COBEM 2021 26th International Congress of Mechanical Engineering

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas
November 22-26, 2021, Virtual Congress, Brazil

COB-2021-0472
ANALYSIS OF THE FATIGUE LIFE PREDICTION OF OVERHEAD CONDUCTORS CONSIDERING THE TIME AND FREQUENCY DOMAIN

Emanuely Ugulino Cardoso
Jorge Luiz de Almeida Ferreira
University of Brasilia, Brasília, Federal District, Brazil
emanuely.ugulino@gmail.com; jorge@unb.br

Abstract. *Overhead conductors are the most requested components in a transmission line, due to the vibratory movement resulting from the action of the wind. In Brazil the highest incidence is associated with aeolian vibration, being the main cause of conductor fatigue failure. Fatigue can be defined as a progressive failure of a component under repetitive, cyclic or oscillating loads and the fatigue damage over time is irreversible and cumulative. One of the main methodologies for life estimation is based on the fatigue curve of the S-N (stress-life) material. In many cases, stresses have random characteristics, and the estimate of fatigue damage depends on the loading history. Thus, the life estimate can be performed in the time domain, from the cycle count in each stress level of the recorded signal, based on rainflow cycle counting, or in the frequency domain based on power spectral density moments of the structure. Considering the results of fatigue tests carried out with the CAA 397.5 MCM - IBIS conductor which are available in the literature, this research aims to reconstruct the stress histories at critical points of the conductor and estimate his life. From the estimates made with the reconstructed signals it will be possible to compare them with the results obtained from the tests and verify the efficiency of the methodology employed. The analyses consider the effect of discretization of the power spectrum and the technique of simulation of stochastic processes in the fatigue life predictions, so that predictions will be made through spectral and time models. From the results it was noticed that when making estimates in the time domain, fatigue life values are more variable than in the frequency domain, because the statistics and spectral measures are less influenced by the signal size, thus, the most accurate models were Rayleigh and Dirlik.*

Keywords: *conductors fatigue, aeolian vibration, power spectrum, rainflow.*

1. INTRODUCTION

Transmission lines are large structures that are subject to loads that are influenced by environmental conditions, with overhead conductors being the most requested components in a transmission line. Therefore, there is a need to understand the mechanism that controls the occurrence of fatigue failure caused by aeolian vibration, which could lead to innovative technologies being developed, making transmission line installation more reliable and economically feasible. Besides the dynamic forces caused by the wind, the conductor is also submitted to static loading, due to the tensile load applied when installing the transmission lines (Kalombo *et al.*, 2020). These and other dynamic forces of nature set power line conductors into cyclic motions, including aeolian vibration, sub-span oscillation and conductor galloping.

Among the wind induced conductor motions the most dangerous is aeolian vibration, considered as the most recurrent phenomenon. The occurrences can be on almost any transmission line at any time, with a fast accumulation of fatigue cycles. Subspan oscillation can hardly produce strand fatigue failures since the accumulation of fatigue cycles is slow and the conductor stress at spacer damper clamps is generally moderate. Galloping is a relative rare event on any given line span or section because of the combination of weather conditions that is required for its occurrence (Cosmai *et al.*, 2017). In this sense, it is important to examine the fatigue behavior and to evaluate the fatigue reliability of the conductive cables under random loads.

Fatigue is a localized damage process of a component produced by cyclic loading. It is the result of the cumulative process consisting of crack initiation, propagation, and final fracture of a component. During cyclic loading, localized plastic deformation may occur at the highest stress site. This plastic deformation induces permanent damage to the component and a crack develops. As the component experiences an increasing number of loading cycles, the length of the crack (damage) increases. After a certain number of cycles, the crack will cause the component to fail (separate) (Lee, 2005b).

When the stress state satisfies the hypothesis of Gaussian stationary ergodic signal and is synthesized by a single Power Spectral Density function (uniaxial stress state), the literature shows approaches that, starting from its Power Spectral Density function (PSD), it is possible to directly obtain an estimation of the damage. The development of a frequency domain approach is an alternative to rainflow, the time domain reference criterion (Braccesi, Cianetti and Tomassini, 2015).

Based on the results of fatigue tests carried out with the CAA 397.5 MCM - IBIS conductor which are available in the tests literature, this paper aims to reconstruct the stress histories active at critical points of the conductor and estimate its useful life. From the estimates made with the reconstructed signals it will be possible to compare them with the results obtained from the tests and verify the efficiency of the methodology employed. The analyses consider the effect of discretization of the power spectrum and the technique of simulation of stochastic processes in the fatigue life predictions, so that predictions will be made through spectral and time models.

Therefore, this research aims to define the discretization of the power spectrum that resulted in better results of fatigue life prediction and also to determine which analysis domain (frequency or time) was more satisfactory. In the frequency domain will be used the models of Rayleigh and Dirlik, already in the time domain will be adopted the rainfow technique. Then, the damages calculated by Palmgren-Miner and Serensen-Kogayev rules are verified to establish which methodology obtained better estimates of fatigue life.

