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SPRINGS WITH PROPORTIONAL STIFFNESS TO THE APPLIED FORCE

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Abstract. *The studies on springs gained notoriety after the publication of "Lectures of Potentia Restitutive, or Spring Explaining the Power of Spring" by the English physicist Robert Hooke, in 1678, which deals with the fundamentals of the elasticity of materials, providing an important basis for understanding the body´s strain as a function of the stress at which they are subjected. Following Hooke's work, solids came to be understood as deformable bodies, similar to springs, which elongation or shortening could be predicted based on the law established by him, that of the proportionality between the applied force and the response of the spring (or the solid) in terms of the presented displacement. To establish the cause-and-effect relationship a proportionality constant called stiffness or spring coefficient was foreseen. In this work, however, the concept that defines a spring stiffness is established no longer as a function of the induced displacement but through the applied force on the system, an assumption that can be, adequately evaluated through the theory of vibration of the mechanical systems. In this regard, a model consisting of a simply supported beam submitted to a normal force that tends to reduce its frequencies is used to verify the assumed hypothesis. Connected to the system, there is a translational spring positioned according to its longitudinal direction, which stiffness is allowed to vary with the intensity of the applied force. The spring, or analogous device, therefore, acts in order to keep the natural frequency of the beam unmodified. The model was computationally elaborated by the finite element method. The results obtained prove the constancy of the natural frequency of the beam, even when the system is under the action of a force that tends to reduce its structural stiffness and, consequently, to change the frequency of vibration of the system.*

Keywords: *variable stiffness; spring; vibration; frequency; finite elements method.*

1. INTRODUCTION

This work deals with the control of beam vibrations as a basis for rotating machines, being in line with important aspects related to the industry; agribusiness; communications; and infrastructure. In this sense, the presented work deals with the control and mitigation of vibrations in mechanical and structural systems. A class of economic-strategic importance for the national civil and military industry are the machine bases, subject to vibrations induced by the supported equipment.

The vibrations can affect the safety of the structure itself, but, in the most general case, they can have harmful effects on the equipment itself and the quality of the manufactured product. They can also make the working environment for operators unsuitable. All branches of industry are subject to these problems, including the extremely sensitive areas of oil exploration, mining and refining, wind energy, and atomic energy.

In this context, in the present work, the effects of non-linearities present in the support systems of rotating machines such as engines or assembly lines were studied. The mathematical basis of the problem lies in the fact that the dynamic characteristics of a structure depend on its stiffness and its mass. With these two parameters, its natural frequencies and vibration modes are determined. However, the initial stiffness, determined in the unloaded situation, can be affected by the presence of loads, the so-called geometric stiffness (Wahrhaftig *et al.*, 2013; Wahrhaftig and Brasil, 2016; Wahrhaftig and Brasil, 2017a). In the case of compression forces, which tend to reduce stiffness and vibration frequencies, these can lead to loss of system stability or resonance.

To avoid resonance or to establish desired operating conditions, there is a need to maintain the frequency of the support system at the level at which it was originally designed. For this, it is necessary to design a device, or spring, of variable stiffness to follow the change in the demand level. This is the investigated hypothesis, which is verified through computer simulation.

2. JUSTIFICATION

Studies on springs gained notoriety with the English physicist Robert Hooke (1678) after the publication of his work “Lectures de Potentia Restitutiva, or Spring Explaining the Power of Springing”, which brings the fundamentals about the elasticity of materials. This work provided an important foundation for the understanding of the deformations that bodies present as a function of the stress to which they are submitted.

Following Hooke's work, solids came to be understood as deformable bodies, similar to springs, whose elongation or shortening could be predicted based on the law established by him, that of proportionality between the applied force and the response of the spring (or of the solid) in terms of the displacement presented, with a constant of proportionality called stiffness or spring coefficient, responsible for establishing the cause-and-effect relationship.

In the present investigation, the concept that defines the stiffness of a spring is established no longer as a function of the displacement presented, but through the force imposed on the system, an assumption that can be properly evaluated through the theory of vibration of systems mechanics.

For this investigation, the computational model of an axially compressed beam was elaborated using the finite element method (FEM). It is worth mentioning that the FEM is a continuous discretization technique, whose domains are divided into small but finite regions, united in nodes, in which the generalized displacements are the unknown terms of the problem. The beam, in this computational model, is being axially loaded by a normal force that tends to reduce its frequencies. Connected to the system, there is a translational spring positioned according to its longitudinal direction, whose stiffness is admitted to vary with the intensity of the applied force. The spring, or similar device, therefore, acts to maintain the natural frequency of the system unchanged.

