

COB-2021-1301

BIDIRECTIONAL EVOLUTIONARY STRUCTURAL OPTIMIZATION FOR STRUCTURES UNDER MULTIPLE DYNAMIC LOADS

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Abstract. *With the advancement of computational resources in engineering, the use of topological optimization tools has become a constant in the day-to-day design of structures and parts. However, there is the need to maintain the level of safety in optimized projects, since the final structure is optimally designed. Only in specific cases, safety can be assured based on a single static loading case, being common the existence of multiple dynamic loads throughout the part's service life. In this work, a code for topological optimization is developed to account for multiple dynamic loading cases, resulting in feasible optimized structures. The code is based on the BESO methodology, where the transient analysis is performed by the Newmark method. A strain energy-based criteria and the concept of dynamic equivalent static loading are used to perform the optimization. Examples from the literature are used to validate the developed algorithm, demonstrating the importance and the effect of these loads on the final topology of the structure.*

Keywords: *topological optimization, multiple and dynamics loads, finite elements BESO methodology, solid mechanics.*

1. INTRODUCTION

Numerical optimization methods have undergone great advances in recent decades, especially topological optimization methods. To find structures capable of supporting specific loads in defined domains and boundary conditions with the least amount of material possible, several topological optimization methodologies have been created and expanded to support increasingly practical problems.

Algorithms such as BESO (Bi-directional Evolutionary Structural Optimization) use the FEM (Finite Element Method) to evaluate portions of the material and simultaneously remove and place material in the domain, in order to achieve the highest possible efficiency in mass-stiffness ratio of the material structure or some other parameter that you want to optimize. In its classic form, given the boundary conditions, a domain, and the loads to which the structure is subjected, BESO is capable of reaching an optimal configuration regardless of the initial configuration, and with good independence from the finite element mesh. The result, however, has limits when applied to real situations, because structures are rarely subjected to only one case of loading during their entire life span.

The growing evolution of additive manufacturing technologies adds possibilities for using optimized structures in real and relevant applications. To obtain the most efficient structure possible in a real situation, however, it should be taken into account that it is unlikely to be subject to just one static and single case of loading. The objective of this work is to expand the BESO methodology to be able to optimize structures subject to transient loads in an efficient way, to obtain topologies that can meet actual optimization demands.

1.2 Literature review

Topological optimization problems with considerations for multiple load cases, as well as problems involving transient loads, have been studied by several authors. Natke (1993) proposes the use of convex modeling of permissible excitations as a means of simplifying a topological optimization problem with dynamic loads, representing it as a non-linear problem suitable to the existing standard computer programming procedures. The structure considered for the optimization would be a system divided into subsystems, which would be added and removed according to their contribution to the structure. The final structure, however, is sensitive to the chosen convex model, and the method of dividing the domain into subsystems limits the scope of what can be solved with this method.

Bendsøe and Sigmund (2002) used an efficient Finite Element Method (FEM) to implement a topology optimization that divides the continuous domain into a finite number of elements. They proposed the Solid Isotropic Material with Penalization (SIMP) methodology to remove elements whose contribution to the compliance of the structure is low, applying a penalty on the density of the element in question, making it proportionally empty (through the use of penalties) for the next iterations. The authors also expand the method to consider multiple loads by the use of a weighted average flexibility for each separate loading case as an objective function. However, it must be taken into account that the final structure also had areas of intermediate densities, and thus, its flexibility is often overestimated.

Zhou and Li (2005) consider multiple loads in a fiber-reinforced composite material model, calculating the optimal fiber orientation in each loading case. Then, in the optimization of the structure itself, they restrict the elastic matrix of the same to better approach the elastic matrices defined by the optimum structure of each loading case.

Huang and Xie (2010) present in their book the Bi-Directional Evolutionary Structural Optimization (BESO) method. The differential of the method is that at each iteration, in addition to removing solid elements that contribute little to the stiffness of the structure, it also fills empty elements whose calculated sensitivity indicates a high contribution of the element. This makes the method robust and computationally efficient, in addition to being simple to implement.

Nedushan *et al.* (2011), proposes the optimization of structures subject to transient loads (time varying) using the concept of equivalent static loads, to simplify the analysis. The authors also choose the maximum strain energy during the loading interval as an objective function, since it varies over time, making the use of flexibility as an objective function much more complicated. They also claim that minimizing the maximum strain energy ensures that at all other time steps the structure will have less strain energy.

Behrou and Guest (2017) solve a topological optimization problem with multiple dynamic loads, using Newmark's methodology to calculate the elementary displacements, velocities and accelerations in each time step and the SIMP method to minimize the sum of the deformation energies of each step.

Several authors, including Iwamura and Faria (2013), Li *et al.* (2014), Jaouadi and Lahmer (2015), Nutu (2015), and Lógó *et al.* (2017) present other methodologies for the topological optimization of structures subject to multiple cases of loading, all different from each other but with valid assumptions, as the one presented in this work.

2. THEORETICAL FUNDAMENTALS

According to James *et al.* (2009), topological optimization can be defined as the process of determining the optimal number and configuration of members within a physical design domain to achieve a specific design objective. The optimization process varies according to the applied methodology, but it is common that for continuous structures each defined structural member of the material is evaluated according to the restrictions and objectives and that, based on this evaluation, whether the configuration and existence (or non-existence) of such a member is decided.