2. FUNDAMENTAL THEORIES

2.1 Time domain approach

Since the mid-1800s, a standard method of analyzing fatigue models has been the stress-based approach. August Wöhler determined a stress curve applied as a function of the number of cycles required for material to break and a fatigue strength limit stress for steels. This method is also known as the tension-life approach or S-N (stress-life), called in the technical literature as S-N Curve or Wöhler Curve (Watanabe, 2014).

To generate data useful for fatigue designs using the stress-life approach, stress-life fatigue tests are usually carried out on several specimens at different fully reversed stress amplitudes over a range of fatigue lives for identically prepared specimens. The fatigue limit is associated with the phenomenon that crack nucleation is arrested by the first grain boundary or a dominant microstructural barrier. Also, it can be overcome by the application of a few overloads, in a corrosive environment, etc. (Lee and Taylor, 2005b).

Equation (1) represents the S-N curve:

$$S_a = B\left(N_f\right)^b, \tag{1}$$

where *b* is the slope of the S-N curve, and *B* is the material parameter determined by experiments.

An alternative form of the S-N curve to predict N_f for a given S_a is given by Eq. (2):

$$N_f = A * S_a^{-m}, \tag{2}$$

where $m = -1/b$ and $A = (1/B)^{-m}$.

Predicting fatigue damage for structural components subjected to variable loading conditions is a complex issue. The first, simplest, and most widely used damage model is the linear damage. This rule is often referred to as Miner's rule (1945). However, in many cases the linear rule often leads to nonconservative life predictions. The results from this approach do not take into account the effect of load sequence on the accumulation of damage due to cyclic fatigue loading (Lee, 2005b).

The universally used linear damage assessment model was first proposed by Palmgren (1924) for application to the Swedish ball bearing industry. Miner demonstrated excellent agreement between the predictions from the linear damage rule and his experimental results. This success led to the Strong association between Miner and the linear damage rule, and the linear damage rule is commonly referred to as Miner's linear damage rule. In this case, the fatigue damage has a unique, linear relation with the cycle ratio regardless of the stress levels (Lee, 2005b). In terms of mathematics, the linear damage rule can be expressed as follows:

$$D_i = \frac{n_i}{N_{f,i}}, \tag{3}$$

Failure is predicted when the sum of damage reaches a “critical value” (called critical damage) and is typically considered to be 1.

The fatigue damage is strongly associated with the cycle ratio, $n_i/N_{f,i}$, where n_i and $N_{f,i}$ are, respectively, the number of applied stress and/or strain cycles and the fatigue life at a combination of stress and/or strain amplitude and mean stress levels. The fatigue life, $N_{f,i}$, can be obtained from baseline fatigue data generated from constant-amplitude loading tests, as the stress-life (S-N) method (Lee and Taylor, 2005a).

Cycle counting is used to summarize irregular load-versus-time histories by providing the number of times cycles of various sizes occur. The ASTM E1049 standard proposes several acceptable procedures for cyclic-counting methods employed in fatigue analysis, among them the rainflow method (ASTM E1049–85, 2017).

2.2 Frequency domain approach

As presented by Bishop (1988), the Power Spectral Density (PSD) of a signal gives an indication of the average power contained in particular frequencies. It can be expressed as a one-sided function *G(f)*. From the one-sided spectral density function of the process, *G(f)*, the spectral moments of order *n*, M_n, are found through Eq (4):

$$M_n = \int_0^\infty f^n G(f) df, \tag{4}$$

The zeroth moment M_0 can then be used as an alternative method of computing the root mean square (RMS) value which is a good indication of the intensity of a process:

$$RMS = \sigma = \sqrt{M_0}, \tag{5}$$

The crossing rates of zero *E[0+]* and peaks *E[P]* can be calculated based on spectral moments, according to Eq. (6) and Eq. (7), respectively:

$$E[0^+] = \sqrt{\frac{M_2}{M_0}}, \tag{6}$$

$$E[P] = \sqrt{\frac{M_4}{M_2}}, \tag{7}$$

By making a relationship between the zero level crossing rate and the peak rate, the irregularity factor is obtained according to Eq. (8). The irregularity factor will essentially measure how many peaks a signal has for each time the signal crosses at the zero stress level (Lee, 2005a).

$$\gamma = \frac{E[0^+]}{E[P]} = \frac{M_2}{\sqrt{M_0 M_4}}, \tag{8}$$

When the irregularity factor tends to one, it means that for each peak, there will be approximately a zero level crossing and the process is said to have a narrow bandwidth. However, when the bandwidth is wide, it is said that the irregularity factor will tend to zero, that is, there will be infinite peaks for each zero level crossing (Lee, 2005a).

The process is said to be narrow band if its spectral density has only a narrow range of frequencies. In contrast, a wide band process is one whose spectral density covers a wide band of frequencies, this bandwidth is called the width parameter (Watanabe, 2014).

Lee (2005a), shows that a narrow band process is smooth and harmonic. For each peak, there is a corresponding upward crossing of zero, which means that *E[0+]* is equal to *E[P]*. However, the wide band process is more irregular.