It should be noted that a device that responds according to the force to which it is subjected may have applications in various fields of engineering, whether in the field of buildings, in the control of actions of a seismic nature or wind, for example, or in mechanical engineering with applications in vehicles for various purposes. An example that characterizes a mechanical system with the direct application of this concept is automotive vehicles. It is known that for automobiles, the comfort condition for their occupants is defined by the quality of their suspension, which is nothing more than a spring-damper system. This condition is related to the load carried by the vehicle. Imagine that the suspension has been calibrated so that the optimum condition is linked to the vehicle occupied only by the driver. If the vehicle adds passengers, it has a different condition from the previous one, as the mass and, consequently, the forces on the suspension have been increased. Now consider that, in addition to passengers, cargo is added to the trunk of the car. Under these conditions, the vehicle's dynamics undergo a new change, modifying the comfort condition that was initially foreseen. It should be conceived, however, that the suspension system is capable of being regulated according to the acting force to maintain the optimum comfort condition. In this condition, no changes would be produced under the pre-established conditions.

3. BIBLIOGRAPHIC REVIEW

Excessive vibrations cause discomfort to users and, in some situations, can cause the failure of structures and mechanical components. According to Santana and Pinto (2019), one of the most common causes of several vibration problems in structures is the dynamic load, characterized by its fundamental properties: the modes and natural frequencies of the structure. According to Brito (2014), vibrations can be natural, such as earthquakes or strong winds in structures, or from human activity, such as industrial machines, heavy traffic on highways and railways, and the construction industry itself, is one of the most significant sources for generating a good part of the vibrations in the urban environment.

In the specific field of variable stiffness mechanisms, Liu *et al*. (2020) consider these to be a type of mechanism that can be used for flexible actuation, so that its dynamic parameters can be adjusted to a specific problem, aiming to achieve the necessary stiffness for each position or displacement of the system, varying according to the external loads. Also, according to Liu *et al*. (2020), a variable stiffness mechanism, improves the safety of machines in operation and can also meet the requirements of different working conditions, being a topic widely studied in the field of robotics and mechanics.

Several studies on the application of devices with variable stiffness have emerged. According to Jutte (2008), springs of variable stiffness are commonly applied in mechanisms that support a variable or unknown load. A known application of a spring of variable stiffness is the car suspension system. On the other hand, Vuong *et al*. (2017) designed a variable stiffness joint for robotic machining applications. Moutinho (2007) presented a system based on changing the stiffness of a building through the use of devices capable of providing a variable, active stiffness, to avoid resonance phenomena. Particularly, in this case, devices were installed in diagonals on certain floors of the building, which constituted of hydraulic cylinders regulated by valves that made it possible to instantly command the blocking or release of the bars, mobilizing or demobilizing their axial stiffness.

Liu *et al*. (2020) classify variable stiffness mechanisms into two groups:

(a) passive, composed of springs with different shapes, in which the spring stiffness is altered according to the equilibrium position so that the stiffness is achieved according to external loads; and

(b) active, which can have their stiffness adjusted without depending on the effect of external loads.

While the first type is considered to be a simple mechanism with some limitations, the active mechanism needs additional energy inputs to perform active stiffness adjustments.

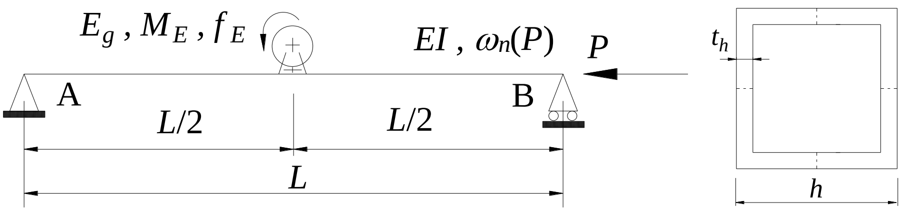
According to Moutinho (2007), the most common way to attenuate vibrations is through the insertion of elements that act on the stiffness of the system. These elements modify the stiffness of the structure by causing a dynamic change in the system and its response to external actions. Among these is active control. Cordeiro (2017) states that, unlike passive control, active control introduces forces directly into the structure to attenuate vibrations. Guimarães (2013) highlights the importance of vibration control for medium and large machines and structures that are subject to cyclic loading, especially over a long period.