As a numerical solution for the structural evaluation of continuous solids in all their volume, the Finite Element Method (FEM) is used. In the FEM, the continuous domain is divided into several subdomains, which are called finite elements. The initial domain is, therefore, divided into a mesh of finite elements so that it is possible to numerically solve the problem in each element and, consequently, in the entire structure. Depending on the problem analyzed, there are several types of elements that can be used. In this work, a four-node isoparametric finite element is used to evaluate plane stress states. The element presents good accuracy and computational cost. Each node allows two translational degrees of freedom (DOF). In the FEM, shape functions are assumed to represent the displacement field in the interior of the finite elements, which are in turn interpolated based on the node values. This element allows full explicit integration that facilitates assemblage of stiffness and mass matrices.

2.1 BESO method

According to Huang and Xie (2010), the BESO method (Bi-directional Evolutionary Structural Optimization) is based on the proposal of evaluating by the FEM the relevant properties of the structure in each finite element and its contribution to the structure as a whole. Based on this, a defined percentage of filled elements with the lowest contribution of the structure is transformed into empty elements, and likewise, empty elements whose contribution to the structure is assessed as high are transformed into filled elements again. This process is repeated until the convergence conditions are reached.

For this work, the convergence conditions are configured in a volume constraint that must be achieved and on the condition that the change in the value of the objective function between consecutive iterations is less than a predefined value.

2.1.1 Objective function

The BESO methodology is widely used to find the most rigid structure (lowest Compliance, C) given that only a fraction of the domain's volume must be composed of full elements. The problem statement can be represented in the form of Equations (1), (2), and (3) as described in Huang and Xie (2010).

$$\text{Minimize } C = \frac{1}{2} \mathbf{f}^T \mathbf{u} \quad (1)$$

$$\text{Subject to } V^* - \sum_{i=1}^{N_e} V_i x_i = 0 \quad \text{with } x_i \in \{1; x_{min}\} \quad (2)$$

where C is the flexibility (Compliance) of the structure, \mathbf{f} is the vector of applied loads, \mathbf{u} the displacement vector, V^* the target volume for the final structure, N_e the number of elements in the FEM mesh, V_i the volume of an individual element, x_i a binary variable that indicates whether the element is full when it is 1 or empty when it is x_{min} (1×10^{-3}), a value very close to 0, used to avoid numerical errors.

Knowing that the flexibility of the structure is nothing more than the sum of the strain energy of each element, through Equation (3) the objective function is calculated in each iteration.

$$C = \frac{1}{2} \sum_{i=1}^{N_e} x_i^p \mathbf{u}_i^T \mathbf{K}_i \mathbf{u}_i, \quad (3)$$

where \mathbf{u}_i is the nodal displacement vector of the i -th element, \mathbf{K}_i is the elemental stiffness matrix and p the penalty factor, which is a parameter used to modify the value of x_i exponentially, so that x_{min} can be as small as is convenient to facilitate the derivative of the objective function w.r.t. x_i and definition of the sensitivity number .

2.1.2 Sensitivity number

In the BESO method's algorithm, the evaluation of the contribution of each finite element to the compliance of the structure is done through the difference in the total strain energy of the structure caused by the removal of the element. This difference is called the sensitivity number and is equal to the derivative of the strain energy of the element itself. The sensitivity number in its basic form can be obtained through Equation (4).

$$\alpha_i^e = -\frac{1}{p} \frac{\partial C}{\partial x_i} = \frac{1}{2} x_i^{p-1} \mathbf{u}_i^T \mathbf{K}_i \mathbf{u}_i, \quad (4)$$

where α_i^e is the elementary sensitivity. Having obtained the sensitivity numbers of each element, the solid elements with less sensitivity and that are below an established threshold are selected to be removed at each iteration, as well as the empty elements with large sensitivity are selected to be filled.

In its basic form, the sensitivity number can cause problems in the final structure, such as the so-called checkerboard pattern, which can be identified in the final structure as a grid formed by interspersed empty and full elements, in a way that cannot be constructed in practice. This is solved by applying a filter to the formula that takes into account the sensitivity of the elements close to the element being evaluated when deciding whether or not to remove the element. This is done by defining the elementary sensitivity as a weighted average of the sensitivities of nearby elements, the weights of each sensitivity being given by the distance of the element being analyzed to each element whose center is within a predefined radius. As explained in Haug and Xie (2010), the shorter the distance, the greater the weight used.

2.2 Consideration of multiple load cases

Topological optimization tends to find the best possible solution for a single case. The topology of the structure is modified to better resist the action of a set of loads simultaneously applied, being perhaps not as efficient, or even unstable, for any other configuration of loads to which the structure could be subjected, which is often not practical or smart from a design standpoint. Structures in which different punctual loads are applied at different times as well as structures with mobile loads cannot be solved without considering these multiple loading cases.

Therefore, during the optimization process, the contribution of each element must be considered for all different loading cases. In practice, small changes need to be made to the objective function and the sensitivity number, as shown by Huang and Xie (2010). A weight value must be assigned to each load case and applied to both equations to obtain a weighted average. Equations (5) and (6) show the objective function and the sensitivity number α (for each element at iteration i) modified to consider multiple loads:

$$f(\mathbf{x}) = \sum_{k=1}^{M_c} w_k C_k \quad (5)$$

$$\alpha_i^e = \frac{1}{2} x_i^{p-1} \sum_{k=1}^{M_c} w_k (\mathbf{u}_i^T \mathbf{K}_i \mathbf{u}_i)_k, \quad (6)$$

where M_c is the number of load cases considered and w_k is the weight assigned to each load case. The sum of the weights must be equal to 1, and the weights assigned to each case must be chosen according to the objectives of the optimization to be carried out, taking into account the importance of a load with respect to the others. In this work, equal importance was assumed for each load case for a better demonstration of the methodology, except for the last case (Problem 3), which considers only the load cases where there are peaks of energy deformation.

