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MODELLING OF FLUID TRANSIENTS IN VISCOELASTIC COMPLIANT PIPES

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Abstract. *In industry, daily operations as valve maneuvers generate unsteady pipe flows. The dynamics of such flows is governed by a cyclic passage of a wavefront which is responsible for local pressure surges. These pressure traces subject the compliant pipe to stresses that are greater than those found under steady-state regime. As those fluctuations depend not only on the fluid and the nature of the flow but also on the mechanical response of the material with which the tubes are made, the accurate analysis of these problems is conditioned to the joint description of all these aspects. This paper arises to present a thermodynamic consistent model capable of handling transient flows in viscoelastic compliant pipes. To accomplish this goal, first, a robust model of unsteady friction on the fluid is applied as the basic framework, afterward, the tube viscoelasticity is advanced invoking a constitutive theory with a strong thermodynamic foundation. Such an approach allows the computation of the rates of dissipation on fluid and on the solid distinctly and accurately. The proposed model results in a quasi-linear hyperbolic system of partial differential equations whose solution is approximated by using the method of characteristics. When compared with experimental data, numerical responses of the present model are found to be largely more accurate than the one which disregards the anelastic nature of the tube. This fact is partially explained by the fact of the rates of energy dissipation presented by the fluid and tube to be at the same order of magnitude throughout the transient.*

Keywords: *unsteady friction, viscoelasticity, internal variables, rate of energy dissipation, pipe flows*

1. INTRODUCTION

Piping systems used to move liquids are present in several areas of engineering, from treated water supply and sewage to the petrochemical industries and nuclear plants. Whatever the type of application, these pipes are susceptible to severe pressure loads and stresses on the walls of the tubes during transient events, whenever there are variations in the amount of movement of the fluid or structure (Rachid and Costa Mattos, 1998). Typical sources of transients are due, but not limited to, stopping and starting pumping, opening and closing valves as well as structural vibrations. As the magnitude of the pressure waves that propagate in the liquid and the phenomena associated with its attenuation and dispersion depend not only on the fluid and the nature of the flow (whether laminar or turbulent) but also on the mechanical response of the material with which the tubes are made of, the accurate analysis of these problems is conditioned to the joint description of all these aspects.

Depending on the type of application, tubes of different materials are used, giving rise to different mechanical responses. Typical industrial pipe systems are made with steel of different carbon contents, which in the scope of applications exhibit linear elastic behavior. On the other hand, sanitary sewer and water supply pipes are constantly made of PVC or polyethylene, which respond to hydraulic loading with a viscoelastic mechanical response (Tijsseling et al., 2012). Recently, the range of application of such materials is growing due to their low cost, high-pressure ratings, lightweight in addition to their temperature, chemical, and abrasion resistance (Covas et al, 2005). The viscoelastic behavior of the tube is intrinsically related to processes of a dissipative nature. Like friction in the fluid, they contribute to dissipating energy in the fluid-structure system. As a result, if accurate analyzes are desired, the inelastic behavior of the pipe material becomes a key factor to be considered in the engineering design so that the effective values of the stresses in the pipe walls can be accessed.

On the other hand, hydrodynamic loads in the fluid are notably influenced by friction in the fluid, and therefore must be consistently taken into account in the analysis. Within the one-dimensional context in which these problems are modelled, considering that the lengths of the tubes exceed their diameters by several orders of magnitude, the attenuation and dispersion of the pressure waves are only adequately represented in the one-dimensional context by using frequency-dependent friction models (Duan et al. 2020). Among the theoretical models capable of reproducing this intrinsically two-dimensional phenomenon are the models based on convolution integral (Vardy and Brown, 2007) and local-balance (Pezzinga, 2000). Even though those models can predict local pressure and averaged velocity responses with good accuracy, they fail to obey the second law of thermodynamics, as demonstrated in Gonzaga Filho (2017). Then, they cannot provide a solid description of the energy dissipation of the fluid motion which might be of importance to the design of efficient transient control techniques (Duan et al., 2017). To overcome this drawback and provide a broader view of the internal structure of the flow, a quasi-one-dimensional unsteady friction model was proposed by Andrade (2019). Unlike other one-dimensional models of the literature, this approach allows the determination of the instantaneous velocity profile in each and every cross-section of the tube. This functionality is achieved thanks to the use of the continuum theory of mixtures (Atkin and Crane, 1976), which treats the liquid as a pseudo structured mixture, formed by cylindrical shells of invariant volumetric fraction that slide one over the other with different speeds. Such peculiarity of the model not only allows to capture with great accuracy the oscillations of pressure, mean-velocity fields, both in phase and in magnitude but also to correctly compute the shear stresses and the rate of energy dissipation during transients. Besides, the authors shown that for both laminar and turbulent flows, the proposed model unconditionally satisfies the second law of thermodynamics (Andrade, 2019).

