26th International Congress of Mechanical Engineering
COBEM 2021
ABCM Associação Brasileira de Engenharia e Ciências Mecânicas
November 22-26, 2021, Virtual Congress, Brazil

COB-2021-0467
GAIN ADJUSTMENT METHOD TO AVOID JACKKNIFE OF A MARV WITH TWO PASSIVE TRAILERS IN BACKWARD MOVEMENTS

Diego Nunes Bertolani
Instituto Federal de Educação, Ciência e Tecnologia do Espírito Santo - IFES campus Guarapari
diego.bertolani@ifes.edu.br

Mario Sarcinelli-Filho
Universidade Federal do Espírito Santo - UFES
mario.sarcinelli@ufes.br

Abstract.
This work deals with a unicycle-like mobile platform pushing two passive trailers connected to it, thus characterizing an articulated chain able to transport a greater amount of cargo. Firstly, it is developed a suitable model for such a vehicle, considering that the point in the middle of the back of the last trailer should follow a specified path, considering a tractor element pushing one or two trailers. After, a controller is developed to guide the system, consisting of a kinematic controller, responsible for the movement of the vehicle, plus a dynamic compensator, to compensate the dynamics of the tractor element, corresponding to a cascade or inner-outer loop control structure. Finally, a strategy to online adjust the control gains is proposed, to avoid jackknife situations, when the difference between the heading angles of two elements of the chain is so big that prevents continuing the navigation. The objective is to obtain a controller easy to implement, scalable to more than two trailers and able to perform a path-following task at relatively high velocity, with a low occurrence of jackknife situations. Finally, simulations are discussed, whose results validate the proposed model and control system, using dynamic parameters of a real tractor element.

Keywords: *multi-articulated robotic vehicles, multibody robotic systems, active-passive systems, nonlinear control, mobile robotics.*

1. INTRODUCTION

Robotics is already present in several branches of industrial activity worldwide nowadays, as well as in the daily life of many people. This technological scenario shows the use of robots in various areas and applications, such as, for instance, industry, agriculture, house keeping, medicine, and in environments that represent risk for human life.

In such a context this work represents a development in the field of mobile robotics, focusing on the control of multi-articulated robotic vehicles (MARV). Such vehicles, provided they embed autonomous navigation systems, are capable of carrying out maneuvers to assist the accomplishment of tasks in several applications of daily life. An example of a real application of a MARV could be cargo transportation in agriculture, with a tractor element pushing one or more trailers, transporting either agricultural inputs or the harvest. This would represent a more efficient transportation, because of the high cargo capacity of the MARV, in comparison with a single vehicle. Another real application is to park a multi-trailer truck with reverse movement.

The greatest complexity of navigating these multi-articulated robots consists of performing maneuvers, in particular during backward movements, which can lead to a situation known as jackknife. Jackknife should be avoided, because it can cause the shock between the elements of the articulated chain, thus precluding the continuity of the navigation. This area of study is explored, for instance, by Beglini *et al.* (2020) and Hejase *et al.* (2018).

In these works, however, jackknife prevention methods are applied to a system with just one passive trailer and car-like traction elements. Therefore, although there are several ways and methodologies in the literature to avoid jackknife, there is still no universal and well-defined solution to this problem, as discussed in (Michałek and Pazderski, 2019), even in forward movements. Some authors, such as Manesis and Stamatis (2009), use mechanical solutions, not control solutions, creating mechanisms that reduce the possibility of jackknife occurrence.

This work proposes a control solution to avoid jackknife, which consists adjust the gains associated to the controller designed to guide the MARV, considering a system with two passive trailers and a unicycle tractor element. The simulated experiments reported and analyzed ahead show that such a strategy is efficient in precluding the composition to go into jackknife, even in unfavorable situations, in which it is necessary to perform quite complex maneuvers. Notice that in such experiments the MARV movement is performed with the tractor element is pushing, not pulling, the trailers. This is, therefore, the contribution of the paper: a simple control solution to help the MARV to avoid jackknife.

To discuss the topics involved, the work is hereinafter split into four sections, starting with Section 2, which discusses the modeling of the robot-trailer(s) system, with the definition of the geometric, angular and velocity parameters of the articulated chain. This section also defines the kinematic, dynamic and complete models of MARV, in addition to giving equations characterizing the velocity transmission between the chain elements. In the sequel, Section 3 presents the control structure implemented and the gain adjustment method proposed to avoid the occurrence of jackknife, whereas Section 4 shows the results obtained. Finally, the main conclusions are highlighted in Section 5.

2. MARV MODEL

The MARV model presented in this work is based on the velocity relation between the elements of the composition. In this section, all the considerations made in the kinematic and dynamic models of the robot are described.

2.1 Kinematic Model

A MARV is composed by a tractor element, in this work the well-known differential-drive mobile robot *Pioneer 3-DX*, and passive elements (elements without any type of motorization and/or traction system), as illustrated in Figure 1, where the rightmost element is the tractor element and the two other are the trailers. Thus, the commands to move the entire vehicle should be imposed on the *Pioneer 3-DX*, because it is not possible to directly control the trailers, which move according to the control signals sent to the tractor element. Such commands are a linear velocity u_0 along the axis X^r and an angular velocity ω_0 around the axis Z^r, defined using the right-hand rule and the axes X^r and Y^r (Martins *et al.*, 2017).