2.3 Consideration of transient response

The traditional BESO method is able to optimize the topology of structures for known static loads. Faced with a dynamic load in which the value of the applied force changes over time, a solution must be applied so that the final topology is suitable for any load applied in a given period. For this, the total period can be discretized and divided into a finite number of time intervals. In each instant, an equivalent static load, which according to Park and Kang (2003) is a static load that generates the same displacement field generated by the dynamic load in a certain time, will be applied, and each one of these loads will be a load case in the BESO method for multiple loads.

Must also take into account the fact that these loads applied successively generate a transient response, different from what would be obtained by applying each load separately. For each instant of time, it is necessary to find the displacement, speed and acceleration of each element of the structure and use this information in the calculation of elementary flexibility.

The response of the structure's excitation by an external force can be modeled using the equation of motion, as follows in Equation (7).

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}_v\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) - \mathbf{f}(t) = 0, \quad (7)$$

where \mathbf{M} , \mathbf{C}_v and \mathbf{K} are, respectively, the mass, damping and stiffness matrix. In addition, $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and \mathbf{u} are the of acceleration, velocity and displacement vectors, respectively, and \mathbf{f} is the vector of external forces. The equation of motion is a second-order differential equation that must be solved for each node in arbitrary steps in the time domain to obtain nodal displacements at each time point. Damping modeling is usually done using the Rayleigh hypothesis (for viscous damping) which indicates that $\mathbf{C}_v = \gamma\mathbf{M} + \delta\mathbf{K}$. The coefficients γ and δ are obtained, in the case of two or more damping ratios (ξ_1, ξ_2) available for two vibration frequencies (ω_1, ω_2), from the solution of the system of equations $\xi_i = 0.5(\omega_i + \gamma/\omega_i)$, $i = 1, 2$.

2.4 Newmark algorithm

Newmark's implicit numerical integration method can be used to obtain the solution of the equation of motion in the time domain. Three recurrence equations are used to obtain the displacement, speed and acceleration in each step, and are represented in Equations (8), (9) and (10), respectively.

$$\mathbf{u}_n = \left[\frac{1}{\alpha(\Delta t)^2} \mathbf{M} + \frac{\beta}{\alpha \Delta t} \mathbf{C} + \mathbf{K} \right]^{-1} \left\{ \mathbf{f}_n + \mathbf{M} \left(\frac{1}{\alpha(\Delta t)^2} \mathbf{u}_{n-1} + \frac{1}{\alpha \Delta t} \dot{\mathbf{u}}_{n-1} + \left(\frac{1}{2\alpha} - 1 \right) \ddot{\mathbf{u}}_{n-1} \right) + \mathbf{C} \left(\frac{\beta}{\alpha \Delta t} \mathbf{u}_{n-1} + \left(\frac{\beta}{\alpha} - 1 \right) \dot{\mathbf{u}}_{n-1} + \left(\frac{\beta}{\alpha} - 2 \right) \frac{\Delta t}{2} \ddot{\mathbf{u}}_{n-1} \right) \right\} \quad (8)$$

$$\dot{\mathbf{u}}_n = \dot{\mathbf{u}}_{n-1} + [(1 - \beta)\ddot{\mathbf{u}}_{n-1} + \beta]\ddot{\mathbf{u}}_n \Delta t, \quad (9)$$

$$\ddot{\mathbf{u}}_n = \frac{1}{\alpha(\Delta t)^2} (\mathbf{u}_n - \mathbf{u}_{n-1}) - \frac{1}{\alpha \Delta t} \dot{\mathbf{u}}_{n-1} - \left(\frac{1}{2\alpha} - 1 \right) \ddot{\mathbf{u}}_{n-1}, \quad (10)$$

where the subscript n indicates the current time step, α and β are parameters related to the stability of the method and Δt the time step. In this work, $\alpha = 0.25$ and $\beta = 0.5$ to ensure stability and Δt is chosen small enough to attain accuracy.

3. METHODOLOGY

The implemented algorithm follows the flowchart shown in Figure 1. The parameters used in the optimization, as well as the boundary conditions, will be chosen according to the problem to be solved.