Using this model as a base and aiming to provide more advanced engineering design methodologies and more effective scientific-technological tools, this work aims the development of a model that incorporates the viscoelastic behavior of the tube. To achieve this objective at the same mathematical structure, this work applies the previously mentioned mechanical friction model proposed in Andrade (2019) aligned with a constitutive theory of the viscoelastic tube based on the Thermodynamics of Irreversible Processes with internal variables (Maugin and Muschik, 1994). In addition to provide the constitutive equations of the pipe, this approach also leads to a straightforward thermodynamic consistent expression of the energy dissipation of the tube. Thus, the proposed modeling can account for isolated and accurate energy dissipation rates in the fluid and in the tube. That is an innovative perspective concerning the existing models in the literature which noted works can be found in Covas et al. (2005), Pezzinga (2016) and Tijsseling et al. (2012).

2. BASIC EQUATIONS

In order to account to the transient friction in fluid transients, a recent mechanical model based on the continuum theory for mixtures (Atkin and Crane) was recently proposed by the authors in Andrade (2019). Although it is essentially one-dimensional, the framework in which the model is based on allows the fluid to be viewed as a structured pseudo-mixture formed by n cylindrical shells of invariant volumetric fraction $\alpha_j\{j = 1, \ldots, n\}$. In this context, these constituents have their own motion along the direction of the pipe center-line with speeds $v_j\{j = 1, \ldots, n)$. A sketch of the pseudo-mixture can be seen in Fig.1. As one can see, each one of these $j - th$ pseudo-mixture constituents is characterized by its radius R_j, thickness ΔR_j and volumetric fraction α_j which are related as the following

$$R_j = R_{j-1} + \frac{\Delta R_{j-1}}{2} + \frac{\Delta R_j}{2} \text{ for } j = 1, \ldots n \text{ with } R_0 \equiv 0 \text{ and } R \equiv R_n + \frac{\Delta R_n}{2}. \tag{1}$$

$$\alpha_j = \frac{2R_j \Delta R_j}{R^2}, \text{ for } j = 1, \ldots, n. \tag{2}$$

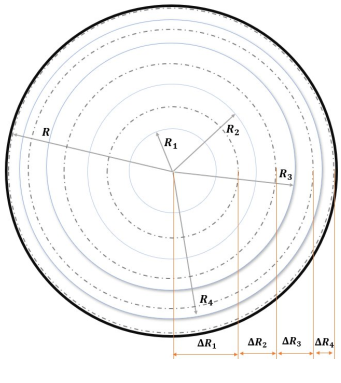

Figure 1. Virtual structured of the mixture for $n = 4$.

This mathematical framework enforces the postulation of the momentum balance of each of the constituents in addition to the one-dimensional mass and momentum balances of the flow (mixture as a whole) to turn the problem mathematically well-posed. As the flows is assumed to be low mach number and the mass density, pressure and velocity of the pseudo-mixture as a whole are defined such that $\rho = \sum_{j=1}^{n} \rho_j$, $p = \sum_{j=1}^{n} p_j$ and $v = \sum_{j=1}^{n} \alpha_j v_j$, respectively, in which $\rho_j = \alpha_j \rho$, v_j, $p_j = \alpha_j p$ stands for these parameters of the $j - th$ constituent of the mixture, the mechanical balances of the proposed model can be expressed as

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A v)}{\partial x} = 0; \tag{3}$$

$$\frac{\partial(\rho v A)}{\partial t} + \sum_{j=1}^{n} 2\pi R\, a_j + A\frac{\partial p}{\partial t} = 0; \tag{4}$$

$$\frac{\partial\left(\rho_j v_j A\right)}{\partial t} + A\frac{\partial p_j}{\partial t} + 2\pi R a_j + m_j A = 0, j = 2, \ldots n. \tag{5}$$

In those equations, a_j represents the reactive contact friction force per unit lateral area that acts on the pipe-fluid interface, being $2\pi R$, the pipe internal perimeter of the circular pipe area $A = \pi R^2$. Further, m_j represents the internal interaction force per unit of volume exerted by the other constituents on the $j-th$ constituent.

The pressure fields found in the transient causes fluid density changes as well as pipe wall deformation, both phenomena must be taken into account to a more realistic mathematical description of unsteady flows in compliant pipes. Because of that, a liquid equation of state in addition to a cross-sectional area strain relationship must be addressed. In this work, the liquid flow is slightly compressible that obeys the following equation of state

$$\rho = \rho_0\left(1 + \frac{p}{K}\right), \tag{6}$$

where K is the fluid bulk modulus and ρ_0 is the undisturbed fluid density. While the pipe is assumed to be subject to small deformations in which the circumferential strain ε_θ can be related to the cross-sectional area as

$$A = A_0\left(1 + 2\varepsilon_\theta\right). \tag{7}$$

Where A_0 is the undisturbed cross-sectional area. Upon substitution of Eq.(7, 6) into the mass balance (Eq. 3), the basic equations of the model can be found:

$$\frac{1}{K}\frac{\partial p}{\partial t} + 2\frac{\partial \varepsilon_\theta}{\partial t} + \rho_0\frac{\partial v}{\partial x} = 0, \tag{8}$$

$$\rho_0\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{2}{R}\sum_{j=1}^{n} a_j = 0, \tag{9}$$

$$\alpha_j\rho_0\frac{\partial v_j}{\partial t} + \alpha_j\frac{\partial p}{\partial x} + m_j + \frac{2}{R}a_j = 0, \quad j = 1, \ldots, n, \tag{10}$$