Benasciutti and Tovo (2005), show that the irregularity factor, which varies between 0 and 1, can be used to characterise the shape and the geometrical frequency distribution of a given spectrum as a function of spectral moments. Being expressed as:

$$\gamma = \frac{M_n}{\sqrt{M_0 M_{2\mathrm{n}}}}, \tag{9}$$

Another parameter that can be analyzed is the bandwidth as defined by Eq. (10):

$$\lambda = \sqrt{1-\gamma^2}, \tag{10}$$

Thus, narrow band processes should have an irregularity factor equal to 1 and a width parameter equal to zero, while wide band processes should have an irregularity factor tending to zero and a spectral width parameter tending to 1 (Watanabe, 2014).

In the papers of Benasciutti and Tovo (2005) and Braccesi, Cianetti and Tomassini (2015), the authors expose that in random processes, it is difficult to determine exactly the fatigue damage that the structure is subjected to. Therefore, it is common to use for models characteristic of Gaussian and stationary processes the concept of damage intensity $\overline{D}$, which estimates the fatigue damage per unit of time. From the amplitude distribution of the counted cycles, $f_{Sa}(S_a)$, it is simple to express the expected damage per unit of time (i.e. the intensity of the damage) under the Palmgren-Miner rule:

$$\overline{D} = \frac{N(T)}{A} \int_0^\infty S_a^m f_{Sa}(S_a) dS_a \,, \tag{11}$$

which depends on constant amplitude fatigue properties (through the $N_f S_a^m = A$ relation) and on the number of counted cycles per unit time, *N(T)*. Thus different models of the amplitude distribution, $f_{Sa}(S_a)$, cause different damage values to be obtained.

For narrow band processes it is reasonable to assume that every peak is coincident with a cycle and predicts an intensity of fatigue damage with a Rayleigh amplitude distribution (Benasciutti and Tovo, 2005).

According to Lee (2005a), if the loading history is stationary narrow band Gaussian and the stress amplitudes follow the Rayleigh distribution:

$$E(S_a^m) = \left(\sqrt{2}\,\sigma_S\right)^m \Gamma\left(\frac{m}{2}+1\right), \tag{12}$$

where $\sigma_S = \sqrt{M_0}$.

In such a case, the expected total fatigue damage D_{NB} of a zero-mean stationary narrow band Gaussian stress process over a time interval can be written as:

$$D_{NB} = \frac{\sum_{i:1}^{k} n_i}{A} E(S_a^m) = \frac{E[0^+]*T}{A}\left(\sqrt{2\,M_0}\right)^m \Gamma\left(\frac{m}{2}+1\right), \tag{13}$$

where $N_{NB} = E[0^+] * T$ is the total number of cycles in time and Γ(.) is the gamma function.

Instead of using the damage correction factor from narrow band stresses to wide band stresses, Dirlik (1985) developed a closed empirical expression for the rainflow amplitude probability density function, $f_{Sa}(S_a)$, with based on extensive Monte Carlo simulations of stress amplitudes, and was verified in theory by Bishop and Sherratt (1986). Bishop and Lack (1996) showed improvements in the results of the fatigue damage calculation using the Dirlik solution when comparing the corresponding time used with time domain calculations (Watanabe, 2014)

According to Benasciutti and Tovo (2005), the approximate expression for the probability density function of ranges S_a of rainflow counted cycles proposed by Dirlik is a result of a best fit over a large set of data from numerical simulations:

The Dirlik damage model for a studied period of time T is demonstrated by Eq. (14):

$$D_{Dirlik} = \frac{E[P]*T}{A}\int_0^\infty S_a^m f_{Sa}(S_a)\,dS_a, \tag{14}$$

The probability density function (PDF) is determined from a series of parameters defined by Dirlik:

$$f_{Sa}(S_a) = \frac{D_1}{\sqrt{M_0}Q} e^{-\frac{Z}{Q}} + \frac{D_2\,Z}{\sqrt{M_0}R^2} e^{-\frac{Z^2}{2R^2}} + \frac{D_3 Z}{\sqrt{M_0}} e^{-\frac{Z^2}{2}}, \tag{15a}$$

where the function parameters are defined as:

$$Z = \frac{S_a}{\sqrt{M_0}};\;\; X_m = \frac{M_1}{M_0}\sqrt{\frac{M_2}{M_4}} = \gamma_1 * \gamma_2;\;\; D_1 = \frac{2(X_m-\gamma^2)}{1+\gamma^2};\;\; R = \frac{\gamma - X_m - D_1^2}{1-\gamma-D_1+D_1^2},$$

$$D_2 = \frac{1-\gamma-D_1+D_1^2}{1-R};\;\; D_3 = 1 - D_1 - D_2;\;\; Q = \frac{1.25(\gamma - D_3 - D_2 R)}{D_1}, \tag{15b}$$

where Z is the normalized stress amplitude *(S_a)* and X_m, $D1$, $D2$, $D3$, R and Q are best-fitting parameters.