4. BASIC HYPOTHESIS OF THE PROBLEM STUDIED

The theoretical basis of this work enters the field of vibration theory, where stiffness and masses are the main calculation parameters. The present investigation, therefore, focuses on mathematical modeling, via computational analysis, of a device that acts similar to a spring that has variable stiffness with the applied force.

This hypothesis is based on the fact that when observing the operation of a spring when applying and removing a force, it is possible to verify its ability to recover the imposed deformation and, in applications of a dynamic nature, to restore the induced oscillatory movement in the system. From this observation, the hypothesis arises that a spring can be introduced in a mechanical or structural system to control its vibrations.

In the context of the dynamics of structures, the mathematical model represented by Figure 1(a) and (b) assumes that the presence of an axial compressive, which decreases the beam's stiffness and consequently its natural vibration frequencies, can lead to unexpected, potentially dangerous resonant regimes. In Figure 1(a), P is the axial force, L is the length of the beam, $f_{E,}$ and $\omega_n(P)$ are the motor frequencies and the natural frequency of the beam, respectively, Eg designates the rotating machine and M_E its mass, EI is the product of the bending stiffness of the beam. In Figure 1(b) h and t_h is the side and wall thickness dimensions of the cross-section of the beam.



(a) (b)

Figure 1. The mathematical hypothesis of the problem: (a) beam model, (b) cross-section.

Taking it as an example, when approaching the present problem through continuous mechanics, Wahrhaftig *et al.* (2018) and Wahrhaftig *et al*. (2020) obtained the frequency of the first natural mode of vibration of the reinforced concrete beam represented in Figure 1, as a function of a compressive force P. It could be verified by them, mathematically, that turning the force as the independent variable of the problem, and calculating the frequency for increasing levels of force, as can be seen in Figure 2, the resonance is induced, generating a resonant regime that can be noticed by the intersection of the presented curves, solid line (beam) and dotted line (rotating machine), which can represent an unwanted design condition. In the opposite direction, the same force that drives the system to resonance can provide a way to move the structure away from the resonant regime, if noticed in the preliminary design stages, as shown in the same Figure 2 by indicating a non-resonant regime. Therefore, the presented solution can be a useful tool in the context of design and structural analysis. However, it should be noted that what is verified by the use of this tool, whether in one or another condition, that is, in a resonant or non-resonant regime, is the imposition that occurs to change the vibration frequency of the supporting system.

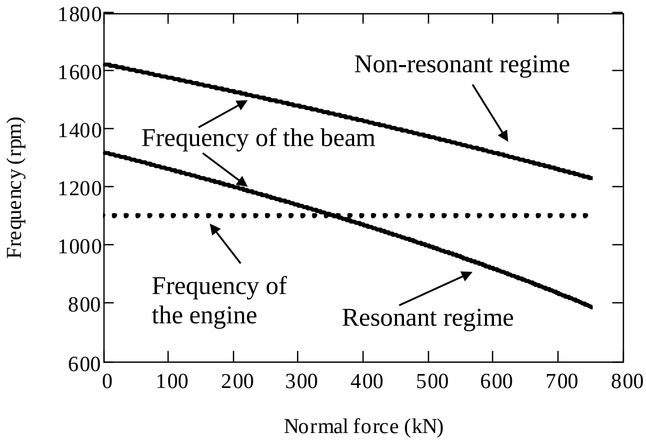


Figure 2. The resonant and controlled regime of vibration.

In contrast to the conditions presented above, the basic hypothesis of the present investigation lies in the imposition that the natural vibration frequency of the base system remains the same. Therefore, this hypothesis can be compared to a model consisting of a simple-supported beam axially loaded by a normal force that tends to reduce its frequencies and that presents a behavior similar to a control system with the presence of a translational spring that acts on the sense of keeping the natural frequency of the beam unchanged. Thus, hypothetically, it would be possible to avoid resonance phenomena and/or calibrate a mechanical/structural system so that it maintains its pre-established operating conditions.