3 CONSTITUTIVE EQUATIONS

3.1 Pseudo-mixture

The constitutive equations of the forces m_j and a_j are addressed by appealing to the material frame indifference principle and the aforementioned kinematic aspects of the structured mixture. The resulting proposed forms for m_j and a_j are given by:

$$m_j = C_{j,j-1}\left(v_j - v_{j-1}\right) + C_{j,j+1}\left(v_j - v_{j+1}\right) \text{ for } j = 1, \ldots, n, \text{ with } C_{1,0} \equiv 0 \text{ and } C_{n,n+1} \equiv 0 \tag{11}$$

$$a_j = \begin{cases} 0, & j = 1, \ldots., n-1 \\ C v_n, & j = n \end{cases}. \tag{12}$$

The material constants of model $C, C_{j,j+1}$ can be found through the assumption that during the fluid transient the viscosity structure remains the same as the one found in the permanent regime (Duan et al., 2020). Thus, the steady-state momentum balances of the $j-th$ constituents (Eq.10) together with an incompressible fully-developed velocity profile may be used to generate a linear system given by

$$\begin{bmatrix} \Delta v_{1,2} & 0 & 0 & \dots & 0 & 0 \\ \Delta v_{2,1} & \Delta v_{2,3} & 0 & \dots & 0 & 0 \\ 0 & \Delta v_{3,2} & \Delta v_{3,4} & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \ddots & 0 & 0 \\ 0 & \vdots & 0 & \Delta v_{n-2,n-1} & \Delta v_{n-1,n} & 0 \\ 0 & 0 & 0 & 0 & \Delta v_{n,n-1} & v_n \end{bmatrix}_{n \times n} \begin{bmatrix} C_{1,2} \\ C_{2,3} \\ C_{3,4} \\ \vdots \\ C_{n-1,n} \\ C \end{bmatrix}_n = \begin{bmatrix} -\alpha_1 \frac{\partial p}{\partial x} \\ -\alpha_2 \frac{\partial p}{\partial x} \\ -\alpha_3 \frac{\partial p}{\partial x} \\ \vdots \\ -\alpha_{n-1} \frac{\partial p}{\partial x} \\ -\alpha_n \frac{\partial p}{\partial x} \end{bmatrix}_n , \tag{13}$$

with $\Delta v_{j,k} \equiv v_j - v_k \; \{j, k \in N\}$, whose solution are the material constants of the model. The aforementioned approach is general for laminar or turbulent flows. This work will be limited to show the material constants for laminar flows as the results presented here are found to be in this flow regime. Nonetheless, it is worthy to mention that the turbulent nature of the flow also can be captured by this methodology as demonstrated in Andrade (2019).

Invoking the classic parabolic laminar velocity profile, the material constants of model results in (Andrade, 2019):

$$C_{j,\,j+1} = \frac{4\mu}{R_{j+1}^2 - R_j^2} \left(\sum_{i=1}^{j} \alpha_i \right), \text{ for } j = 1, \dots, n-1, \tag{14}$$

$$C_{j,j+1} = \frac{2\mu\, R}{R^2 - R_n^2} \left(\sum_{i=1}^{j} \alpha_i \right), \text{ for } j = n, \tag{15}$$

in which μ is the fluid viscosity.

3.2 Tube

The constitutive equations that describe the pipe stress-strain relations are needed to treat properly the circumferential strain that appears in the mass balance of the model (see Eq. 8). When the tube material has inelastic behavior, micro-structural rearrangements of the material occurs. Such behavior is intrinsically tied with dissipative effects. In order to provide consistent constitutive equations for this kind of compliant piping, the thermodynamics of irreversible processes with internal variables is invoked (Maugin and Muschik, 1994). The basic assumption is that the state of the body at a given material point and at a given time is completely defined by a set of state variables. Within the general framework of thermodynamics of irreversible processes, one internal variable is introduced for each dissipative mechanism. To each internal variable, one associated evolution law is postulated in such a way that the second law of thermodynamics is automatically satisfied. In this theory, two thermodynamical potentials, the Helmholtz free energy and a pseudo-dissipation potential, are sufficient to define a complete set of constitutive equations.