To calculate the fatigue damage using the Palmgren-Miner rule, the load distribution of Eq. (15a) is replaced in Eq. (14) of damage intensity that derives the following expression:

$$D_{Dirlik} = \frac{E[P]*T}{A}\left(\sqrt{M_0}\right)^m \left[D_1 Q^m \Gamma(1+m) + \left(\sqrt{2}\right)^m \Gamma\left(1+\frac{m}{2}\right)(D_2|R|^m + D_3)\right], \tag{16}$$

where $N_{DK} = E[P] * T$ is the total number of cycles in time.

This distribution is an explicit function of the two γ_1 and γ_2 bandwidth parameters and, when compared to previous expressions, a further dependence on M_1 spectral moment is clearly introduced (Benasciutti and Tovo, 2005).

2.3 Serensen–Kogayev hypothesis

Among different proposals for fatigue damage cumulation models, the Serensen-Kogayev hypothesis, as well as the Palmgren-Miner hypothesis, sums up the fatigue damage in a linear manner, but determines damage differently (Kurek and Łagoda, 2020). According to Macha and Niesłony (2007) the damage formula takes the form:

$$D(T_0) = \begin{cases} \frac{1}{b_{SK}A}\sum_{i=1}^{k} n_i S_{ai}^m & \text{for} \quad S_{ai} \geq a_{SK}S_{af} \\ 0 & \text{for} \quad S_{ai} < a_{SK}S_{af} \end{cases}, \tag{17}$$

where b_{SK} is the Serensen-Kogayev coefficient that characterizes amplitude spectrum random loading, defined by the Eq. (18):

$$b_{SK} = \frac{\sum_{i=1}^{k} S_{ai}t_i - a_{SK}S_{af}}{S_{a\,max} - a_{SK}S_{af}}, \tag{18}$$

where a_{SK} is the coefficient accounting for amplitudes below the fatigue limit, for presentations $a_{SK} = 0.5$ was adopted and S_{af} is the stress amplitude (fatigue limit) for N equal to 2 x 10^7 cycles; $S_{a\,max}$ is the maximum amplitude of counted cycles and t_i is the frequency of levels S_{ai} occurrence for observation time T_0, defined by the Eq. (19):

$$t_i = \frac{n_i}{\sum_{i=1}^{k} n_i}, \tag{19}$$

When the value of factor b_{SK} is assumed to be 1, Serensen-Kogayev's hypothesis is equivalent to Palmgren-Miner's, and in this sense, this hypothesis considers the fatigue damage caused by part of the stress amplitudes below the fatigue limit (Kurek and Łagoda, 2020). According to Karolczuk and Kluger (2020) the Serensen-Kogayev rule assumes that the total sum of damages *D* is not equal to 1, but depends on the number of loading cycles. For both rules, the calculated number of cycles to failure is a function of the damage *D* accumulated under the observed number of cycles:

$$N_f = \frac{n}{D}, \tag{20}$$

According to Macha and Niesłony (2007) it could be restated in the light of the spectral aspect under an assumption that amplitudes are approximated by means of the Rayleigh distribution, the formula for fatigue life *T*, expressed in seconds, and coefficient b_{SK}:

$$T = \frac{A * b_{SK}}{E[P] * (2\mu_\sigma)^{\frac{m}{2}} * \Gamma\left(\frac{m+2}{2}, \frac{a_{SK}^2 S_{af}^2}{2\mu_\sigma}\right)}, \tag{21}$$

$$b_{SK} = \frac{\sqrt{2\mu_\sigma} * \Gamma\left(\frac{3}{2}, \frac{a_{SK}^2 S_{af}^2}{2\mu_\sigma}\right) exp\left(\frac{a_{SK}^2 S_{af}^2}{2\mu_\sigma}\right) - a_{SK}S_{af}}{S_{a\,max} - a_{SK}S_{af}}, \tag{22}$$

where $\mu_\sigma = M_0$ is the variance of the stress history and the fatigue life in cycles is $N_f = E[P] * T$.

2.4 Time signal reconstruction

From the data acquired in experimental tests, such as the RMS value, cable excitation frequency and the bandwidth, the signal for a given time *t* can be reconstructed taking into account the PSD of the component's stress response. The simulated random processes are asymptotically Gaussian processes as the number of terms of sine or cosine functions approaches infinity (Yang, 1973).

A stationary random process with zero mean and a one-sided spectral density *G(f)* can be simulated by *S(t)* as follows:

$$S(t) = \sum_{n=1}^{\infty} A_n \cos(2\pi f_n t + \phi_n), \tag{23}$$

where A_n e ϕ_n represent, respectively, the amplitude and phase of the frequency spectrum of the function *S(t)* and f_n the frequency of the component.

The amplitude of the sinusoidal signal stress in the time domain is obtained from the area of each PSD, according to Eq. (24):

$$A_n = \sqrt{2 * \Delta f_n * G(f_n)}, \tag{24}$$

The ordered pairs *(t, S(t))* will form the desired random sign, having a stationary and Gaussian characteristic, since its mean is null and the standard deviation is equivalent to the RMS value.