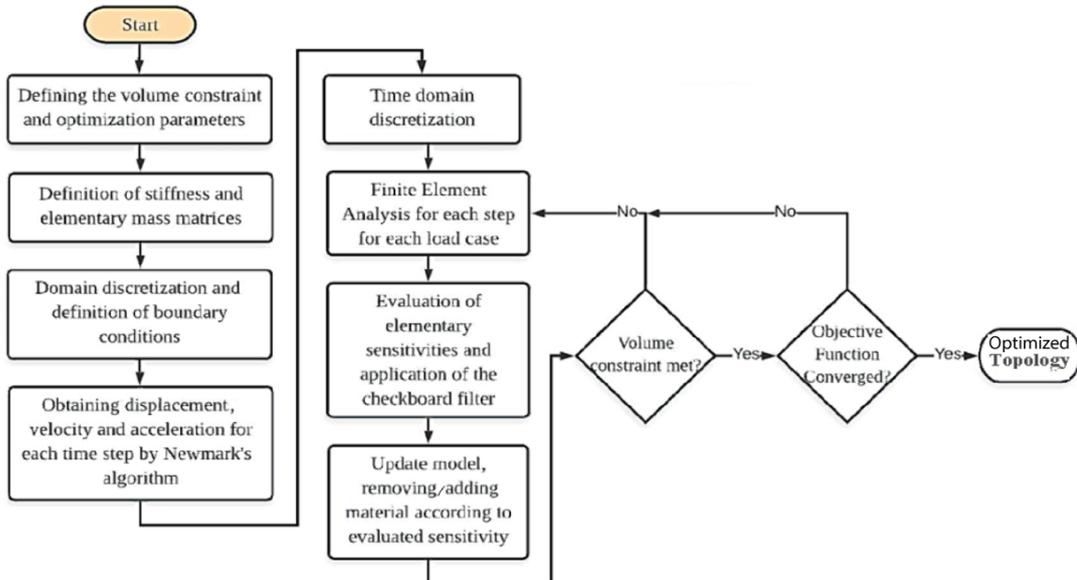


Figure 1. Flowchart of the BESO method adapted to include multiple loading cases.

As an expansion of the use of the BESO method for multiple loads, the algorithm used here tries to solve optimization problems of structures subject to dynamic loads. This is done by discretizing the time interval for applying the forces in a finite number of steps and considering the structure of each of these steps as being under a separate equivalent static loading condition.

To obtain the equivalent static forces, the Equation (7) of motion is solved for each degree of freedom (DoF) of the initial structure in the instant of time being considered. Using Newmark's implicit numerical integration method, accelerations, velocities and displacements are obtained, for each instant of time, in each DoF.

From this, it is possible to use the displacements field to obtain the equivalent static force in each DoF, for each instant of time, using Equation (11).

$$\mathbf{F}_e^{i,n} = \mathbf{K}\mathbf{u}^{i,n}, \quad (11)$$

where superscripts i and n represent the i -th degree of freedom in the n -th time, respectively and \mathbf{F}_e is the equivalent static force for this DoF and the present instant of time.

Thereafter, each time step is used, previously discretized and subjected to a loading whose response is described by the calculated equivalent forces, as a separate load case and the BESO methodology is applied for multiple load cases. That is, the weighted average of the Compliance for each loading case is used as an objective function of optimization, the elementary sensitivity is calculated from the weighted average of the sensitivities for each case and the decision is made to remove or add elements based on calculated sensitivity.

3.1 Proposed problems

Based on the described methodology, two problems from the literature and one proposed by the authors of this article are analyzed. The first is a multiple static load case proposed by Behrou and Guest (2017), the second problem is a multiple dynamic load cases, also reported by the same authors, and lastly a situation considering only the time instants when strain energy Peaks (DEP) happens. These examples will be referred to, respectively, as Problem 1, Problem 2, Problem 3.

Problem 1 proposes to compare the results obtained with the application of a different methodology, which follows a different basis for optimization. Problem 2 seeks to apply the method proposed here to a dynamic situation and compare the result with that of another method found in the literature. Finally, Problem 3 proposes to compare the result with Problem 2, using less computational time. As far as possible, the problems proposed in this work uses the parameters reported in the literature, allowing direct comparisons of results.

3.1.2 Problem 1 – Cantilever beam under multiple static loads

Problem 1 was taken from section 4.1 of Behrou and Guest (2017). The proposal is the optimization of a cantilever beam defined in a rectangular domain of 270×90 , 4-node isoparametric elements. In this problem, the multiple loads are assumed static. The objective function is the minimization of the compliance under two static load cases. The target volume fraction is 0.35. This Problem is meant to serve as a reference for the following problems that will consider time-varying loads. So, the beam is assumed carrying two different static load cases, as shown in Figure 2. The parameters chosen for optimization are presented in Table 2.

In this case, \bar{f}_1 and \bar{f}_2 are both equal to f_{max} , a constant load value that is applied at point A and point B, respectively, as shown in Figure 2. This value will represent the maximum load value of a time-varying load in the following Problem 2.

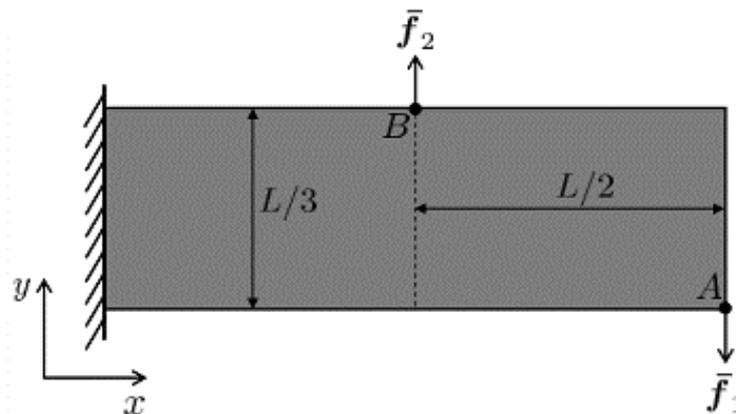


Figure 2. Domain, boundary conditions and loads of Problems 1 and 2. (Adapted from Behrou and Guest, 2017).