The mechanical behavior of the pipe is assumed to be described by the theory of linear viscoelasticity. In such theory, the volume invariability of the body requires that the volumetric response of the anelastic response be null restricting its viscoelastic nature to pure shear. This assumption turns the application of the so-called generalized Kelvin-Voigt model suitable in those materials. This model is formed by m units of Kelvin-Voight elements within shear elastic constant G_i and coefficients of viscosity η_i $(i = 1, \dots, m)$ coupled in series with a string with elastic a instantaneous shear modulus of elasticity G_0. Each of these Kelvin-Voigt units is associated to a second-order strain tensor $\boldsymbol{\varepsilon}^i$, for $i = 1, \dots, m$ so that the inelastic strain tensor $\boldsymbol{\varepsilon}^a$ is equal to the summation over i of $\boldsymbol{\varepsilon}^i$.

Under the umbrella of this theory, the thermodynamic state of an isothermal viscoelastic tube of densityρ_t is supposed to be identified by the total strain and anelastic strain tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^a$ in addition to a set of internal variables identified as the strain tensor $\boldsymbol{\varepsilon}^i$ of each Kelvin-Voigt unit. In this context, the Helmholtz free energy potential can be expressed as a decomposition of the elastic and anelastic strain energy densities W_e, W_a :

$$\rho_t \Psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^a, \boldsymbol{\varepsilon}^i) = W_e\,(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a) + W_a\left(\boldsymbol{\varepsilon}^i\right). \tag{16}$$

Where W_e and W_a can be established as

$$W_e\ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a) = \frac{1}{2}\mathbf{C}((\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a):(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a), \tag{17}$$

$$W_a = \frac{1}{2}\frac{2E_i}{3}\ \boldsymbol{\varepsilon}^i : \boldsymbol{\varepsilon}^i \tag{18}$$

being **C** the classic symmetric and positive definite fourth order tensor of theory of elasticity. Meanwhile W_a represents the inelastic strain density that is function solely of the internal variable $\boldsymbol{\varepsilon}^{\mathrm{i}}$.

Partial differentiation of the Helmholtz potential in terms of the state variables $\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^a, \boldsymbol{\varepsilon}^i\right)$ defines respective thermodynamic forces $\boldsymbol{\sigma}\ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a)$ and $\mathbf{B}^{\varepsilon^{\mathrm{i}}}(\boldsymbol{\varepsilon}^i)$:

$$\boldsymbol{\sigma}\ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a) = \rho_t \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} = \rho_t \frac{\partial W_e}{\partial \boldsymbol{\varepsilon}} \tag{19}$$

$$\mathbf{B}^{\varepsilon^i} = -\rho_t \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}^i} = -\rho_t \frac{\partial W_a}{\partial \boldsymbol{\varepsilon}^i}. \tag{20}$$

As one can see, the first is the usual thermodynamical relation found in elastic materials, which comprehends a constitutive equation for the purely elastic response of the solid. Meanwhile the second represents a thermodynamic force related to the inelastic deformation of the material. Such a law of state is still insufficient to describe the anelastic behavior of the material since the relations between the internal variable with the anelastic strain tensor and the thermodynamic force $\boldsymbol{\sigma}\ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a)$ is still unknown. Such a lack is completed by the introduction of a scalar-differentiable pseudo-potential of dissipation $\Phi\ (\boldsymbol{\sigma}, \mathbf{B}^{\varepsilon^i})$ which provides these relations through evolution laws of state. Those laws are found by the partial derivatives of this potential according to their dependent variables as such

$$\frac{\partial \boldsymbol{\varepsilon}^a}{\partial t} = \frac{\partial \Phi(\boldsymbol{\sigma}, \mathbf{B}^{\varepsilon^i})}{\partial \boldsymbol{\sigma}}; \tag{21}$$

$$\frac{\partial \boldsymbol{\varepsilon}^i}{\partial t} = \frac{\partial \Phi(\boldsymbol{\sigma}, \mathbf{B}^{\varepsilon^i})}{\partial \mathbf{B}^{\varepsilon^i}}; \tag{22}$$

If the pseudo-potential of dissipation is convex, positive, homogeneous and null at the origin, this approach provides always positive values of energy dissipation regardless initial and boundary conditions or external forces applied on the body (Maugin and Muschik, 1994). Therefore, the choice of pseudo-potential based on the restrictions imposed in the prior statement together with the proper inelastic strain energy unveils the final form of the set of thermodynamic consistent constitutive equations of the inelastic material.