However, as, ordinarily, a spring has a single value for its stiffness, this device would not be able to cope with changes in the level of such applied force. Faced with this evidence, a new hypothesis, naturally, emerges, that of the need to work with a spring, or similar device, of variable stiffness with the applied force, capable of maintaining the dynamic characteristics of the system unaltered. In the mathematical model of a beam with this consideration, Figure 3, the spring is pierced by an arrow, representing that its elastic constant, or stiffness, is variable.

Figure 3. Vibration model of a beam having attached a spring with variable stiffness.

5. COMPUTATIONAL SIMULATION

It is important to note that a beam element containing 6 degrees of freedom can be represented according to Figure 4, where x and z are the local reference axes; l, u, and v are the length and axis offsets of the element; u_1, v_1, θ_1, u_2, v_2, θ_2 are the displacements related to the translational and rotational movement of nodes 1 and 2, respectively. In terms of this type of element and following the development presented by Filho (1975), Ritto *et al*. (2008), Wahrhaftig (2017b), and Rao (2018), the stiffness and mass matrices are obtained by Eq. (3) to (6), where the bending stiffness matrix, Eq. (3), depends on elastic properties and beam geometry (section and length); and the geometric stiffness, Eq. (4), depends on the normal force and length of the beam. The matrix that considers the presence of an externally applied spring, Eq. (5), is a matrix where its components contain the values of the translational and rotational stiffnesses concerning the element's degrees of freedom. In the present case, only one translational spring stiffness value was added to the system, just to the right node of the element located in contact with the free support, as indicated in Figure 5. The consistent mass matrix of a beam element is given by Eq. (6).

Thus, the modeling performed in the present study for the analysis of vibration control with a variable stiffness device was done using the finite elements method (FEM), taking planar beam-frame elements, and based on a non-linear hypothesis, whose use lies in the use of the three portions of the stiffness of the structure, therefore consisting in the linearization of a non-linear problem. The model represented in Figure 7 was developed at SAP 2000 program (2019) and has 33 elements, 67 nodes, totalizing 198 degrees of freedom. The boundary conditions assumed in the simulation were two end supports, both hinged, with the left one being fixed and the right one is unrestrained. It is worth mentioning that the assembly of the total matrices of the structure, stiffness, and mass, about the global reference axes of the system (X, Z), which considers the superposition of the degrees of freedom of each 2D beam element, was performed automatically by the used program.

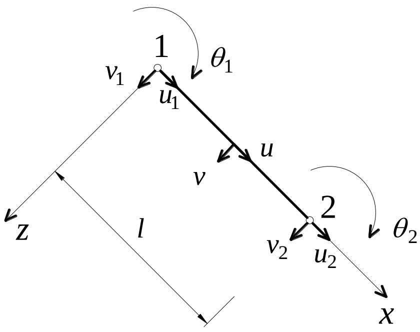


Figure 4. Common one-dimensional bending element with 6 degrees of freedom.

A computational model, Figure 5, therefore, was designed to represent the hypothesis contained in the beam in Figure 3 and took into account the following material and geometric parameters:

- Modulus of elasticity (E): 205 GPa
- Poisson's coefficient (ν): 0.33
- Density: 8750 kg/m^3
- Acceleration of gravity: 9.81 m/s^2
- Length (L): 2 m
- Cross section dimensions: t_h = 1.5 mm e h = 50 mm.

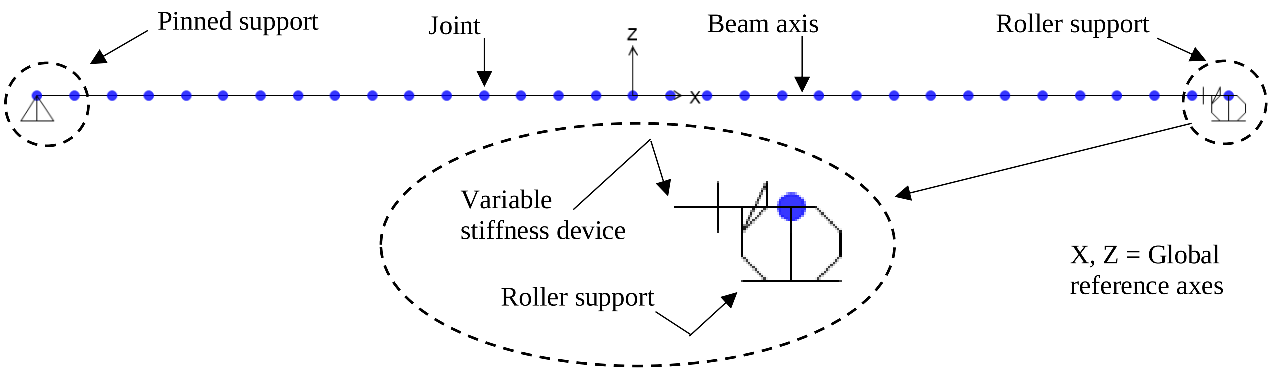


Figure 5. Finite elements computational model.

