused on accessing the ability of this settlement on providing accurate results. To do so, we compare the model predicted responses for head and the rate of energy dissipation with those obtained with a 2D $k-\omega$ turbulence model by Riasi et al. (2013). Finally, a comparison regarding the CPU computational time is accessed by taking as reference the numerical run carried out by Duan et al. (2009) with its computational efficient version of the 2D five-region turbulent model of Vardy and Hwang (1991).

Table 1. Main characteristics of the Holmboe and Rouleau (1967) experiment used in the comparison of the proposed model results with those obtained by Riasi et al. (2013).

	$Re\,(-)$	L(m)	D(m)	$\rho_0\left(\frac{\text{kg}}{\text{m}^3}\right)$	$\mu\left(\frac{\text{Ns}}{\text{m}^2}\right)[10^{-3}]$	$a\left(\frac{\text{m}}{\text{s}}\right)$
Holmboe and Rouleau (1967)	6166.00	36.09	0.025	1000.00	0.86	1350.00

Table 2. Main characteristics of the Duan *et al.* (2009)numerical simulation.

	$Re\,(-)$	L(m)	D(m)	$\rho_0\left(\frac{\text{kg}}{\text{m}^3}\right)$	$\mu\left(\frac{\text{Ns}}{\text{m}^2}\right)[10^{-3}]$	$a\left(\frac{\text{m}}{\text{s}}\right)$
Duan *et al.* (2009)	6600.00	37.20	0.022	1000.00	1.00	1276.00

All these simulations used as reference for comparison purposes with the results predicted by the proposed model refer to a quick valve closure in a reservoir-pipe-valve installation, as described in the previously. The main features of the installations are presented in Tables 1, 2 for the Riasi et al. (2013) and Duan *et al.* (2009) results, respectively.

Figure 5 shows that the proposed model can predict head histories quite similar to those of Riasi et al. (2013). It is also worth noting that the Homboe and Rouleau's experimental data are also fairly well reproduced by the models, as it can be seen in Fig. 5.

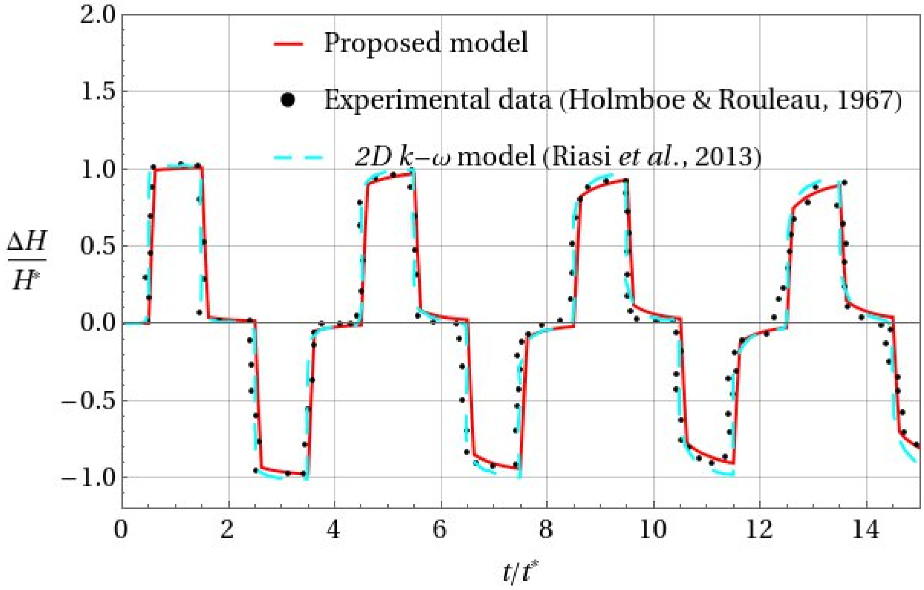

Figure 5. Normalized pressure head histories at mid-length of the pipe for the Holmboe and Rouleau(1967) experiment. Comparison between the responses of the present model and the 2D $k-\omega$ model of Riasi et al (2013) .

In order to evaluate the model in a broader sense, this work compared the normalized rate of energy dissipation history throughout the whole pipe length predicted by the proposed model and by the 2D model of Riasi *et al* (2013). Since the energy dissipation evolves several nuances of the model as the instantaneous velocity profiles as will be seen in the following.

As suggested by Costa Mattos *et al.* (1995), when dealing with the continuum mixture theory, the second law of thermodynamics should be postulated for the constituent and for the mixture as whole. In this context the local rate of energy dissipation per unit of length d of the proposed fluid flow can be stated as (Andrade, 2019)

$$d = \sum_{j=1}^{n} C_{j,j+1}\left(v_{j+1}-v_j\right)^2 + \frac{2}{R}C\left(v_n\right)^2 \geq 0, \tag{20}$$

As one may note in Fig. 6, an overall agreement between the responses of rate of energy dissipation is clearly evidenced between the models. Assuming that the 2D $k-\omega$ model is capable to reproduce this physical phenomenon, one can infer that the one-dimensional proposed model is able to not only properly describe the main flow parameters, such as the head, but also the rate of energy dissipation, which takes internal aspects of the flow structure into account (see Eq. (20)). The success of this achievement stems on the proper description of internal momentum transfer mechanisms, which are inherently linked to the dissipative and dispersive effects in the whole pipe extension. The discrepancies between the proposed model and the 2D models may be mainly explained by the application of a rather simple turbulence structure and pre-transient turbulent viscosity values (section 2).