Assuming that the tube is isotropic, pseudo-potential can be stated as (Maugin and Muschik, 1994)

$$\Phi(\boldsymbol{\sigma}, \mathbf{B}^{\varepsilon^i}) = \sum_{i=1}^{m} \frac{1}{2}\ \frac{3}{2E_i\tau_i}\left(\mathbf{B}^{\varepsilon^i} + \boldsymbol{\sigma}_{dev}\right) : \left(\mathbf{B}^{\varepsilon^i} + \boldsymbol{\sigma}_{dev}\right). \tag{23}$$

Where τ_i and E_i are the relaxation time and Young's modulus of the $i-th$ Kelvin-Voigt unit and $\boldsymbol{\sigma}_{dev}$ is the deviatoric part of the thermodynamic force $\boldsymbol{\sigma}$. Together with the anelastic and elastic thermodynamic forces, this pseudo-potential generates the following constitutive equations of the pipe

$$\boldsymbol{\sigma} = \frac{\nu_0}{\left(1+\nu_0\right)}\ \mathrm{tr}\ \boldsymbol{\sigma} + \frac{E_0}{\left(1+\nu_0\right)}\left(\boldsymbol{\varepsilon} - \sum_{i=1}^{m} \boldsymbol{\varepsilon}^i\right), \tag{24}$$

$$\frac{\partial \boldsymbol{\varepsilon}^i}{\partial t} = \frac{3}{2}\ \frac{1}{E_i\ \tau_i}\boldsymbol{\sigma}_{dev} - \frac{\boldsymbol{\varepsilon}^i}{\tau_i}, for\ i = 1, \ldots, m \tag{25}$$

$$\frac{\partial \boldsymbol{\varepsilon}^a}{\partial t} = \sum_{i=1}^{m} \frac{\partial \boldsymbol{\varepsilon}^i}{\partial t} \qquad (24)$$

Where ν_0, E_0 are the Poisson coeficient and instantaneous Young's modulus of the tube.

4. GOVERNING EQUATIONS

The equations of the unsteady flow model for viscoelastic pipes are derived by combining the basic and constitutive equations described in sections 3 and 4, respectively. Nevertheless, to establish the model equations, some additional hypotheses regarding the state of stress in the pipe are still required. Assuming that the pipe is anchored at both ends and is subjected to an internal pressure loading only, it results, neglecting shear stresses in the pipe wall and its inertia, that the mean circumferential stress component σ_θ is the only independent stress component in the tube. By considering it and taking into account the averaged values of the radial and circumferential stress components σ_r, σ_θ for thick-walled tubes (Tijsseling, 2007), the governing equations of the model can be stated as

$$\left[\frac{1}{\rho_0\, a^2}\right]\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} + 2\left(\sum_{i=1}^{m}\frac{\partial \varepsilon_\theta^i}{\partial t}\right) = 0;$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{2}{R\rho_0}\sum_{j=1}^{n} a_j = 0 \tag{27}$$

$$\frac{\partial v_j}{\partial t} + \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\alpha_j\rho_0}\left\{m_j + \frac{2}{R}a_j\right\} = 0, \quad j = 2,\dots,n \;;$$

$$\left(\frac{\partial \varepsilon_\theta^i}{\partial t}\right) + \frac{\varepsilon_\theta^i}{\tau_i} - \frac{R}{e}\frac{\psi}{E_i\tau_i}p = 0, \quad i = 1, \dots, m.$$

Where

$$a = \left\{\rho_0\left[\frac{1}{K} + \frac{2}{E_0}\left(\frac{R}{e} + \frac{\left(1+\frac{e}{R}\right)}{\left(2+\frac{e}{R}\right)} + \nu_0 - \nu_0^2\left(\frac{R}{e}\frac{1}{1+\frac{1}{2}\frac{e}{R}}\right)\right)\right]\right\}^{-1/2}; \tag{28}$$

is the wave-front velocity and the scalar ψ is given by

$$\psi = \left(\frac{R^2}{e(e+2R)} + \frac{6R^2(e+R)^2\ln\left[1+\frac{e}{R}\right]}{e^2(e+2R)^2}\right) - \nu_0\left(\frac{R}{e}\frac{1}{1+\frac{1}{2}\frac{e}{R}}\right). \tag{29}$$

The model is formed by a set of $(2+(n-1)+m)$ partial differential equations for the $(2+n-1+m)$ dependent variables $p, v, v_j\{j = 2, \dots n\}$, $\varepsilon_\theta^i\ \{i = 1, \dots m\}$. A suitable prescription of the initial conditions (at $t = 0$) and boundary conditions complete the mathematical formulation of the model.

Equations (27a) and (27b) are the balances of mass and linear momentum for the virtual structured mixture as a whole, respectively. The Eqs. (27c) and (27d) are the momentum equations for each constituent and the evolution laws of the circumferential anelastic strain.