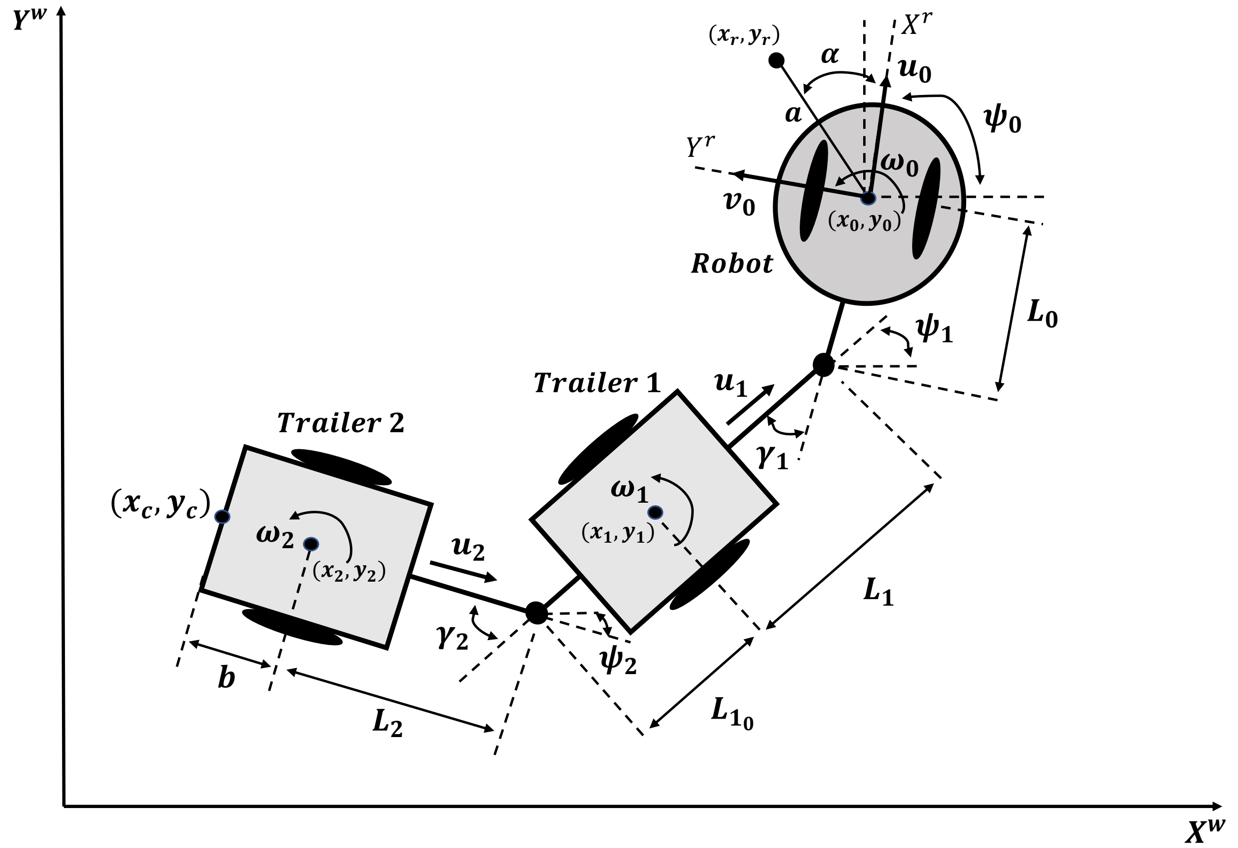


Figure 1. MARV with two trailers, its geometric/angular parameters, and the definition of velocities.

From Figure 1 the following parameters of the MARV are characterized:

- $u_{0,1,2}$ = linear velocity of the element $0,\ 1,\ 2$, oriented according to the respective longitudinal axis;
- $\omega_{0,1,2}$ = angular velocity of the element $0,\ 1,\ 2$, positive counterclockwise;
- $\psi_{0,1,2}$ = orientation of the element $0,\ 1,\ 2$, related to the global coordinate system;,

- $\gamma_{1,2}$ = relative angle between the consecutive elements 1 and 2, positive counterclockwise;
- $L_{0,1_0}$ = distance between the point in the middle of the baseline linking the two wheels of an element and the hitch point of the subsequent one (if this distance is equal zero one has an on-axle hitching. Otherwise the hitching is off-axle);
- $L_{1,2}$ = distance between the coupling point and the point in the middle of the baseline linking the wheels of the trailers;
- $(x_{0,1,2}, y_{0,1,2})$ = coordinates of the point in the middle of the baseline linking the two wheels of an element, related to the global coordinate system;
- (x_r, y_r) = coordinates of the one possible point of interest for control out of the tractor, related to the global coordinate system;
- (x_c, y_c) = coordinates of the point of interest for control, which is the point in the middle of the back part of trailer 2, in the global coordinate system;
- b = distance between the point in the middle of the axle linking the wheels of trailer 2 and the point of interest for control.

As mentioned, the point of interest for control is the one of coordinates (x_c, y_c). Thus, the coordinates of the point being controlled in the navigation plane should be calculated starting from the coordinates of the point in the middle of the virtual axle linking the wheels of the tractor element, namely (x_0, y_0). Therefore, considering the chain in Figure 1 one can write

$$x_1 = x_0 - L_0 \cos\psi_0 - L_1 \cos\psi_1 \tag{1}$$

$$y_1 = y_0 - L_0 \sin\psi_0 - L_1 \sin\psi_1, \tag{2}$$

thus getting the coordinates of the point in the middle of the axle linking the wheels of trailer 1 related to the similar point of the robot. Now, from (1) and (2) one obtains

$$x_c = x_1 - L_{1_0} \cos\psi_1 - (L_2 + b)\cos\psi_2 \tag{3}$$

$$y_c = y_1 - L_{1_0} \sin\psi_1 - (L_2 + b)\sin\psi_2, \tag{4}$$

which are the coordinates of the point in the middle of the rear of trailer 2 related to the point in the middle of the axle linking the wheels of trailer 1.

Finally, still analyzing Figure 1 the angular relationships

$$\gamma_1 = \psi_0 - \psi_1 \tag{5}$$

$$\gamma_2 = \psi_1 - \psi_2, \tag{6}$$

can also be established.