Description	Parameter	Values	Units
Young's Module	E	200	[Pa]
Poisson's ratio	N	0,33	[-]
Volumetric fraction	V_{max}	0,35	[-]
Volumetric evolution rate	ER	0,02	[-]
Minimum radius allowed	r_{min}	4	Finite elements
Length	L	0,9	[m]
Penalty	P	3	[-]
Material density	P	7800	[kg/m ³]
Minimum allowable density	ρ_{min}	0,0001	[kg/m ³]
Finite element thickness	E	0,01	[m]
Maximum applied force	f_{max}	10000	[N]

Table 2. Parameters used in the optimization of Problem 1.

3.1.3 Problem 2 – Cantilever beam under multiple dynamic loads cases

Behrou and Guest (2017) also proposed problem 2. In this case, it is assumed time varying multiple loads. In order to compare the results with the reference paper, there is no damping in this example. The same domain and boundary conditions as in Problem 1 are used, as shown in Figure 2. The two forces \bar{f}_1 and \bar{f}_2 , however, assume time-dependent values, as shown in Figure 3.

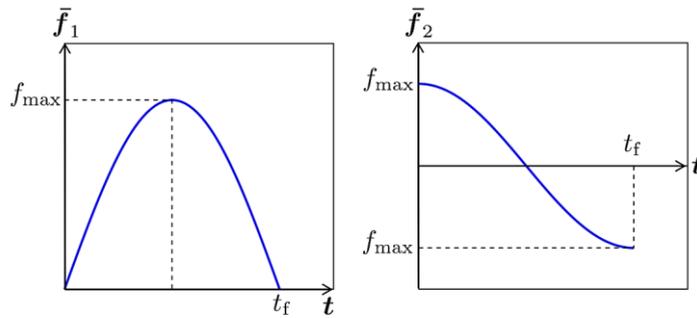


Figure 3: Time history for the applied forces (Adapted from Behrou and Guest, 2017).

The values assumed by the forces follow Equation (12):

$$\bar{f}_1 = f_{max} \sin(t) \quad \text{and} \quad \bar{f}_2 = f_{max} \cos(t), \quad (12)$$

Each force is applied separately from the other, configuring two different load cases. It is assumed the total time for the analysis of π seconds and $N=20$ time steps.

3.1.4 Problem 3 – Cantilever beam under multiple dynamic loads cases considering only DEP

In order to compare the result with the previous Problems, this case is proposed in this paper. The same domain and boundary conditions as in Problem 2 are used. As in Problem 2, the two forces \bar{f}_1 and \bar{f}_2 assume time-varying, as shown previously in Figure 3. However, in order to reduce the processing time of the algorithm, it is proposed here to consider only the instants of time where there are peaks in the Compliance of the structure (DEP), which are more relevant to the objective function being minimized, as shown in Figure 7(b). It is expected that this will reduce the computational costs when dealing with time-varying loads and numerical integration. The algorithm collects the instants of time when the peaks occurs and performs the optimization considering only those instants of time. The parameters used in the optimization are the same as in Problem 2.

4. RESULTS

In this section, the result obtained for each problem presented in the Methodology section will be compared with the results presented in the literature. This includes the final topology, objective function values and convergence parameters.

4.1 Results for Cantilever beam under multiple static load cases (Problem 1)

Figure 4 (a) shows the final topology found by Behrou and Guest (2017) for Problem 1. The authors do not present the convergence history or final values of the objective function in their article. In Figure 4 (a) the white color represents the voids. Figures 4 (b) and 5 represent the topology obtained in this paper and its convergence history for the same problem. In Figure 5, the red circles represent the volumetric fraction and the blue squares represent the objective function.

Analyzing Figures 4 (a) and 4 (b), the two topologies present clear differences, and it can be assumed that they are due to the difference in the methodologies used as well as in the objective function chosen to be minimized. While in this paper, the BESO methodology was used to minimize an objective function composed of the weighted average of the Compliance of the two load cases, Behrou and Guest (2017) use the SIMP methodology to minimize the sum of the strain energy corresponding to the responses of structures subjected to each of the two load cases.

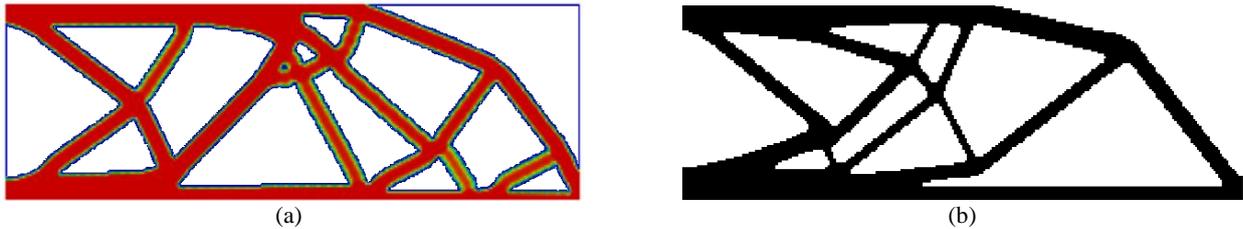


Figure 4. (a) Topology taken from Behrou and Guest (2017) and (b) obtained in this paper, for Problem 1.