5. RATE OF ENERGY DISSIPATION

5.1 Fluid

As suggested by Costa Mattos et al. (1995), when dealing with the continuum mixture theory, the second law of the thermodynamics should be postulated for the constituent and for the mixture as whole. By assuming isothermal transformations, it can be proved that, based on the assumptions made so far, the second law for the $j-th$ constituent reads as:

$$d_j = \frac{\partial \rho_j}{\partial t}\left(\frac{p_j}{\rho_j} - \rho_j\frac{\partial \Psi_j}{\partial \rho_j}\right) + a_j v_j\frac{P}{A} \;\geq\; 0, \quad \text{for } j = 1,\dots,n, \tag{30}$$

while that for the mixture is expressed as:

$$d_m = \sum_{j=1}^{n} m_j v_j \ \geq \ 0. \tag{31}$$

In Eq. (30), $\Psi_j = \Psi(\rho,\theta)$ stands for the Helmholtz free energy per unit volume of the $j-th$ constituent, which is supposed to be a differentiable function of the mixture density ρ and the absolute temperature θ. The terms d_j and d_m at the left-hand side of inequalities given by Eq. (30) and (31) represent the local rate of energy dissipation per unit length associated with the $j-th$ constituent and with the mixture as a whole, respectively. To ensure possible thermodynamic processes, they must be satisfied for all admissible transformations the constituents may be subjected to. As a result, the first term in Eq. (30) must vanish, by resulting $p_j = \rho_j^2 \, \partial\Psi_j / \partial\rho_j$ or, equivalently, $p = \rho^2 \, \partial\Psi / \partial\rho$ for the mixture. Summing up the rate of energy dissipation associated with the $j-th$ constituent, for $j = 1,\ldots,n$, in Eq. (30) we can come up with the overall rate of energy dissipation of the mixture by taking Eq. (31) into account:

$$d_f = \sum_{j=1}^{n} C_{j,j+1} \left(v_{j+1} - v_j \right)^2 + \frac{2}{R} C \left(v_n \right)^2 \geq \ 0, \tag{33}$$

Observe that positive-valued material constants are a sufficient condition that automatically satisfies the inequalities in Eq. (30-33). Hence, as the material constants are always positive (see Eqs. (14,15)), the proposed model is thermodynamically consistent a priori.

5.2 Tube

According to Maugin and Muschik (1994), assuming that the pipe is subjected to isothermal processes, the second law of thermodynamics can be stated as

$$\boldsymbol{\sigma} : \frac{\partial \boldsymbol{\varepsilon}}{\partial t} - \rho_t \, \frac{\partial \Psi}{\partial t} \geq \ 0 \, . \tag{34}$$

With the previously assumption of that the viscoelastic behavior of the problem is merely deviatoric, the second law can be rewritten as

$$\left(\boldsymbol{\sigma} - \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} \right) : \frac{\partial \boldsymbol{\varepsilon}}{\partial t} \ + \ \left(\boldsymbol{\sigma}_{dev} - \frac{\partial \Psi}{\partial t} \right) : \boldsymbol{\varepsilon}^a \ - \rho_t \, \frac{\partial \Psi}{\partial t} \geq \ 0. \tag{35}$$

The law of state given by Eq.(19) turns null the first right handed term of this inequality meanwhile the equation of state given by Eq.(20) aligned with evolutionary laws expressed in Eqs. (25-26) turns the expression of the local rate of energy dissipation per unit of length of the pipe d_t to be

$$d_t = \ 2\Phi \ = \sum_{j=1}^{m} \frac{2}{3} E_i \tau_i \left(\ \frac{3}{2} \ \frac{1}{E_i \ \tau_i} \boldsymbol{\sigma}_{dev} \ - \frac{\boldsymbol{\varepsilon}^i}{\tau_i} \ \right) : \left(\ \frac{3}{2} \ \frac{1}{E_i \ \tau_i} \boldsymbol{\sigma}_{dev} \ - \frac{\boldsymbol{\varepsilon}^i}{\tau_i} \ \right) \geq \ 0 \tag{36}$$

One may note that this expression is strictly positive always. Thus, the proposed model provides both fluid and pipe dissipation to be positive-valued scalars. Then,the thermodynamic consistency of the model is ensured.

6 NUMERICAL PROCEDURE

The mechanical model represented by the Eq.(27) forms a quasi-linear hyperbolic system of partial differential equations. The classic method of characteristics may be employed to such a system to produce the following set of compatibility equations (Whitham, 1974):

$$-\frac{1}{\rho_0 a} \frac{dp}{dt} + \frac{dv}{dt} \ + \frac{2}{R} \frac{a_n}{\rho_0} + 2a \sum_{i=1}^{m} \left[\frac{\varepsilon_\theta^i}{\tau_i} - \frac{R}{e} \frac{p\psi}{E_i \tau_i} \right] \ = 0, \ \text{ along } \ C^- \ \equiv \ \frac{dx}{dt} = - a, \tag{37}$$

$$\frac{1}{\rho a}\frac{dp}{dt}+\frac{dv}{dt}+\frac{2}{R}\frac{a_n}{\rho_0}\;-2a\sum_{i=1}^{m}\left[\frac{\varepsilon_\theta^i}{\tau_i}-\frac{p\psi D}{2eE_i\tau_i}\right]=0,\ \text{along } C^+\ \equiv\ \frac{dx}{dt}=+a,$$