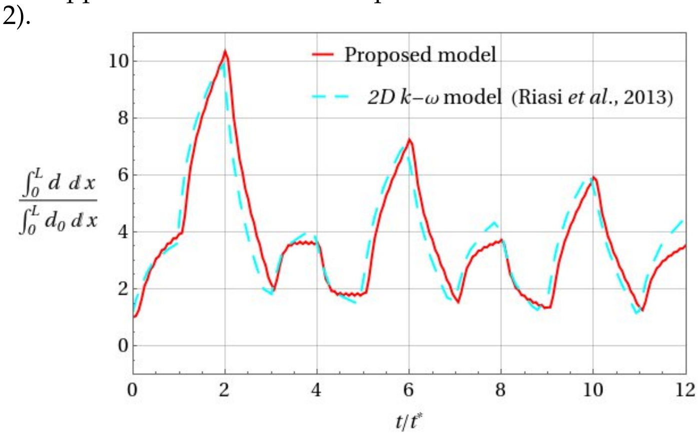

Figure 6 – Normalized rate of energy dissipation histories in the whole pipe extension. Comparison between the responses of the present model with and the 2D $k-\omega$ model of Riasi et al. (2013).

Normalized head responses at the valve during the first 2 seconds ($t/t^* \cong 70$) after the transient is generated are presented in Fig. 7 in accordance with the predictions of the 2D five-region model of Duan *et al.* (2009) and the proposed model. As it can be seen, the responses basically matched. The simulation carried out with the proposed model in a personal computer (Core i3 -7th Gen) took less than one-tenth of a second. Although Duan et al (2009) have used a different personal computer (P4 2.0 GHz), a rough comparison can still be made. Based on the CPU time of about 4.3 seconds reported by the authors, the proposed model is about 50 times faster. Besides, in same work, they also claim that their model is capable of doing a specific fluid transient simulation about 2 times faster than the 2D model of Pezzinga (2000) and 300 times faster than the WFB model of Vardy et al. (1993). Thus, we can conclude that the CPU time spent by the proposed model is very encouraging, enabling its usage to practical situations which demands fast computational responses, such as those of real-time applications. Not only the mean velocity and pressure responses of the transient are actually computed with a fair precision but also the velocity profiles, wall shear stress and rate of energy dissipation are also well described. By gathering the good capabilities intrinsically present in the 2D and 1D models within a same context, the proposed model becomes a promise tool for a diverse range of applications in the pipeline industry.

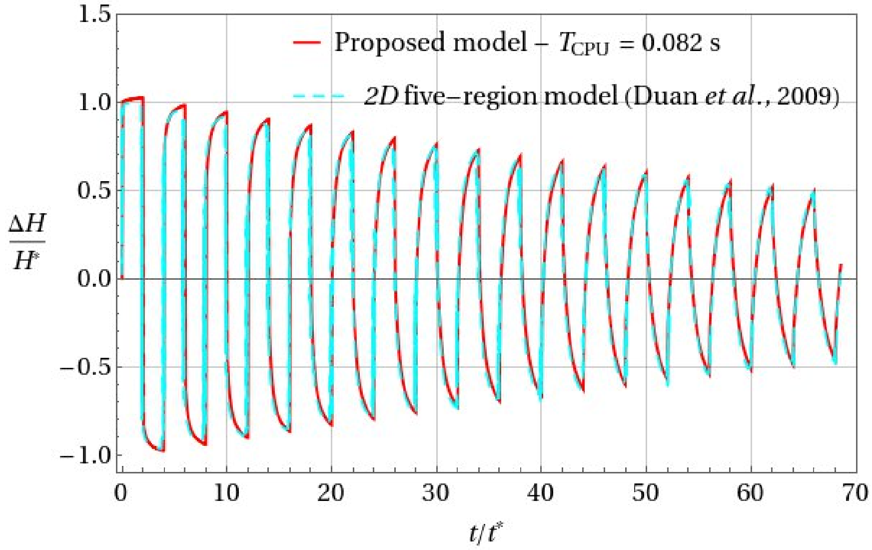

Figure 7 – Normalized head responses next to the valve for the 2D five region model of Duan *et al* (2009) and the proposed model.

5 CONCLUSION

This work presented an unsteady friction model based on the theoretical framework of continuum theory of mixtures for describing turbulent pipe flows in the transient regimes. In this model, the stream is assumed to be a pseudo-mixture of n constituents of invariant volume fractions, giving rise to a hyperbolic system of $n+1$ quasi-linear partial differential equations, in which $n>2$ stands for the number of constituents in the virtual mixture.

As the arrangement of the constituents in the mixture is totally arbitrary, an analysis of the distribution of constituents was carried out in pursuit of one that could grant precision and low computational times. The obtained results revealed that allocating the constituents in the region close to the wall provides more efficient results than applying an evenly distribution. The reason for that choice is that the severe velocity gradients which prevail in the transient regime are mainly located to the flow region close to the wall. This arrangement has the distinctive characteristic of rendering fast enough simulations required when real-time applications are sought.

Yet the model is one-dimensional in essence, the comparisons carried out with some 2D turbulent models available in the literature have shown that the proposed model can provide an excellent agreement for the pressure responses and predict quite reasonable descriptions of the the rate of energy dissipation histories for the sake of any further hydraulic application.

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5. RESPONSIBILITY NOTICE