The frequencies and natural modes of vibration are obtained, using FEM, by solving a problem of eigenvalues and eigenvectors, as shown in Eq. (1),

$$([K(P)] - \{\omega_n^2(P)\}[M])\varphi_n \quad = 0, \tag{1}$$

where $[M]$ is the total generalized mass matrix, $[K(P)]$ is the total generalized stiffness matrix of the system, which is given by:

$$[K(P)] = [\mathrm{K}_0] + [\mathrm{K}_G(P)] + [\mathrm{K}_m]. \tag{2}$$

In Eq. (1), $\{\omega_n(P)\}$ represents the natural frequencies of the system, in rad/s, associated with the force P and φ vibration modes. Therefore, the vibration mode φ_{ni} will be related with the frequency $\{\omega_{ni}(P)\}$. Considering a beam element with 6 degrees of freedom, the stiffness matrices, $[\mathrm{K}_G(P)]$, $[\mathrm{K}_0]$, $[\mathrm{K}_m]$, and mass $[M]$, where P is the independent variable of the vibration problem, are, respectively:

$$\mathrm{K}_G(P) = \frac{P}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{6}{5} & \frac{L}{10} & 0 & -\frac{6}{5} & \frac{L}{10} \\ & & \frac{2L^2}{15} & 0 & -\frac{L}{10} & -\frac{L^2}{30} \\ Symetric & & & 0 & 0 & 0 \\ & & & & \frac{6}{5} & -\frac{L}{10} \\ & & & & & \frac{2L^2}{15} \end{bmatrix}, \quad (3)$$

$$\mathrm{K}_0 = E\begin{bmatrix} \frac{A}{L} & 0 & 0 & -\frac{A}{L} & 0 & 0 \\ & \frac{12I}{L^3} & \frac{6I}{L^2} & 0 & -\frac{12I}{L^3} & \frac{6I}{L^2} \\ & & \frac{4I}{L} & 0 & -\frac{6I}{L^2} & \frac{2I}{L} \\ Symetric & & & \frac{A}{L} & 0 & 0 \\ & & & & \frac{12I}{L^3} & -\frac{6I}{L^2} \\ & & & & & \frac{4I}{L} \end{bmatrix}, \quad (4)$$

$$\mathrm{K}_m = \begin{bmatrix} 0 & 0 & 0 & k_m & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ Symetric & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{bmatrix}, \quad (5)$$

$$M = \frac{\rho A l}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22l & 0 & 54 & -13l \\ & & 4L^2 & 0 & 13L & -3l^2 \\ Symetric & & & 140 & 0 & 0 \\ & & & & 156 & -22l \\ & & & & & 4l^2 \end{bmatrix}. \quad (6)$$

In Eqs. (3)-(6) l is the length of the element, I is the moment of inertia of the section related to the considered motion, ρ is the material´s density, and A is the cross-sectional area of the bar element. It is worth mentioning that the engine mass was added to the system in the central position of the model. Therefore, the total mass matrix must incorporate this value in the node corresponding to this degree of freedom. With the considerations made above, the frequency of the beam's first natural mode of vibration to $P = 0$ is 22.81 Hz, with Hz = $\omega_n(P)/2\pi$, with this mode described in Figure 6.

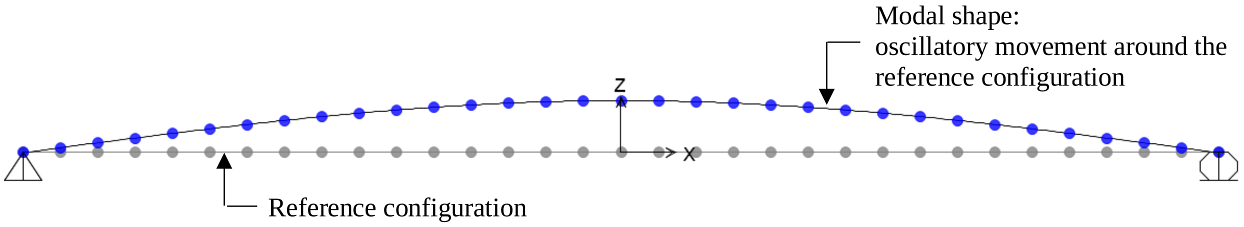


Figure 6. First vibration mode.