In this work it was considered the kinematic model of the differential drive robot dealt with in (Martins *et al.*, 2008, 2017) and (Resende *et al.*, 2013), with the difference that here the point of interest for control may be anywhere, even out of the robot platform, although rigidly coupled to it, and is rotated of an angle α with respect to the axis X^r of the robot reference system, as shown in Figure 1. Under these consideration, the kinematic model is given by

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\psi}_0 \end{bmatrix} = \begin{bmatrix} \cos\psi_0 & -\sin\psi_0 & -a\sin(\alpha+\psi_0) \\ \sin\psi_0 & \cos\psi_0 & a\cos(\alpha+\psi_0) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ v_0 \\ \omega_0 \end{bmatrix}, \tag{7}$$

where, as illustrated in Figure 1,

- $\dot{x}_r$ is the linear velocity of the robot in the axis X^w;
- $\dot{y}_r$ is the linear velocity of the robot in the axis Y^w;
- $\dot{\psi}_0$ is the angular velocity of the robot, positive counterclockwise;
- a is the distance between the point of interest for control and the point in the center of the virtual axle linking the robot wheels;

- α is the angle between u_0 and the line that links the point of interest for control and the point in the middle of the virtual axis linking the driven wheels of the robot;
- u_0 is the linear velocity of the robot, oriented to is front;
- v_0 is the lateral velocity of the robot, orthogonal to u_0; and
- ω_0 is the angular velocity of the robot.

Notice that such a model admits that the robot has a velocity v_0 in the axis Y^r of its own reference system. Nonetheless, due to the nonholonomic restriction inherent to the differential drive robot such a velocity should be forced to be $v_0 = 0$, as it is shown ahead.

The model described in (7) applies to the robot. As in this work the aim is to control the movement of the last trailer of the configuration, it was decided to adopt this same kinematic model for the passive elements of the chain. Such a strategy is suitable, since the same movement restrictions of the robot ($v_0 = 0$) applies for the trailers. The only difference is that the robot has traction capability whereas the trailers do not have. Thus, the system to be controlled should be understood as a system composed of three robots with the same kinematic model.

Therefore, what is proposed is to consider the point of interest for control in the last trailer, at a distance b of the center of its axle, and generate control actions that make such a point, (x_c, y_c), to perform the desired movement. However, this strategy considers that the last trailer can actively follow the desired trajectory/path, which in practice is not true, since such element of the composition is passive. Therefore, the necessary control action should be transferred to the tractor element (the element zero), the active element of the composition, so that it can generate the control action necessary to carry out the desired movement of the point (x_c, y_c).

The velocity relation between the elements of the articulated chain, starting from the last passive element to the tractor element, is based on the works of Morales *et al.* (2012) and Morales *et al.* (2013). Based on such works one can write

$$u_1 = u_2 \cos\gamma_2 + \omega_2 L_2 \sin\gamma_2 \tag{8}$$

$$\omega_1 = \frac{u_2 \sin\gamma_2}{L_{1_0}} - \frac{\omega_2 L_2 \cos\gamma_2}{L_{1_0}}, \tag{9}$$

thus obtaining the velocities of trailer 1 from the velocities of trailer 2. Continuing following the articulated chain towards the tractor element, one can write

$$u_0 = u_1 \cos\gamma_1 + \omega_1 L_1 \sin\gamma_1 \tag{10}$$

$$\omega_0 = \frac{u_1 \sin\gamma_1}{L_0} - \frac{\omega_1 L_1 \cos\gamma_1}{L_0}, \tag{11}$$

thus obtaining the velocities of the tractor element from the velocities of trailer 1, and then having the velocities of the element zero as a function of the velocities of trailer 2.

For backward movements, it is considered that $\alpha = 180°$ in the kinematic model of the last trailer.

2.2 Dynamic Model

The dynamic model of the tractor element used in this research has already been developed in Martins *et al.* (2008), Martins *et al.* (2017) and Cruz and Carelli (2008). Such a model can be written as

$$\begin{bmatrix} \dot{u}_0 \\ \dot{\omega}_0 \end{bmatrix} = \begin{bmatrix} \frac{\theta_3^0}{\theta_1^0}\omega_0^2 - \frac{\theta_4^0}{\theta_1^0}u_0 \\ -\frac{\theta_5^0}{\theta_2^0}u_0\omega_0 - \frac{\theta_6^0}{\theta_2^0}\omega_0 \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1^0} & 0 \\ 0 & \frac{1}{\theta_2^0} \end{bmatrix} \begin{bmatrix} u_{0_{ref}} \\ \omega_{0_{ref}} \end{bmatrix}, \tag{12}$$

where the signals $u_{0_{ref}}$ and $\omega_{0_{ref}}$ are provided by the controller to be implemented, whereas θ_1^0, θ_2^0, θ_3^0, θ_4^0, θ_5^0 and θ_6^0 are dynamic parameters associated to moments of inertia, electrical parameters of the motors, internal forces and torques, among other features. Equations characterizing each one of such parameters can be found in (Martins *et al.*, 2017) and (Cruz and Carelli, 2008). However, to find the exact values or to measure such parameters is a non-trivial task. Thus, an identification procedure was performed to estimate them, as in (Martins *et al.*, 2017), getting estimates $\hat{\theta}_i$ of $\theta_i^0, i = 1, \cdots, 6$, which are

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \\ \hat{\theta}_4 \\ \hat{\theta}_5 \\ \hat{\theta}_6 \end{bmatrix} = \begin{bmatrix} 0.23882 \\ 0.23938 \\ 0.0038418 \\ 0.9435 \\ -0.007849 \\ 0.92249 \end{bmatrix}. \tag{13}$$

In the simulated results presented ahead these values are used as the real parameters of the model of the tractor element or element zero (namely $\theta_i^0 = \hat{\theta}_i$).