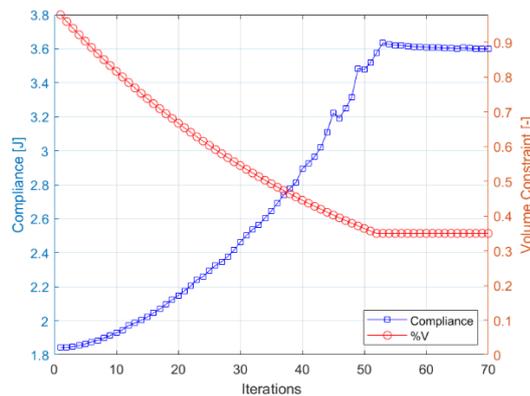


Figure 5. Weighted average Compliance [J] and volume fraction × Iterations in this work, for Problem 1.

4.2 Results for Cantilever beam under multiple dynamic load cases (Problem 2)

The topology found by Behrou and Guest (2017) for Problem 2 is found in Figure 6 (a), and its strain energy × time graph in Figure 7 (a). Figures 6 (b) and 7 (b) present the same results found with the use of the method proposed in this paper.

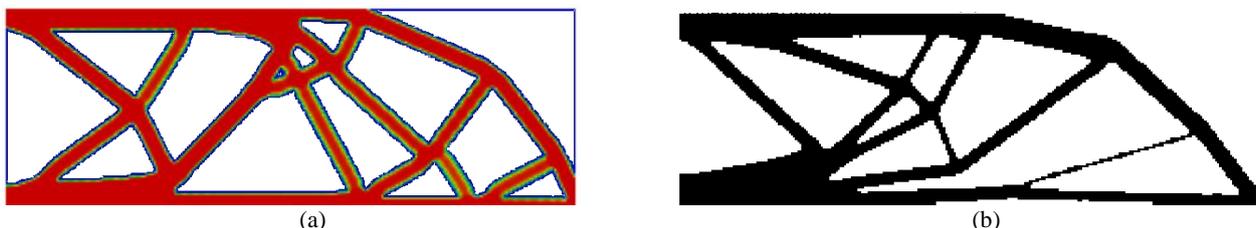


Figure 6. (a) Final topology from Behrou and Guest (2017) and (b) obtained in this paper, for Problem 2.

Figure 7 (a) and (b) present the Compliance of the initial structure represented in blue, and in red, the Compliance after optimization.

Solving this problem, a total number of 40 loading cases were considered that comes from the two dynamic forces applied and discretized in 20 time steps. The topology found by the implemented algorithm clearly differs from that presented by Behrou and Guest (2017). The methodology used leads to different optimal topologies, however comparing the Compliance of the structure over time in both methods, the structures obtained by this paper seems to be capable to

offer more reduction in the final Compliance. In the proposed algorithm a peak value lower than 20 Nm in the strain energy is obtained, whereas in the Behrou and Guest (2017) this value was approximately 20 Nm. Again, the present formulation follows BESO framework and Behrou and Guest (2017) follows SIMP formulation that allows intermediate density elements.

Differences in results are explained by the following reasons: (i) Behrou and Guest (2017) overestimate the final strain energy values, due to the presence of elements of intermediate densities, inherent to the SIMP; (ii) the implemented algorithm optimizes the structure taking into account each instant of time as a separate load, in opposition to the choice of Behrou and Guest (2017) that minimizes the sum of the deformation energies during the entire transient response; (iii) in this paper, it is used the mean Compliance for each load case as objective function, while Behrou and Guest (2017) use the strain strain energy of the structure itself.

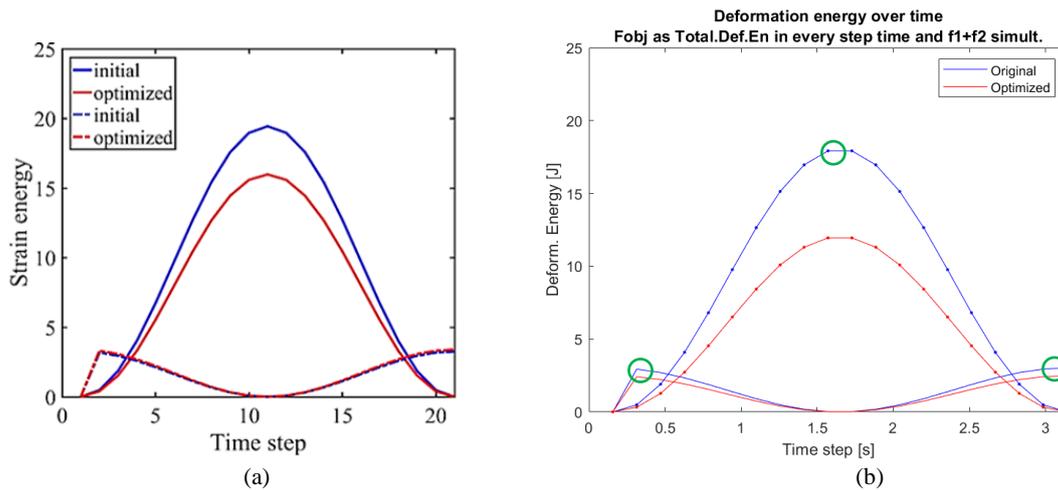


Figure 7. (a) Compliance [J] \times Time step[s], taken from Behrou and Guest (2017) for Problem 2; (b) obtained in this paper, the relevant DEP (strain energy Peaks) are highlighted with green circles.