3. METHODOLOGY

The study carried out by Watanabe (2014) proposed, through experimental tests, to analyze the fatigue life of overhead conductors subjected to random loading, and then to verify the predictions of life and damage from spectral and time methods. Watanabe performed 13 tests with different stress levels for CAA 397.5 MCM (IBIS) conductor, with a frequency band of 0.9 Hz centered on the closest dominant frequency of 22 Hz.

Based on the tests carried out by Watanabe (2014), this work aimed to perform life predictions, with different methods of analysis, in order to compare the results obtained with the experimental life and verify the models that are most accurate. This work was developed computationally using the Matlab software and the methods used for life prediction were two spectral models, Rayleigh and Dirlik, and two time models based on rainflow, in which one model used the rainflow function provided by the software itself and the other was performed the implementation of the entire rainflow structure. Furthermore, the Serensen-Kogayev hypothesis was also implemented for the rainflow and spectral methods.

For the development of the life prediction analysis, in addition to the test frequency data, the respective RMS values are required, as well as the parameters of the S-N curve of the conductor. Thus, to determine the power spectrum, reconstruct the signal in time and perform the life predictions, the necessary information was obtained from the work of Watanabe (2014). Table 1 presents the parameters of the S-N curve for the analyzed conductor.

Table 1. Parameters of the S–N curve for an IBIS conductor.

Parameter	Value
Log (A)	11,56
m	3,538

Among the tests performed by Watanabe (2014), this work analyzed test 8, whose information is in Table 2.

Table 2. Experimental fatigue test data.

Test	RMS (MPa)	Experimental Life N_{exp} (x 10^6)
8	13,6	3,44

According to Rapuano and Harris (2007), Gasior and Gonzalez (2004) and Martins (2018) the samples in the frequency domain are equally spaced, with a frequency range that determines the resolution of the transformation frequency defined as:

$$\Delta f = \frac{F_S}{N}, \tag{25}$$

where F_s is the sample frequency and N is the number of points in the spectrum.

When considering the entire frequency range, the size of the frequency bins is defined, which remains constant, such as:

$$\Delta f_b = \frac{k * F_S}{N}, \tag{26}$$

where k represents the frequency acquisition rate.

To construct the power spectrum, information from the experimental fatigue test is needed, such as the RMS values, sample frequency and the frequency band, whose values are fixed. However, it is also necessary to determine the frequency acquisition rate parameters and the number of points in the spectrum, whose values can be chosen. With this information, the power spectrum is constructed and Rayleigh and Dirlik models use it as a basis for making the life predictions, as defined in section 2.2.

The reconstruction of the signal in time occurs from the power spectrum as discussed in section 2.4. and, through the reconstructed signal, the time models make use of the rainflow technique, for counting and identifying cycles, when making fatigue life predictions, according to section 2.1.

The Serensen-Kogayev hypothesis, according to section 2.3, was also used in the analysis of fatigue life prediction by both the rainflow and spectral method to compare with the other models which obtained better accuracy in relation to the experimental life. Unlike the other analyses used this hypothesis does not consider a critical damage and presents a correction factor when compared to the Palmgren-Miner rule.

From this information, 20 fatigue life prediction analyses were defined, applied to the spectral and time models previously defined, considering 5 fixed Δf_b values, which determine the size of the frequency bins, and varying the k and N parameters, which interfere in the power spectrum and, therefore, in the reconstruction of the signal in time. The test has a sample frequency of 22.45 Hz, with an average value adopted for the critical damage of 0.5. The analyses consider the effect of power spectrum discretization and the stochastic process simulation technique on fatigue life predictions. Thus, Table 3 presents the parameters values that were adopted to construct the power spectrum.

Table 3. Definition of values for the construction of the power spectrum.

Parameters of the analyses								Δf_b
k	N	k	N	k	N	k	N	
2	2048	4	4096	8	8192	16	16384	0,0219
2	1024	4	2048	8	4096	16	8192	0,0438
2	512	4	1024	8	2048	16	4096	0,0877
4	512	8	1024	16	2048	32	4096	0,1754
8	512	16	1024	32	2048	64	4096	0,3508

Finally, with the fatigue life prediction results obtained through all the analyses, the error in relation to the experimental life obtained by Watanabe (2014) was determined through Eq. (27) and, thus, the model that presented the best prediction was defined. It was also analyzed the influence of the size of the frequency bins on the results and the relationship with the parameters of the frequency acquisition rate and the size of the power spectrum.

$$Err = \frac{|N_f - N_{exp}|}{N_{exp}}, \qquad (27)$$

where N_f is the predicted life through the proposed life prediction models and N_{exp} is the experimental life.

4. RESULTS AND DISCUSSION

The fatigue life predictions and the errors were obtained from the spectral and time models considering the 20 proposed analyses. Tables 4, 5 and 6 present all analyses with Δf_b = 0.0219, Δf_b = 0.0439 and Δf_b = 0.0877, respectively.