2.3 Complete Model

The complete model of the differential drive robot adopted as element zero in this work is obtaining by joining its kinematic and the dynamic models, characterized in (7) and (12), respectively. Considering the nonholonomic constraint $v_0 = 0$ and assuming $a = 0$ (the point of interest for control of the element zero is in the middle of the virtual axis linking its wheels) and $\alpha = 0°$ in (7), such a model is defined as

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\psi}_0 \\ \dot{u}_0 \\ \dot{\omega}_0 \end{bmatrix} = \begin{bmatrix} u_0 cos(\psi_0) \\ u_0 sin(\psi_0) \\ \omega_0 \\ \frac{\theta_3^0}{\theta_1^0}\omega_0^2 - \frac{\theta_4^0}{\theta_1^0}u_0 \\ -\frac{\theta_5^0}{\theta_2^0}u_0\omega_0 - \frac{\theta_6^0}{\theta_2^0}\omega_0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\theta_1^0} & 0 \\ 0 & \frac{1}{\theta_2^0} \end{bmatrix} \begin{bmatrix} u_{0_{ref}} \\ \omega_{0_{ref}} \end{bmatrix}. \tag{14}$$

In such a complete model the kinematic part represents the kinematic model of a unicycle vehicle with the point of interest for control located in the middle of the virtual axle linking the wheels. This strategy does not compromise the control here discussed, because there is no singularity in the vehicle, since the effective point of interest for control is in the back of the last trailer, and there is no inversion of the kinematic matrix for the active element, only for the element being controlled, for which such a point is at a distance b of the baseline of its wheels. Notice from (14) that the kinematics and the dynamics are absolutely decoupled (see (Martins *et al.*, 2008) for details).

3. THE IMPLEMENTED CONTROLLERS

The implemented controllers and their particularities are now discussed. The objective is to present the structure and control law of each algorithm developed in the following subsections.

3.1 Kinematic controller

The control is based on the position and orientation of the last trailer of the articulated chain, but the control actions necessary to move the MARV are imposed on the element zero, which is the only element with traction capability. Thus, what is desired is to guide the movement of the last trailer through maneuvers carried out by the tractor robot.

The Kinematic controller is based on the strategy of inverse kinematics. In this work, in particular, it is considered only the inverse kinematic model of the last trailer, which is the element to be controlled. In addition, the purely kinematic controller does not take into account the dynamic effects present in the vehicle, assuming perfect velocity tracking, which means that the reference velocity is instantaneously reached.

The proposed kinematic control law is

$$\begin{bmatrix} u_{2_{ref}} \\ v_{2_{ref}} \\ \omega_{2_{ref}} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} V_x \\ V_y \\ V_\psi \end{bmatrix} = \begin{bmatrix} \cos(\psi_2) & \sin(\psi_2) & b\sin(\alpha) \\ -\sin(\psi_2) & \cos(\psi_2) & -b\cos(\alpha) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_\psi \end{bmatrix}, \tag{15}$$

where $\mathbf{A}^{-1}$ is the inverse of the kinematics matrix in (7), with the distance b instead of a (see Figure 1).

As the point of interest (x_c, y_c) is outside the axle of the last trailer, at a distance b of such axle (see Figure 1), it is possible to use the extended kinematics of (15), thus obtaining

$$V_x = \dot{x}_d + K_1 \tanh(K_2\tilde{x}), K_1, K_2 > 0, \tag{16}$$

$$V_y = \dot{y}_d + K_1 \tanh(K_2\tilde{y}), K_1, K_2 > 0, \text{and} \tag{17}$$

$$V_\psi = -\frac{V_x}{b\cos(\alpha)}\sin(\psi_2) + \frac{V_y}{b\cos(\alpha)}\cos(\psi_2), \tag{18}$$

where $\tilde{x} = x_d - x_c, \tilde{y} = y_d - y_c, \dot{x}_d$ is the desired velocity along the axis X^w, and x_d is the desired coordinate in the axis X^w for the point of interest. As for $\dot{y}_d$ and y_d, they are similar to $\dot{x}_d$ and x_d, now regarding the axis Y^w.

Then, the control actions, knowing that $v_{2_{ref}} = 0$, due to the nonholonomic characteristic of the vehicle, are

$$u_{2_{ref}} = V_x\cos(\psi_2) + V_y\sin(\psi_2) + V_\psi b\sin(\alpha), \text{ and} \tag{19}$$

$$\omega_{2_{ref}} = V_\psi. \tag{20}$$

The control actions thus calculated apply to the last trailer. To transmit these control actions to the element zero, the velocity transmission equations (8) and (9) are used to find the references for the trailer 1 ($u_{1_{ref}}$ and $\omega_{1_{ref}}$), and the equations (10) and (11) are used to finally get the control commands ($u_{0_{ref}}$ and $\omega_{0_{ref}}$) to be applied to the element zero.

3.2 Dynamic controller

Figure 2 illustrates the dynamic control strategy adopted. This controller has a structure similar to the purely kinematic controller, since it is based in the inverse dynamics technique.