4.3 Comparison between results of static and dynamic problems (Problem 1 and Problem 2)

Since Problem 1 and Problem 2 have the same optimization parameters and boundary conditions except for the applied forces, it is convenient to compare the results obtained for both problems. In Figures 8 (a) and 8 (b), the topology obtained here in the optimization of Problem 1 and Problem 2 are displayed, for better visualization, respectively. In Figures 9 (a) and 10 (b), the convergence history for the same problems, in the same order. In Figure 9, red circles represent the volumetric fraction and blue squares, the objective function.

It is interesting to note that despite the fact that in both cases all forces are applied at the same points, in Problem 1 there is a load case for each force, and in Problem 2 there are 20 load cases for each force as the forces vary over time. It should also be remembered that the Compliance obtained for each problem is the weighted average of the Compliance evaluated for each load case. This means that some load cases can generate larger strain energy when applied to the structure than others. This is quite clear in Figure 7 (b), which shows how the strain energy of the structure varies over time. In the same Figure, it is also possible to note that the strain energy generated by the application of the force f_1 at maximum intensity is much greater than the strain energy generated by the application of the force f_2 at its maximum intensity.

Figures 8 (a) and 8 (b) presents minor differences, but visible. This suggests that, even for forces applied to the same points, optimizing the topology of a structure for the highest applied load brings a different result than optimizing the same structure subject to forces whose intensities change over time.



Figure 8. Topology found in this work as a result for: (a) Problem 1; (b) Problem 2.

As for Figures 9 (a) and 9(b), the convergence history shows that the number of iterations for both cases is quite similar. However, the computational time doubles for Problem 2, with respect to Problem 1. This is due to the need to evaluate acceleration, velocity and displacements using Newmark scheme, and due to the large number of load cases considered.

Some tests performed previously showed that using twice as many load cases for Problem 2, the proposed code takes approximately twice as long to arrive at the same solution, indicating that the computational time scales with the number of time steps. This highlights the need for a procedure to overcome this difficulty.

Finally, the difference found in the results of the objective functions indicates that a structure optimized for a single load case will be better prepared for it than a structure optimized for several cases. In general, the fewer cases are considered, the better the result for exactly these cases. On the other hand, any other loading situation that can be applied during its service life may overstress the structure, as the structure will not be prepared for this. In this way, optimizing for more load cases offers safety for the design.

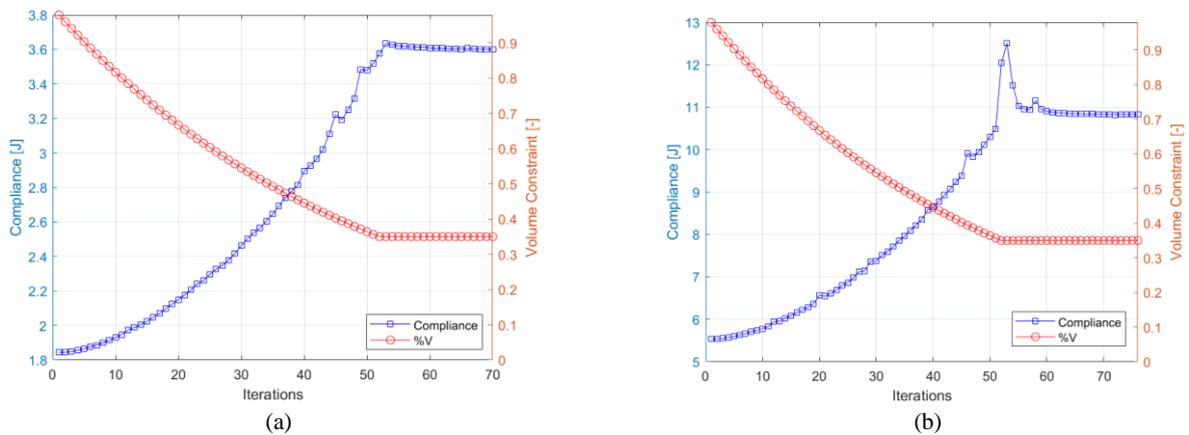


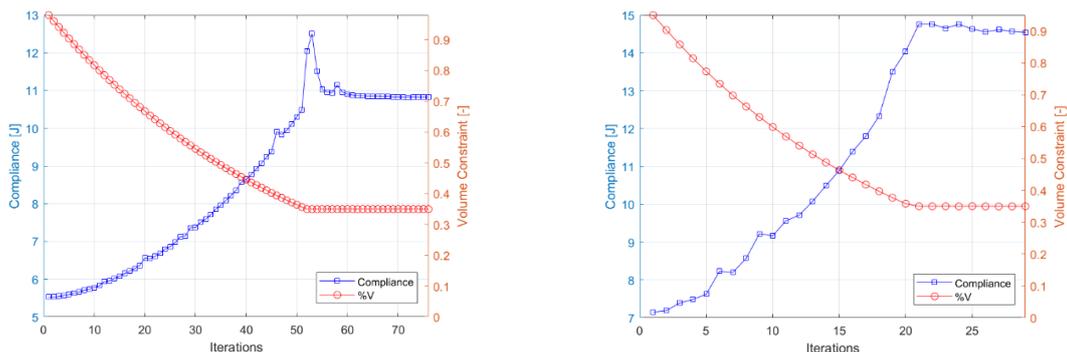
Figure 9. Compliance (weighted average of Compliance [Nm] for each load case) and Volumetric fraction \times Iteration obtained by the author using the proposed algorithm for: (a) Problem 1; (b) Problem 2.