Table 4. Results of the analyses considering Δf_b = 0,0219.

Δf_b = 0.0219	Fatigue life prediction models (x 10^6)			
Parameters	Rayleigh	Dirlik	Matlab Rainflow	Rainflow Implemented
k = 2, N = 2048	3,19	3,19	4,39	4,38
k = 4, N = 4096	3,15	3,15	3,02	3,01
k = 8, N = 8192	3,22	3,22	3,63	3,63
k = 16, N = 16384	3,11	3,12	3,22	3,22

Table 5. Results of the analyses considering Δf_b = 0,0438.

Δf_b = 0.0438	Fatigue life prediction models (x 10^6)			
Parameters	Rayleigh	Dirlik	Matlab Rainflow	Rainflow Implemented
k = 2, N = 1024	3,35	3,35	4,55	4,56
k = 4, N = 2048	3,34	3,34	3,98	3,99
k = 8, N = 4096	3,36	3,37	3,79	3,77
k = 16, N = 8192	3,38	3,45	2,82	2,76

Table 6. Results of the analyses considering Δf_b = 0,0877.

Δf_b = 0.0877	Fatigue life prediction models (x 10^6)			
Parameters	Rayleigh	Dirlik	Matlab Rainflow	Rainflow Implemented
k = 2, N = 512	3,33	3,33	4,31	4,32
k = 4, N = 1024	3,34	3,34	4,15	4,15
k = 8, N = 2048	3,32	3,33	3,47	3,47
k = 16, N = 4096	3,33	3,36	4,09	4,05

Tables 7 and 8 present all the analyzes for the four models considering Δf_b = 0.1754 and Δf_b = 0.3508.

Table 7. Results of the analyses considering Δf_b = 0,1754.

Δf_b 0.1754	Fatigue life prediction models (x 10^6)			
Parameters	Rayleigh	Dirlik	Matlab Rainflow	Rainflow Implemented
k = 4 N = 512	3,34	3,34	4,44	4,44
k = 8 N = 1024	3,34	3,36	4,38	4,36
k = 16 N = 2048	3,34	3,52	3,2	3,06
k = 32 N = 4096	3,33	4,57	5,05	3,63

Table 8. Results of the analyses considering Δf_b = 0.3508.

Δf_b 0.3508	Fatigue life prediction models (x 10^6)			
Parameters	Rayleigh	Dirlik	Matlab Rainflow	Rainflow Implemented
k = 8 N = 512	4,95	4,99	6,19	6,18
k = 16 N = 1024	4,95	5,27	6,43	6,08
k = 32 N = 2048	4,95	7,06	8,63	6,05
k = 64 N = 4096	4,95	1,36	1,84	6,03

The results of fatigue life from the Serensen-Kogayev hypothesis for the rainflow and spectral method are presented in Tables 9 and 10.

Table 9. Results of fatigue life prediction analyses considering Serensen-Kogayev hypothesis.

Fatigue life prediction - Serensen-Kogayev hypothesis (x 10^6)								
Parameters Δfb = 0.0219	Rainflow	Spectral	Parameters Δfb = 0.0438	Rainflow	Spectral	Parameters Δfb = 0.0877	Rainflow	Spectral
k = 2 N = 2048	1,39	1,71	k = 2 N = 1024	1,44	1,99	k = 2 N = 512	4,94	3,67
k = 4 N = 4096	2,2	2,34	k = 4 N = 2048	2,57	2,63	k = 4 N = 1024	2,31	2,5
k = 8 N = 8192	1,21	1,67	k = 8 N = 4096	2,84	2,69	k = 8 N = 2048	1,8	2,3
k = 16 N = 16384	1,19	1,68	k = 16 N = 8192	3,29	3,1	k = 16 N = 4096	2,58	2,72

Table 10. Results of fatigue life prediction analyses considering Serensen-Kogayev hypothesis.

Fatigue life prediction - Serensen-Kogayev hypothesis (x 10^6)					
Parameters Δfb = 0.1754	Rainflow	Spectral	Parameters Δfb = 0.3508	Rainflow	Spectral
k = 4 N = 512	3,92	3,43	k = 8 N = 512	6,19	5,89
k = 8 N = 1024	2,63	2,85	k = 16 N = 1024	6,13	5,89
k = 16 N = 2048	1,36	2,34	k = 32 N = 2048	6,12	5,89
k = 32 N = 4096	3,18	3,11	k = 64 N = 4096	6,12	5,88

From the fatigue life predictions and knowing the experimental life value, the error of each analysis was determined in order to verify which models were more accurate. In addition, an analysis was performed on the influence of the factor

Δf_b on the life prediction. Thus, Figure 1 shows the error of the spectral models (Rayleigh and Dirlik) and time models (Matlab Rainflow and Rainflow Implemented) with respect to the factor Δf_b and Figure 2 shows the error of the fatigue life prediction results by the Serensen-Kogayev hypothesis.