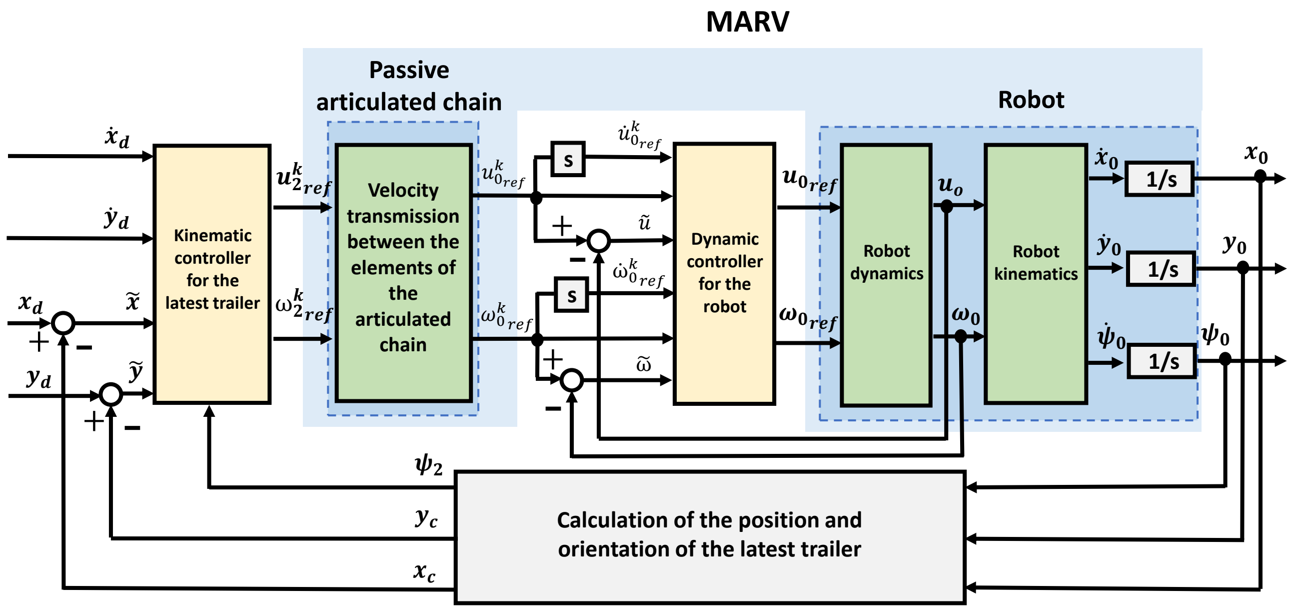


Figure 2. Dynamic controller for a robot with two trailers.

The kinematic controller generates velocity references for the robot ($u_{0_{ref}}$ and $\omega_{0_{ref}}$) and it is assumed that these velocities will be immediately assumed, without any problems or interference. In practice, however, this is not what happens. The dynamic effects, such as friction, mass, inertia, internal forces, torques, disturbances, among others, are present, compromising the desired velocity tracking, making the robot to face problems in navigation. To perform the dynamic control of the robot is an attempt to improve the maneuverability of the entire articulated chain. When the tractor robot tracks more precisely the reference velocities it is expected that when transmitting such velocities along the passive elements the point of interest for control will perform a more accurate movement.

In this work only the dynamic compensation of the active element of the composition is implemented, since it is the only one that has traction and that is capable of correcting the tracking problems directly. As a consequence, all passive elements will be affected by the correction imposed to the robot dynamics.

Therefore, the control law implemented by the dynamic controller, based on (Martins *et al.*, 2008), is

$$\begin{bmatrix} u_{0_{ref}} \\ \omega_{0_{ref}} \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 & 0 \\ 0 & \hat{\theta}_2 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\omega_0^2 & u_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_0 u_0 & \omega_0 \end{bmatrix} \times \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 & \hat{\theta}_4 & \hat{\theta}_5 & \hat{\theta}_6 \end{bmatrix}^T, \tag{21}$$

with

$$\delta_1 = \dot{u}_{0_{ref}}^k + k_u \tilde{u}, \qquad k_u > 0, \tag{22}$$

$$\delta_2 = \dot{\omega}_{0_{ref}}^k + k_\omega \tilde{\omega}, \qquad k_\omega > 0, \tag{23}$$

$$\tilde{u} = u_{0_{ref}}^k - u_0, \text{ and} \tag{24}$$

$$\tilde{\omega} = \omega_{0_{ref}}^k - \omega_0. \tag{25}$$

Analyzing the scheme of Figure 2, it can be seen that the kinematic controller continues to generate velocity references for the last trailer ($u_{2_{ref}}^k$ and $\omega_{2_{ref}}^k$). Again, velocity references are transmitted to the robot, and these signals, $u_{0_{ref}}^k$ and $\omega_{0_{ref}}^k$, will be handled by the dynamic controller, together with feedback data, according to equations 22, 23, 24 and 25, before being applied to the robot. This strategy, in which the kinematic controller generates references for the dynamic controller, is called cascade control or inner-outer loop control structure (Cao and Lynch, 2016).

3.3 Gain adjustment method proposed

Jackknife is a situation naturally happening in the backward movement of MARVs. The fact is that just translating the composition backwards already causes a shock between the members of the composition. This is why the backward navigation is so complex to perform, since normally any control error may cause this problem. Thus, the method here proposed intends to implement a natural task of a person driving an articulated vehicle, both in backward and forward movements, which would be to decrease the velocity if the chain is in accentuated curves or in large rotations and increase the velocity when navigating in a straight line. This velocity variation can be performed in the controllers by changing the gains, which causes changes in the control signals. Hence, the proposed method consists of modifying the gains of the kinematic and dynamic controllers according to

$$K_1 = \frac{K_1}{1 + \lambda_{K_1}\omega_{0_{ref}}}, \tag{26}$$

$$k_u = \frac{k_u}{1 + \lambda_{k_u}\omega_{0_{ref}}}, \quad \text{and} \tag{27}$$