Therefore, after calculating the natural frequency of the beam in the unloaded condition, compressive forces were applied to the system to reduce its value, following the hypothesis given in Eq. (1). At the same time, the stiffness of the device (spring) was varied to maintain the beam frequency unchanged. The results regarding the applied forces and the corresponding spring stiffnesses (k_m) are shown in Table 2.

It can be observed that the results obtained confirm the constancy of the beam's natural frequency even facing a force capable of altering it. This is due to the adjustment made in the spring stiffness as the axial compressive force was applied, making the system's natural frequency not changeable. Therefore, it is verified that the control device acts efficiently to nullify the effect of geometric stiffness, $[K_G(P)]$, as shown in the chart of Figure 7. In that graph, it is seen that the spring stiffness varies linearly with the applied force to maintain the beam's natural frequency at 22.81 Hz. It is interesting to mention that the force of 45 kN is close to that which produces the collapse of the system by equilibrium bifurcation (buckling).

Table 1. Forces and spring stiffness.

Force (kN)	**Spring stiffness (kN/m)**	**Frequency (Hz)**
0	0	22,81
5	11600000	22,81
10	23300000	22,81
15	34845000	22,81
25	58200000	22,81
30	69800000	22,81
35	81500000	22,81
40	9330000	22,81
45	105000000	22,81

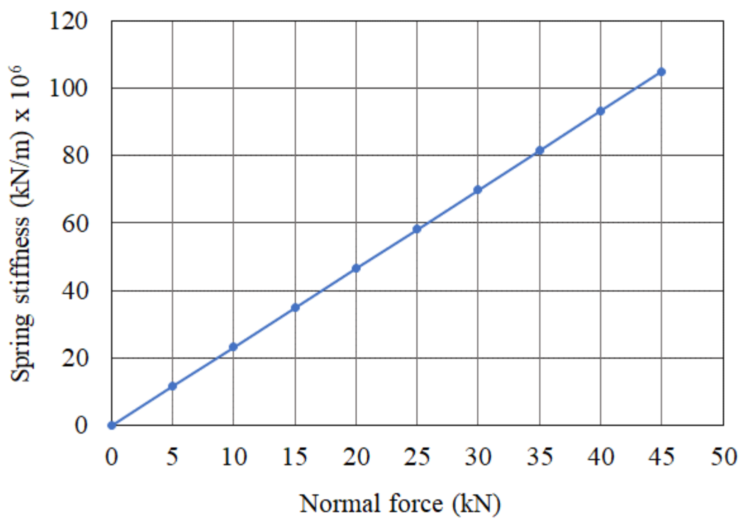


Figure 7. Variable stiffness as a function of applied force (constant beam frequency).

6. CONCLUSIONS

The present work objective was to computationally evaluate the hypothesis of designing variable stiffness devices, as a function of the applied force, in the control of vibrations. For this purpose, a simple-supported beam was simulated, via finite element method, being axially stressed by a normal compression force, which tends to reduce its natural frequency and whose intensity was the independent variable of the vibration problem. To evaluate this hypothesis, a translational spring was added to the beam at one of its ends, whose stiffness was admitted varying with the applied force, and its value was calibrated to maintain the natural frequency of the system unchanged. The results obtained confirm the mathematical hypothesis assumed and the feasibility, theoretically, of the use of variation stiffness devices in the control of vibrations.

For future studies, simulations in the field of mechanical and civil engineering are expected, as well as experimental laboratory tests with physical models.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Brito, L. A., 2014. "Assessment of the main sources of vibration in the urban environment". *Built Environment.* Vol. 14. pp. 233-249. https://doi.org/10.1590/S1678-86212014000400016.

Cordeiro, D.B., 2017. *Study of the effect of implementing a semi-active vibration control system at the pedestrian bridge of the transparent Building*. Master's Thesis, Faculty of Engineering, University of Porto, Porto, Portugal.