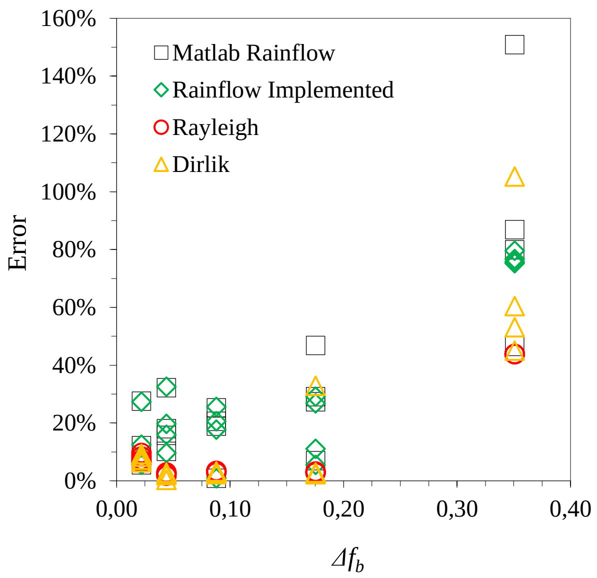

Figure 1. Errors in fatigue life prediction.

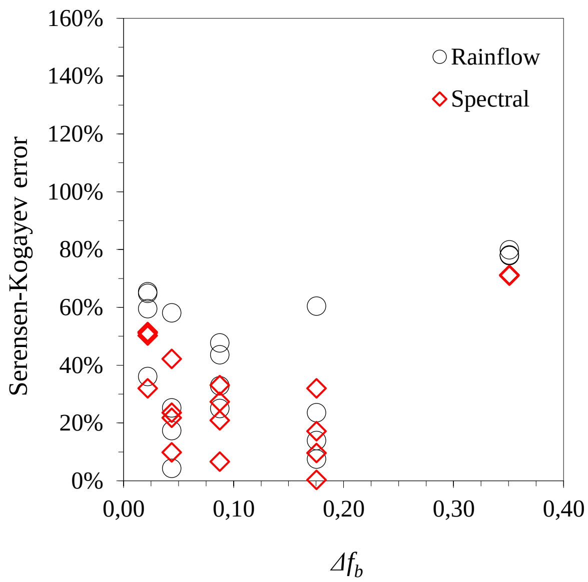

Figure 2. Errors with the Serensen-Kogayev hypothesis.

Thus, both spectral models presented predictions similar to each other and the smaller the size of the frequency bins, the more accurate the fatigue life predictions will be. However, as the Δf_b factor increases, the errors tend to increase and the Dirlik model is more sensitive to these changes, because the damage calculation considers the peak rate and, due to the size of the bins being larger, a smaller amount of information will be analyzed to identify the peaks. As the Rayleigh model considers in the calculation the zero crossing rate, the value is not as variable as the peak rate.

Therefore, both time models had similar fatigue life predictions, as the errors are close. Analogously to spectral models, the greater the Δf_b factor, the greater the error tends to be in the analyses, and the model that used the rainflow routine provided by Matlab presented greater variations in these situations because it did not store the amount of signal peaks when calculated the damage, being necessary to use the value calculated by the spectral models.

For the test analyzed, the spectral models were more accurate than the time models, since the errors obtained were smaller. In addition, when considering the factor Δf_b, the Rayleigh and Dirlik models showed greater stability in the predictions, such that the cable fatigue life predictions have similar results. On the other hand, the time models have greater variations for the same value of Δf_b due to the reconstruction of the time signal occurring randomly.

The predictions made by the Serensen-Kogayev hypothesis for both the rainflow and spectral methods showed values close to each other and when comparing with the other models used, life predictions based on this hypothesis tend to present lower life values, because the correction factor b_{SK} and the analysis only of stress amplitudes that are greater than $a_{SK}S_{af}$ minimize fatigue life.

Therefore, from the test analyzed and the results obtained, it was observed that the spectral models are more adequate in predicting fatigue life, also having as an advantage the lower computational cost. Due to the need that time models have to analyze the entire signal in time, identifying peaks and valleys, the computational cost is high and most of the stored information of the signal is neglected, because the interest is only in the values of peaks and valleys (Bishop, 1988).

Fatigue life predictions in spectral and time models depend on the size of the frequency bins, in which the smaller sizes provide smaller errors in the predictions, i.e., the smaller the value Δf_b the more information is analyzed. However, there is a minimum bin size that any reduction will not provide significant improvement in the face of computational cost, and the advantage of using increasingly smaller bins is that data sampling also increases and, therefore, errors tend to decrease (Martins, 2018). Thus, as the value of Δf_b is the main controller in life predictions, it is not necessary to choose parameters of k and N so high in view of the computational cost, being known that when maintaining the same value of Δf_b the results tend to be equal.

5. CONCLUSIONS

The analysis of the effect of the power spectrum discretization and of stochastic processes simulation techniques on the fatigue life predictions of overhead conductors was the main objective of this paper. For this, the experimental results reported by Watanabe (2014) were used to perform the prediction analysis in the time and frequency domains. Thus, to verify the accuracy of the methods in the time and frequency domain, the results obtained for the life estimated in fatigue were compared with the life obtained experimentally. The main conclusions of the present work can be summarized as follows:

• Both spectral methods (Rayleigh and Dirlik) present fatigue life predictions similar to each other, as well as both time methods (Rainflow Matlab and Rainflow Implemented) and both analyses using the Serensen-Kogayev hypothesis.