$$k_\omega = \frac{k_\omega}{1 + \lambda_{k_\omega}\omega_{0_{ref}}}, \tag{28}$$

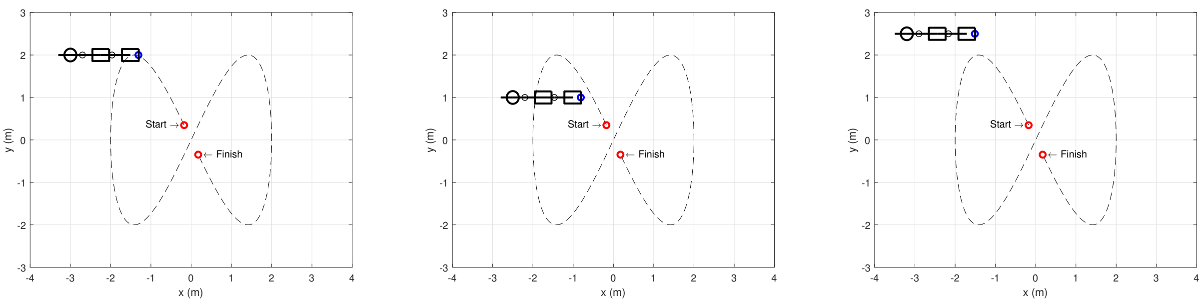
where λ_{K_1}, λ_{k_u} and λ_{k_ω} are adjustment values to change the gains of each controller. Notice that the K_2 gain is not changed, because its influence is limited by the hyperbolic tangent in the control law of Equations 16 and 17.

The method is based on the correction of the gains always taking into account the value of the MARV control signal referring to rotation ($\omega_{0_{ref}}$), according to Equations 26, 27 and 28. This consideration is simplified by the fact that translation also causes jackknife. But, if in fact the orientation control signal is large, it is a clear sign that the vehicle is performing a complex maneuver and is a strong indication of a risky region for jackknifing.

Finally, this method was applied, during navigation, when $\omega_{0_{ref}} > 20°/s$, $\dot{\psi}_0 > 20°/s$, $\dot{\psi}_1 > 20°/s$, or $\dot{\psi}_2 > 10°/s$. A lower bound was adopted for the angular velocity of the last trailer ($\dot{\psi}_2$) because normally large rotations in the last element of the chain cause even greater rotations in the previous elements in order to correct this situation. Thus, the last trailer cannot be allowed to spin too fast, as this will certainly cause the entire composition to go into jackknife. Finally, it is worthy mentioning that the values chosen here were defined using the trial-and-error method.

4. RESULTS

In this section, simulated results are presented, considering the gains adjustment proposed to avoid jackknife. For this, three initial MARV configurations were considered, always starting with all its elements aligned, but in different positions with respect to the desired path, according to Figure 3.



(a) Over the path (b) Inside the path (c) Outside the path

Figure 3. Position of the point of interest for control of the MARV with respect to the path to be followed. The navigation starts with the the point of interest for control moving to the point of the path closest to it, from where the path-following starts. Notice, therefore, that each initial condition demands different maneuvers to start following the specified path.

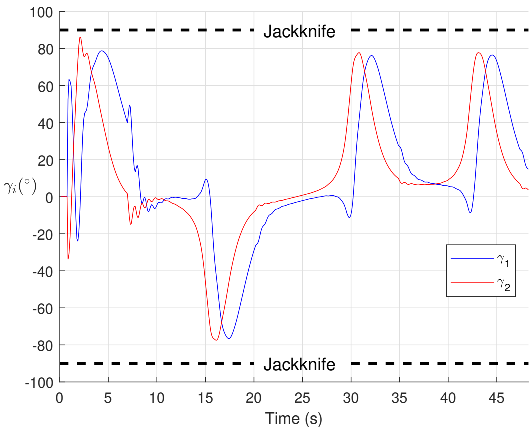
As for the simulation environment, it is used a Matlab® framework called *AuroRa* (Pizetta *et al.*, 2016). It contains the basic graphic support, and allows including the models and controllers for any robotic vehicle. Therefore, such a framework was reconfigured to add the MARV model and the controllers here proposed.

Regarding Figure 3, it can be seen that all the initial configurations considered, although in different positions in the navigation plane, are oriented in opposition to the evolution of the path to be followed. Thus, it is required a great effort, regarding the initial maneuver to place the point of interest for control on the path, which is done considering the point of the path closest to it, and to guide the entire chain in the correct direction of navigation. Therefore, it is demanded a very costly movement for the entire articulated chain, at least in the initial moments, which could easily cause jackknife. In Figure 3(a) the point of interest for control is already over the path. As for Figure 3(b), such a point is inside the path, forcing the chain to rotate counterclockwise to follow the desired path. Finally, in Figure 3(c) this point is outside the path, forcing a clockwise rotation.

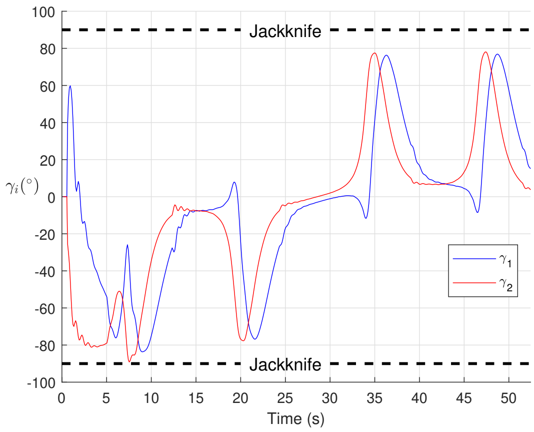
The defined path is a Bernoulli lemniscate with radii $r_x = r_y = 2,0m$. The desired velocity for following the path is equal to $-0,35m/s$, due to the backward movement. The physical dimensions of the MARV are: $L_0 = 0,30m$, $L_1 = 0,455m$, $L_{1_0} = 0,28m$, $L_2 = 0,455m$, $b = 0,20m$. Moreover, as $L_0 > 0$ and $L_{1_0} > 0$ all the hitching are off-axle hitching. The dynamic and kinematic controller gains are $k_u = k_\omega = 4$ and $K_1 = K_2 = 0,8$, respectively, whereas the sampling time is $100\ ms$. The parameters λ_{K_1}, λ_{k_u}, and λ_{k_ω} are $0,015$, $0,2$ and $0,1$, respectively.