Filho F. V., 1975. Matrix analysis of structures (Static Stability, Dynamics). Technological Institute of Aeronautics, Almeida Neves (Ed.), Rio de Janeiro.

Guimarães, M. P., 2013. Semiactive vibrations control by nonlinear control force (in Portuguese). Doctoral Dissertation, University of Brasilia, Brasilia, Brazil.

Hooke, R., 1678. "Lectures de Potentia Restitutiva, or of Spring Explaining the Power of Springing Bodies", Printed for John Martyn to the Royal Society, London, UK.

Jutte, V.C., 2008. *Generalized synthesis methodology of nonlinear springs for prescribed load-displacement functions*. Doctoral Dissertation, University of Michigan, Michigan, United States.

Liu, H., Zhu, and D., Xiao, J., 2020. "Conceptual design and parameter optimization of a variable stiffness mechanism for producing constant output forces". *Mechanism and Machine Theory*, Vol. 154, 104033. https://doi.org/10.1016/j.mechmachtheory.2020.104033.

Moutinho, C.M.R., 2007. *Vibration control in civil engineering structures* (in Portuguese). Doctoral Dissertation, Faculty of Engineering, University of Porto, Porto, Portugal.

Rao, S. S., (2018) The Finite Element Method in Engineering, 6th Edition, Elsevier, Butterworth-Heinemann. https://doi.org/10.1016/C2016-0-01493-6.

Ritto, T. G., Sampaio, R. and Cataldo, E., 2008. "Timoshenko beam with uncertainty on the boundary conditions". Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 30, n. 4, pp. 295-303. https://doi.org/10.1590/S1678-58782008000400005.

Santana, A.L.S., and Pinto, J.F.A., 2019. "Study of vibrations with multiple sources of excitements in mixed steel and concrete structures and performance assessment considering human comfort" (in Portuguese). *V Scientific seminar of UNIFACIG*, Minas Gerais, Brazil.

SAP2000, 2019. "Integrated Software for Structural Analysis and Design, Analysis Reference Manual". *Computer Structures.*, Inc, Berkeley, USA.

Vuong, N.D., Li, R., Chew, C.M., Jafari, A., and Polden, J., 2017. A novel variable stiffness mechanism with linear spring characteristic for machining operations. *Robotica*, Vol. 35(7), pp. 1627-1637. https://doi.org/10.1017/S0263574716000357.

Wahrhaftig, A.M., and Brasil, R.M.L.R., 2016. "Representative experimental and computational analysis of the initial resonant frequency of largely deformed cantilevered beams, *International Journal of Solids and Structures*, Vol. 102–103, pp. 44-55. https://doi.org/10.1016/j.ijsolstr.2016.10.018.

Wahrhaftig, A.M., and Brasil, R.M.L.R., 2017(a). "Vibration analysis of mobile phone mast system by Rayleigh method". *Applied Mathematical Modelling*, 42, pp. 330-345. https://doi.org/10.1016/j.apm.2016.10.020.

Wahrhaftig, A.M., and Brasil, R.M.L.R., and Nascimento, L.S.M.S.C., 2018(a). "Analytical and Mathematical Analysis of the Vibration of Structural Systems Considering Geometric Stiffness and Viscoelasticity*". Numerical Simulations in Engineering and Science*, Vol. (1), pp 349-369. https://doi.org/10.5772/intechopen.75615.

Wahrhaftig, A.M., and Brasil, R.M.L.R.F, 2017(b). "Initial undamped resonant frequency of slender structures considering nonlinear geometric effects: the case of a 60.8 m-high mobile phone mast". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*. Vol. 39, pp.725–735. https://doi.org/10.1007/s40430-016-0547-1.

Wahrhaftig, A.M., Brasil, R.M.L.R., and Balthazar, J.M., 2013. "The first frequency of cantilevered bars with geometric effect: a mathematical and experimental evaluation". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 35, pp. 457–467. https://doi.org/10.1007/s40430-013-0043-9.

Wahrhaftig, A.M., Brasil, R.M.L.R.D.F., Groba, T.B., Rocha, L.M.L., Balthazar, J.M., and Nascimento, L.S.M.S.C., 2020. "Resonance of a rotary machine support beam considering geometric stiffness". *Journal of Theoretical and Applied Mechanics*, Vol. 58(4), pp. 1023-1035. https://doi.org/10.15632/jtam-pl/126681.

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