4.4 Comparison between dynamic load cases using all and DEP only time steps (Problem 2 and Problem 3)

Problem 2 and Problem 3 have the same optimization parameters and boundary conditions, with the exception that Problem 3 considers only the DEP instants. It is convenient to compare the results obtained for the two problems. Figures 10 (a) and 10 (b) show the topology obtained here in the optimization of Problem 2 and Problem 3, respectively. Figures 11 (a) and 11 (b) presents the convergence history for the same problems. In Figure 11, red circles represent the volumetric fraction and blue squares, the objective function.



Figure 10. Final topology found by this paper: (a) All load cases considered; (b) only load cases in DEP instants.



(a) (b)

Figure 11. Compliance weighted average [J] and volumetric fraction \times Iterations obtained in this paper for: (a) Problem 2 and (b) Problem 3.

Comparing the topology for Problem 2 and Problem 3, there is a great similarity between the final topologies. However, there was a large time saving for Problem 3 when compared to Problem 2, since there are fewer load cases to be considered (only the most important).

Reasonable differences are noticed between the objective functions, the compliance value in Problem 2 stabilizes around 11 Nm (Figure 11a) - and in Problem 3 this value stabilizes around 14.5 Nm (Figure 11b), indicating that the topology generated in Problem 3 is less robust to the load configuration applied to these problems.

Finally, the difference found in the results indicates that the optimized structure using all the time instants will be better prepared than the optimized structure using only the peaks of strain energy. However, the processing time required when using all the time instants is much longer than when using only the most relevant time instants. At this point, designer should think about the trade-off that exists between accuracy and computational costs.

5. FINAL REMARKS

The present work expanded the BESO methodology of topological optimization to be able to optimize structures subject to time-varying loads. With the implementation of a code in the MATLAB software based on the code provided by Huang and Xie (2010), an algorithm was developed capable of optimizing two-dimensional beams subjected to time-varying dynamic loads. Using examples from the literature as a comparison, the methodology presented here proved to be able to perform the optimization in order to achieve effective results with a relatively simple code.

The results obtained show that, comparing the strain energy over time resulting from the optimization performed in this work with the optimization by Behrou and Guest (2017), the obtained results present lower strain energy along time, which means that the optimized structure is stiffer for the transient loads applied.

The comparison between the results of Problems 2 and 3 reinforces that the optimization made for the DEP of a structure is similar but not equivalent to the optimization using all time-steps, however a trade-off between accuracy and computational cost should be taken into consideration.

In addition, the comparison between the results of Problems 2 and 3 proposes that a transient analysis that considers only the moments of time when there are peaks of strain energy can result in very similar topologies requiring less processing time.

6. REFERENCES

- Behrou, R., Guest, J. K. "Topology optimization for transient response of structures subjected to dynamic loads". 18th. AIAA/ISSMO Multidisciplinary Anal. and Opt. Conf. 2017. doi: 10.2514/6.2017-3657.
- Bendsøe, M. P., Sigmund, O. "Topology Optimization". Springer-Verlag, Berlin, 2nd. Edition, 370p. 2002.
- Huang, X., Xie, Y.M. "Evolutionary topology optimization of continuum structures". Wiley, 223 p., 2010.
- Iwamura, R. S., Faria, A. R. "Topology optimization of multiple load case structures". In: Proc. of MECSOL 2013. Porto Alegre, Brazil, 2013.
- James, K.A., Hansen, J.S., Martins, J. R. R. A. "Structural topology optimization for multiple load cases using a dynamic aggregation technique", *Engineering Optimization*, 41(12):1103-1118, 2009. doi: 10.1080/03052150902926827.
- Jaouadi, Z., Lahmer, T. "Topology optimization of structures subjected to multiple load cases by introducing the Epsilon constraint method". Internal Report. T. Lahmer, K. Gürlebeck Editors. Institut für Strukturmechanik, 2015.
- Li, H., Gao, L., Li, P. "Topology optimization of structures under multiple loading cases with a new compliance-volume product". *Engineering Optimization*, 46(6):725-744, 2014. doi: 10.1080/0305215X.2013.800054
- Lógó, J., Balogh, B., Ppintér, E. "Topology optimization considering multiple loading". *Computers & Structures*. (207): 233-244, 2018. doi: 10.1016/j.compstruc.2017.03.018.
- Natke, H.G., Soong, T.T. "Topological structural optimization under dynamic loads", *Transactions on the Built Environment*. 2:67-78, 1993.
- Nutu, E., "Multiple load case topology optimization based on bone mechanical adaptation theory". *U.P.B. Sci. Bull., Series D*, 77(4), 2015.
- Park, G., Kang, B. "Validation of a Structural Optimization Algorithm Transforming Dynamic Loads into Equivalent Static Loads". *J. of Opt. Theory and App.*, 118:191-200, (2003).
- Zhou, K., Li, X. "Topology optimization of structures under multiple load cases using a fiber-reinforced composite material model". *Comp. Mech.* 38:163-170. doi: 10.1007/s00466-005-0735-9.

7. RESPONSIBILITY NOTICE

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