It is worthy emphasizing that the simulations presented in this work reflect a good approximation of the behavior of a real MARV. Moreover, a real MARV will be shortly available to run real experiments to validate the strategy here proposed to avoid jackknife.

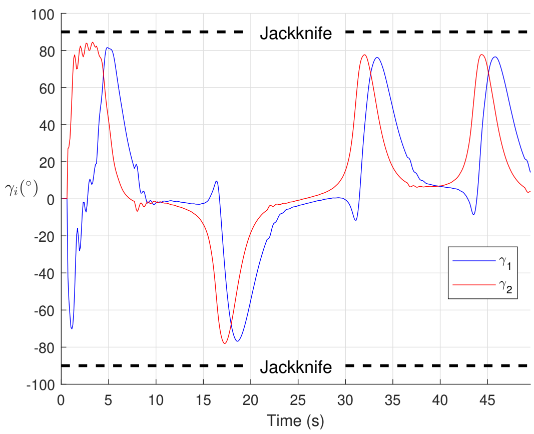
It was considered that the system is in *jackknife* condition when γ_1 or γ_2 is greater than or equal to $90°$. The reason is that due to constructive aspects of the elements of the articulated chain there is no shock between each one and its subsequent, with such consideration. Figure 4 shows the evolution of the relative angles between the elements of the MARV composition, during the accomplishment of the path-following task, with the jackknife avoidance method, for each initial configuration of Figure 3.



(a) Over the path



(b) Inside the path



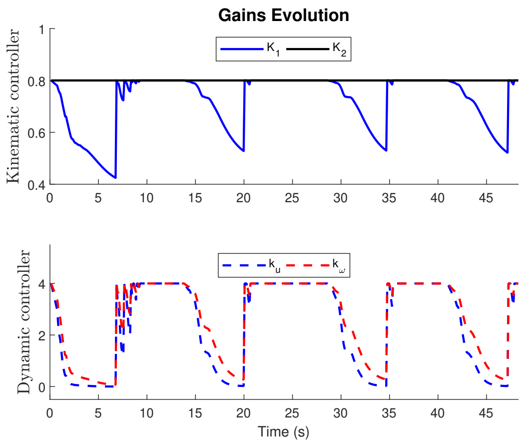
(c) Outside the path

Figure 4. Evolution of the relative angles between the elements of the MARV composition, during the accomplishment of the path-following task, with the jackknife avoidance method, for each initial configuration of Figure 3. The horizontal dashed lines correspond to the bounds above and below which jackknife occurs.

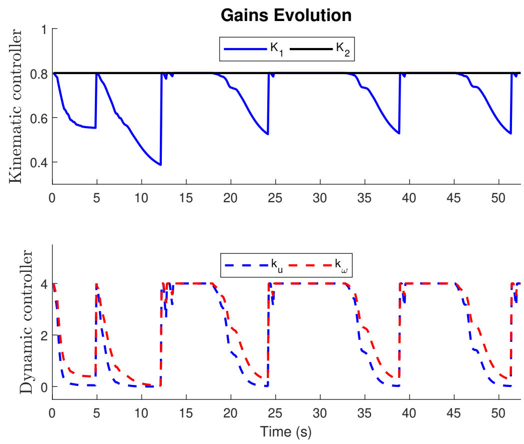
Analyzing such a figure, the first conclusion is that the system does not present jackknife in any of the maneuvers, allowing concluding that the proposed method is effective in avoiding jackknife. Not all movements could be performed without including such a method. It is also noticed that the angles γ_1 and γ_2 evolve to values very close to $90°$, especially at the beginning of the movement, as expected due to the initial chain configurations with respect to the desired path.

The method proposed to avoid jackknife changes the gains of the controllers according to the movement of the MARV. Therefore, it is very important to check how these gains change during the maneuver. Figure 5 shows that the variation of the gain K_1 of the kinematic controller, which decreases whenever the MARV is performing more accentuated rotations during its movement and increases again to its maximum value, fixed at $K_1 = 0,8$, when the path is smooth.

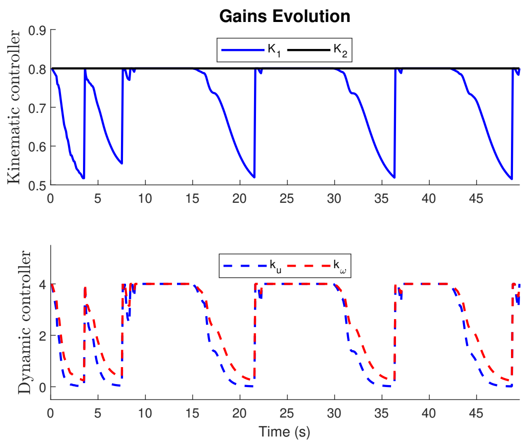
The proposed method also changes the dynamic controller gains (k_u and k_ω), which have a behavior similar to the gain (K_1), decreasing in the curves and increasing in the smoother parts of the path. This allows the vehicle to turn more smoothly, avoiding jackknife, even at borderline driving velocities. Another observation is that it was found that velocities greater than $-0,35m/s$, for this vehicle with two trailers, are not possible to be developed with precision, because of dynamic effects that are no longer corrected.



(a) Over the path



(b) Inside the path



(c) Outside the path

Figure 5. Behavior of the gains along the accomplishment of the proposed task for each initial configuration adopted.

The video available in `https://youtu.be/96pWA338HTc` shows the movement made by the MARV in the three situations explored here. In the video, it can also be seen that navigation is not possible without adopting the proposed method (jackknife situations occur when not using the proposed method). In addition, it can be seen that with the adoption of the proposed method the vehicle is able to perform the entire navigation, showing small errors in following the desired path.