$$-\rho_0\frac{dv}{dt}\ +\ \rho_0\frac{dv_j}{dt}+\frac{m_j+\frac{2}{R}a_j}{\rho_0\alpha_j}-\frac{2}{R}\frac{a_n}{\rho_0}\ =\ 0,\text{along } C^+\equiv\ \frac{dx}{dt}=0,$$

$$\frac{d\varepsilon_\theta^i}{dt}\ +\ \frac{\varepsilon_\theta^i}{\tau_i}-\ \frac{R}{e}\frac{p\psi}{eE_i\tau_i}=0,\text{along }\frac{dx}{dt}=0,$$

in which the characteristic equations of the model C^-, C^+, C^0 are equal to the eigenvalues of the problem $+a, -a, 0$, respectively. To find an approximated solution of equations (37), an integration in time along those characteristics equations can be employed in a discrete grid of the dependent variables (x, t)

One may observe that the first two compatibility equations are associated with non-null wave speeds, defining characteristics curves through which disturbances propagate from one side to the other of the spatial domain. The remaining compatibility equations are associated with non-propagating characteristics ($\frac{dx}{dt}=0$) and are intrinsically related to dispersive and/or dissipative effects. As one may expect, those equations are intrinsically related to the friction on the fluid (Eq.(27c)) and the dissipative phenomena due the inelastic deformation Eq.(27d)).

7. MODEL VALIDATION

The previous sections was devoted to the circumvented presentation of the mechanical modeling. Now, the accuracy of such an approach is accessed by comparing the numerical results of the head H fluctuations next to the valve with the data found in the experiment of Covas et al.(2005). The experimental apparatus that is composed by constant-pressure reservoir from which water flows at a steady-state velocity v_0 in a compliant pipe of length L that reaches a downstream valve located at $x = L$. Initially, this valve is fully open, then the transient is generated by a rapid valve closure maneuver taking place in $0.13s$. This settlement can be mathematically described by the following boundary conditions

$$p\,(x=0,t)\,=p_R, \tag{38}$$

$$v\,(L,t)\,=\begin{cases} v_0\left(1-\dfrac{t}{0.13}\right) & \text{if } 0\le t<0.13 \\ 0 & \text{if}\quad t\ge 0.13\end{cases}, \tag{39}$$

in which p_R is the reservoir pressure. The main features of the experimental setup can be found in Table 1 and the viscoelastic characteristics of the polyethylene tube used in experimental facility of Covas et al. (2005) are specified in Table 2.

Table 1 - Main characteristics of the Covas et al. (2005) experiment

L [m]	R [m]	e [m]	$\rho_0\left[\frac{\text{Kg}}{\text{m}^3}\right]$	μ [Pa.s]	$v_0\left[\frac{\text{m}}{\text{s}}\right]$	ν_0	$E_0\left[10^9\text{Pa}\right]$
271.8	0.0253	0.0063	998.2	0.001	0.028	0.46	1.43

Table 2 - Coefficients of the Kelvin-Voigt units of Covas et al.(2005) experimental tube

Unit	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$E_i\ \left[10^9\text{Pa}\right]$	7.17	161.29	8.71	2.92	10.78
$\tau_i\,[s]$	0.05	0.5	1.5	5	10

To enrich the analysis, the proposed model disregarding the effects of the viscoelasticity of the pipe is also shown in the same figure. As one can see, the elastic model is unable to represent the experimental data. This model overestimates the values of the head the in the whole time span in addition to be not in phase with the experimental responses. On the contrary, the experimental and the proposed modelling responses have good agreement regarding phase or magnitude.