• Among the spectral and time methods, the ones that had more adequate predictions when compared to the experimental life were the spectral methods, with smaller errors.

• Regarding the main control factor of the life prediction methods, defined as the size of the frequency bins (Δf_b), it is observed that the smaller its value the more accurate the life predictions will be, as a greater amount of information will be analyzed, independent of the parameters of frequency acquisition rate (k) and the size of the power spectrum (N).

• Spectral methods besides presenting the best results for fatigue life prediction have the advantage of lower computational cost, as they do not need to store as much data as in time methods.

• The results from the Serensen-Kogayev hypothesis were not so accurate, as in some cases they further reduced fatigue life.

6. REFERENCES

ASTM E1049–85, 2017, “Standard practices for cycle counting in fatigue analysis”, *ASTM International*, West Conshohocken, United States of America.

Benasciutti, D. and Tovo, R., 2005, “Spectral methods for lifetime prediction under wide-band stationary random processes”, *International Journal of Fatigue*, Vol. 27, pp. 867-877.

Bishop, N.W.M., 1988, “The use of frequency domain parameters to predict structural fatigue”, Ph.D. thesis, University of Warwick, United Kingdom.

Braccesi, C., Cianetti, F. and Tomassini, L., 2015, “Random fatigue. A new frequency domain criterion for the damage evaluation of mechanical components”, *International Journal of Fatigue*, Vol. 70, pp. 417-427.

Cosmai, U. et al., 2017, “10 - Conductor Motions”, In: Papailiou, K.O editor, CIGRE Green Books: Overhead Lines, Ed. Springer, Malters, Switzerland, pp. 559-711.

Gasior, M. and Gonzalez, J.L., 2004, “Improving FFT frequency measurement resolution by parabolic and gaussian interpolation”, CERN-AB-Note-2004-021-BDI.

Kalombo, R.B. et al., 2020, “Fatigue life estimation of an all aluminium alloy 1055 MCM conductor for different mean stresses using an artificial neural network”, *International Journal of Fatigue*, Vol. 140.

Karolczuk, A. and Kluger, K., 2020, “Application of life-dependent material parameters to lifetime calculation under multiaxial constant- and variable-amplitude loading”, *International Journal of Fatigue*, Vol. 136.

Kurek, M. and Łagoda, T., 2020, “Fatigue damage cumulation in materials under random loading based on an energy model”, *Fatigue Fract Eng Mater Struct*., pp. 1-11.

Lee, Y.L. and Taylor, D., 2005a, “3 - Cycle counting techniques”, In: Lee, Y.L, Pan, J., Hathaway, R.B., Barkey, M.E., editors, Fatigue Testing and Analysis, Ed Butterworth-Heinemann, United States of America, Burlington, pp. 77–102.

Lee, Y.L. and Taylor, D., 2005b, “4 - Stress-based fatigue analysis and design”, In: Lee, Y.L, Pan, J., Hathaway, R.B., Barkey, M.E., editors, Fatigue Testing and Analysis, Ed Butterworth-Heinemann, Burlington, United States of America, pp. 103–180.

Lee, Y.L., 2005a, “10 - Fatigue analysis in the frequency domain”, In: Lee, Y.L, Pan, J., Hathaway, R.B., Barkey, M.E., editors, Fatigue Testing and Analysis, Ed Butterworth-Heinemann, Burlington, United States of America, pp. 369–394.

Lee, Y.L., 2005b, “2 - Fatigue damage theories”, In: Lee, Y.L, Pan, J., Hathaway, R.B., Barkey, M.E., editors, Fatigue Testing and Analysis, Ed Butterworth-Heinemann, Burlington, United States of America, pp. 57–76.

Martins, A.R., 2018, “Observations on the 21cm line of hydrogen: a phenomenological adjustment for extracting baryon acoustic oscillations from the angular power spectrum” (in Portuguese), dissertation, University of Sao Paulo, Brazil.

Niesłony, A. and Macha, E., 2007, “Spectral method in multiaxial random fatigue”, In: Pfeiffer, F. and Wriggers, P., editors, Lecture Notes in Applied and Computational Mechanics, Ed Springer, Vol. 33, 147 p., Opole, Poland.

Rapuano, S. and Harris, F.J., 2007, “An introduction to FFT and time domain windows”, *IEEE Instrumentation & Measurement Magazine*, Vol. 10, pp. 32-44.

Watanabe, L., 2014, “Analysis of the fatigue life of overhead conductors subjected to random loading” (in Portuguese), thesis, University of Brasilia, Brazil.

Yang, J.N., 1973, “On the normality and accuracy of simulated random processes”, *Journal of Sound and Vibration*, Vol. 26, pp. 417-428.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.