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TRANSIENT ANALYSES OF VISCOELASTIC SYSTEMS BASED ON AN IMPROVED FRACTIONAL DERIVATIVE MODEL AND A REDUCTION TECHNIQUE

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Abstract. *This paper aims to analyze the transient response of a sandwich bi-dimensional frame with a viscoelastic core. Time domain analysis of viscoelastic systems is often cumbersome, and, even with the development of new models, the use of a model reduction technique is usually required. A recurrence based fractional derivative model is used to describe the viscoelastic layer mechanical behavior, the system is discretized using the finite element method, a model reduction technique is applied, and finally, the equation of motion of the system is evaluated using Newmark's method. Through this procedure the time domain responses for the treated and untreated structures are shown. The complete and reduced model are compared in terms of computational time and dynamical responses, which shows the efficiency and effectiveness of the model reduction technique in describing the dynamical responses of viscoelastic systems.*

Keywords: *viscoelastic material, model reduction technique, vibration control, fractional calculus, finite element method.*

1. INTRODUCTION

Viscoelastic materials are widely used in passive vibration control, and several researchers have been working on modeling and applying these materials to dynamic systems (Khoshraftar, 2016; Rao, 2013; Filho et al., 2016; Gonçalves; Rosa; Lima, 2019). Modeling the time domain response of such materials is cumbersome and several models have been proposed. Fractional derivative models (FDM), first proposed by Bagley and Torvik (Bagley; Torvik, 1979; Bagley; Torvik, 1983), and then studied by other researchers such as Galucio et al. (2004) and Schmidt and Gaul (2001), has shown to be the most powerful tool in describing the dynamical properties of such materials (Zhou et al., 2016).

However, when it comes to practical time domain applications, the FDM proposed by Bagley and Torvik, Galucio and colleagues, and Schmidt and Gaul have some limitations as discussed by Nunes (2020). Therefore, based on the work of Schmidt and Gaul (2001), researchers from the Federal University of Uberlândia developed an improved fractional derivative model to describe the viscoelastic transient behavior (Nunes, 2020; Filho et al., 2021; Nunes; Lima, Cunha-Filho, 2021). This model is based on recurrence terms and has shown to make the analysis of those structures more efficient, when compared with other fractional derivative models (Nunes, 2020).

Nonetheless, even with the development of more efficient models, the time domain analysis of viscoelastically treated structures still requires the use of a model reduction technique, especially those complex systems with a large number of degrees of freedom (DOF) and industrial interest. Therefore, this paper aims to apply this improved FDM and a new model reduction technique in the transient analysis of a viscoelastically treated bi-dimensional frame.

2. MODELLING

Based on a four parameters fractional derivative model (Bagley; Torvik, 1979; Makris, 1997), a recurrence-based FDM for the viscoelastic material behavior is shown for shear and elongation in Eqs. (1)-(2), as described by Nunes and Lima (2021) and Nunes (2020).

$$\tau_t = \sum_{j=0}^{N_l} \beta_{j+1}^{G} \gamma_{t-j\Delta t} \tag{1}$$

$$\sigma_t = \sum_{j=0}^{N_l} \beta_{j+1}^E \varepsilon_{t-j\Delta t} \tag{2}$$

Where τ_t and $\gamma_{t\text{-}j\Delta t}$ are shear stress and deformation at time steps t and t-jΔt, $\sigma_{t\text{-}j\Delta t}$ and $\varepsilon_{t\text{-}j\Delta t}$ are normal stress and deformation at time steps t and t-jΔt. β^G_{j+1} and β^E_{j+1} are the recurrence terms described in Eqs. (3)-(4).

$$\beta_{j+1}^G = \frac{2G_\infty \Delta t^{-\alpha}}{1+a_G\Delta t^{-\alpha}} A_{j+1} - \sum_{i=0}^{j} \frac{a_G \Delta t^{-\alpha}}{1+a_G\Delta t^{-\alpha}} A_{i+1} \beta_{j+1-i}^G \tag{3}$$

$$\beta_{j+1}^E = \frac{E_\infty \Delta t^{-\alpha}}{1+a_E\Delta t^{-\alpha}} A_{j+1} - \sum_{i=0}^{j} \frac{a_E \Delta t^{-\alpha}}{1+a_E\Delta t^{-\alpha}} A_{i+1} \beta_{j+1-i}^E \tag{4}$$

$$\beta_1^G = \frac{2G_0 + 2G_\infty \Delta t^{-\alpha}}{1+a_G\Delta t^{-\alpha}} \tag{5}$$

$$\beta_1^E = \frac{E_0 + E_\infty \Delta t^{-\alpha}}{1+a_E\Delta t^{-\alpha}} \tag{6}$$