5. CONCLUSIONS

It can be concluded that the method implemented to prevent jackknife situations presented good behavior, as it managed to perform movements that would not be possible without its use. Indeed, all the simulations presented, which consider different initial positions for the point of interest for control, all of them generating much complex maneuvers to start the navigation, showed no jackknife occurrence, leading to the expectancy that when applied to a real MARV, the next step of the work, this methodology will be efficient. It is important to note that the method does not guarantee that all possible maneuvers will be performed, and in certain configurations the jackknife can still occur. This work points to the direction of reducing the effects caused by the unwanted movement of the articulated chain, proposing a solution that is not generic, but that helps in cases of extreme difficulty in maneuvering MARVs.

The method is easy to implement and can be replicated for vehicles with more than 2 trailers, so it is quite advantageous to avoid possible physical damage when working with real test platforms. In fact, this method reflects a little of the logical behavior of a person when driving a vehicle like this, which would be to decrease velocities in accentuated curves and during turning moments of the elements and to increase velocities when navigating in a safer linear region.

A stability study is still needed to prove in which regions this methodology is valid, that is, it is still necessary to check why in some cases, even with this method, jackknife occurs. This would also involve a study of the controllers used in MARV to verify which would be the best gains and how these gains should evolve so that any maneuver that one wants to perform does not present jackknife. However, this task is not within the scope of this work. The validation of this method in a real test MARV also remains as future work, although it is about becoming a reality, using a lab-scale MARV.

6. ACKNOWLEDGEMENTS

The authors would like to thank Universdade Federal do Espírito Santo (UFES) and Instituto Federal de Educação, Ciência e Tecnologia do Espírito Santo (IFES), Campus Guarapari, for the support given to the project. Dr. Mario Sarcinelli-Filho also thanks CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico, an agency of the Brazilian Ministry of Science, Technology, Innovations and Communications, and FAPES - Fundação de Amparo à Pesquisa e Inovação do Espírito Santo, an agency of the State of Espírito Santo, for the financial support to this research.

7. REFERENCES

Beglini, M., Lanari, L. and Oriolo, G., 2020. “Anti-jackknifing control of tractor-trailer vehicles via intrinsically stable mpc”. In *2020 IEEE International Conference on Robotics and Automation (ICRA)*. pp. 8806–8812. doi:10.1109/ICRA40945.2020.9197012.

Cao, N. and Lynch, A.F., 2016. “Inner–Outer Loop Control for Quadrotor UAVs With Input and State Constraints”. *IEEE Transactions on Control Systems Technology*, Vol. 24, No. 5, pp. 1797–1804. doi:10.1109/TCST.2015.2505642.

Cruz, C.D.L. and Carelli, R., 2008. “Dynamic model based formation control and obstacle avoidance of multi-robot systems”. *Robotica*, Vol. 26, No. 3, pp. 345–356. doi:10.1017/S0263574707004092.

Hejase, M., Jing, J., Maroli, J.M., Bin Salamah, Y., Fiorentini, L. and Özgüner, U., 2018. “Constrained backward path tracking control using a plug-in jackknife prevention system for autonomous tractor-trailers”. In *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*. pp. 2012–2017. doi:10.1109/ITSC.2018.8569262.

Manesis, J.S. and Stamatis, 2009. “Anti-jackknife state feedback control law for nonholonomic vehicles with trailer sliding mechanism”. *Int. J. Systems, Control and Communications*, Vol. 1, No. 3.

Martins, F.N., Celeste, W.C., Carelli, R., Sarcinelli-Filho, M. and Bastos-Filho, T.F., 2008. “An adaptive dynamic controller for autonomous mobile robot trajectory tracking”. *Control Engineering Practice*, Vol. 16, No. 11, pp. 1354–1363. ISSN 09670661. doi:10.1016/j.conengprac.2008.03.004.

Martins, F.N., Sarcinelli-Filho, M. and Carelli, R., 2017. “A Velocity-Based Dynamic Model and Its Properties for Differential Drive Mobile Robots”. *Journal of Intelligent & Robotic Systems*, Vol. 85, No. 2, pp. 277–292.

Michałek, M.M. and Pazderski, D., 2019. “Forward tracking of complex trajectories with non-Standard N-Trailers of non-minimum-phase kinematics avoiding a jackknife effect”. *International Journal of Control*, Vol. 92, No. 11, pp. 2547–2560. ISSN 13665820. doi:10.1080/00207179.2018.1448117.

Morales, J., Mandow, A., Martinez, J.L., Mertínez, J.L. and Garcia-Cerezo, A.J., 2012. “Driver assistance system for backward maneuvers in passive multi-trailer vehicles”. *IEEE International Conference on Intelligent Robots and Systems*, pp. 4853–4858. ISSN 21530858. doi:10.1109/IROS.2012.6385799.

Morales, J., Martinez, J.L., Mandow, A. and Garcia-Cerezo, A.J., 2013. “Steering the last trailer as a virtual tractor for reversing vehicles with passive on-and off-axle hitches”. *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 12, pp. 5729–5736. ISSN 02780046. doi:10.1109/TIE.2013.2240631.

Pizetta, I.H.B., Brandao, A.S. and Sarcinelli-Filho, M., 2016. “A hardware-in-the-loop platform for rotary-wing unmanned aerial vehicles”. *Journal of Intelligent & Robotic Systems*, Vol. 84, No. 1-4, pp. 725–743.

Resende, C.Z., Carelli, R. and Sarcinelli-Filho, M., 2013. “A nonlinear trajectory tracking controller for mobile robots with velocity limitation via fuzzy gains”. *Control Engineering Practice*, Vol. 21, No. 10, pp. 1302–1309. ISSN 0967-0661. doi:https://doi.org/10.1016/j.conengprac.2013.05.012.

8. RESPONSIBILITY NOTICE