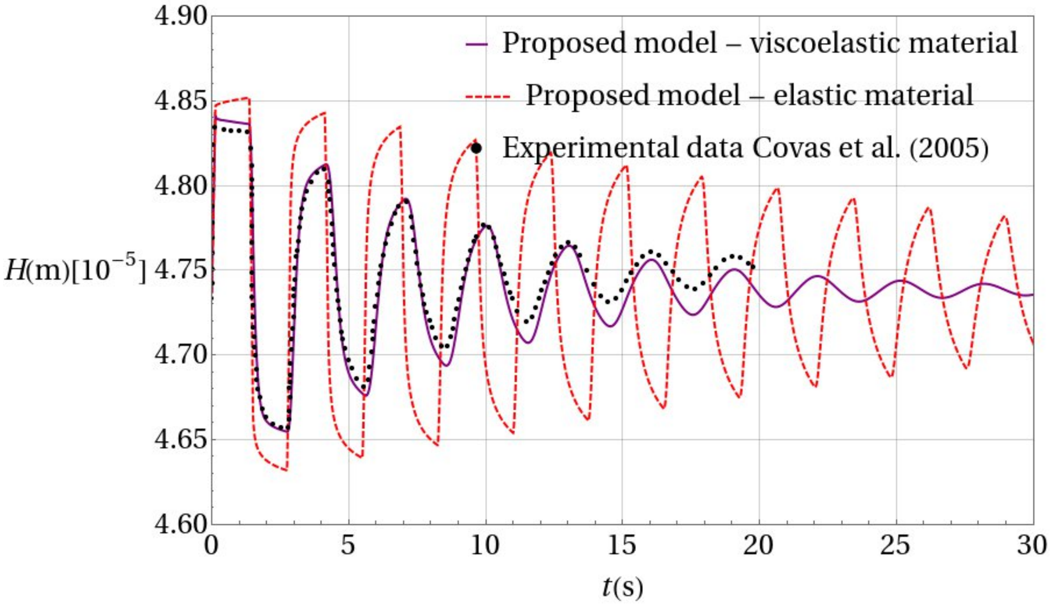

Figure 2 − Pressure head histories at the pipe mid-length of Covas et al.(2005) experiment. Comparison between the present model with and without taking into account the tube viscoelasticity.

Figure 3 shows the local rate of energy dissipation in the mid-length of the pipe normalized by the dissipation on the fluid in the steady state d_{f0} through time. As one may note, the energy dissipation on the fluid and on the pipe are of same order of magnitude. Thus, assuming the pipe to be elastic in this case is a rather coarse approximation. Thus, one of the main reasons behind the differences found in the head responses shown in Fig.2 relies on the additional energy dissipation brought by the inelastic deformation of the pipe.

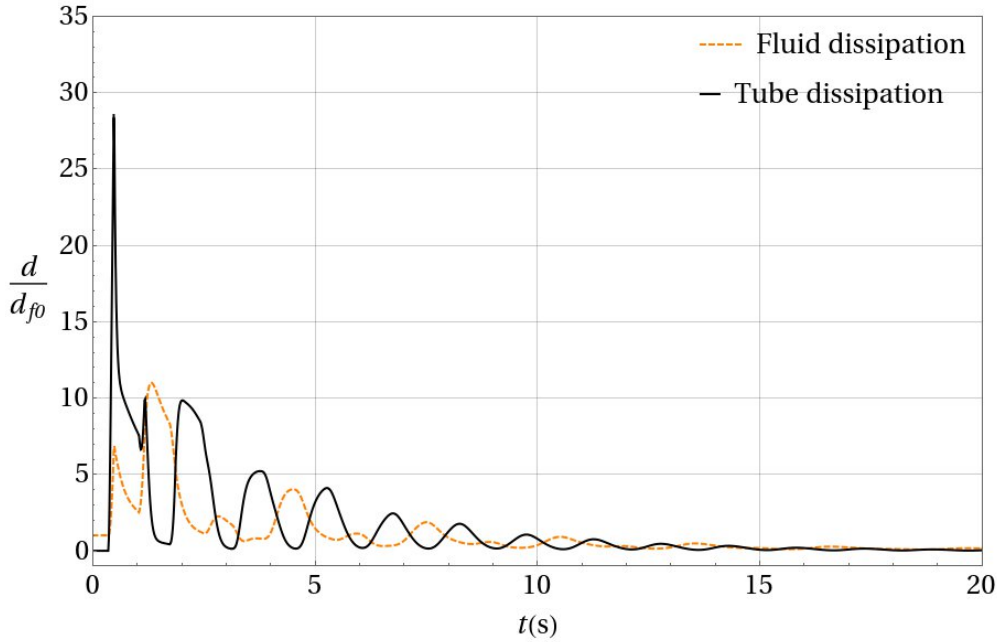

Figure 3. Normalized rates of energy dissipation on the fluid and tube at the mid-length of the pipe versus time.

5 CONCLUSION

It has been presented in this work an extension of fluid transients analysis in order to handle the viscoelastic behavior of the tube. The model is based on a thermodynamic consistent framework grounded in a transient flow model previously developed by the authors with the inclusion of treatment for the viscoelastic responses of the tube. Such a treatment invokes the generalized Kelvin-Voigt mechanical model in addition to the thermodynamics of irreversible processes with internal variables to find the tube constitutive equations and rates of energy dissipation. The final set of partial differential equations is solved by an approximation based on the method of characteristics. Besides its capability of predicting head histories with fair accuracy, the thermodynamic framework on which the model is based allows computing the rates of dissipation on fluid and on the solid distinctly and accurately. Such characteristic is a new feature to the authors' knowledge in the literature and may be useful for further theoretical and practical analysis in the field.

The interference of the viscoelasticity in the pressure responses seems to be of importance and the assumption of a linear elastic tube when they are not can result in responses that are really distant from reality. This preliminary result is important, but a broad study of the influence of such an effect in the diverse range of transient variables to a deep understanding of the phenomenon is still a need.

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