E_0 and E_∞ are the elongation modulus associated with low and high frequencies, G_0 and G_∞ are the shear modulus associated with low and high frequencies, a_E and a_G are parameters of the model associated with elongation and shear, and α is the fractional derivative order. It can be seen that this fractional derivative model requires seven parameters, which are found by curve fitting, however, only four (shear or elongation) are really necessary since elongation and shear parameters are related to each other by the elastic-viscoelastic correspondence principle.

The damping properties of viscoelastic materials come from the memory effect, i. e., previous stress and strain states affect the current behavior of the material. Fractional calculus is a powerful tool in modeling memory phenomenon, however, classical FDM present a self-dependence on stress field. The great advantage of this improved model is to remove this self-dependence, and as discussed by Nunes (2020), this new approach is able to considerably improve computational cost.

The system is then discretized into finite elements. Based on the Classical Laminate Theory (CLT), the displacement fields for each layer are described in terms of a four degrees of freedom model – u, axial displacement; w, transversal displacement; θ, rotation and β, shear deformation of the viscoelastic layer. Linear interpolating functions are used for u and β, cubic functions for w, and θ is defined as w's first derivative. Equations (7)-(10) describe the displacement fields, where {q(e)} is the DOF vector and N(x) are interpolating function. Strain is related to displacement fields as its first derivative.

$$w(x,t) = \left[N_w(x)\right]\left\{q_{(e)}\right\} \tag{7}$$

$$u^{(1)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} \tag{8}$$

$$u^{(2)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} + \left(z - \frac{h_1}{2}\right)\left[N_\beta(x)\right]\left\{q_{(e)}\right\} \tag{9}$$

$$u^{(3)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} + h_2\beta(x,t)\left[N_\beta(x)\right]\left\{q_{(e)}\right\} \tag{10}$$

Kinetic energy is described in Eq. (11) and mass matrix is defined in Eq. (12), where k is an index referred to each layer, ρ is the density, b is the width, h is the thickness of each layer, li is the finite element length and A is the transversal area of each layer. Potential energy is defined as the sum of its components for each layer. For the elastic layers, Eq. (13) shows the deformation energy and Eq. (14) describes the stiffness matrix, where E represents the Young's modulus. For the viscoelastic layer the constitutive law shown in Eqs. (1) and (2) are used and the deformation energy is written in Eq. (15). In this case, it is not possible to define a single stiffness matrix and one for each time step that will be updated by the recurrence terms is used. Stiffness matrix is shown in Eq. (16).

$$T_e = \frac{1}{2}\{\dot{q}_e\}^T[M_e]\{\dot{q}_e\} \tag{11}$$

$$[M_e]=\sum_{k=1}^{3}\rho_k\left(b\int_{z_k}^{z_k+h_k}\int_0^{li}\left[N_u^{(k)}\right]^T\left[N_u^{(k)}\right]dxdz + A_k\int_0^{li}\left[N_w^{(k)}\right]^T\left[N_w^{(k)}\right]dx\right) \tag{12}$$

$$V_e^{(e)} = \frac{1}{2}\{q_e\}^T\left[K_e^{(e)}\right]\{q_e\} \tag{13}$$

$$\left[K_e^{(e)}\right] = b\sum_{k=1,\ 3} E_k\int_{z_k}^{z_k+h_k}\int_0^{li}\left[N'^{(k)}_u\right]^T\left[N'^{(k)}_u\right]dxdz \tag{14}$$

$$V_e^{(v)} = \frac{1}{2}\{q_e(t)\}^T\sum_{j=0}^{N_l}\left[K_e^{**(v)}\right]_j\{q_e(t-j\Delta t)\} \tag{15}$$

$$\left[K_e^{**(v)}\right]_j = b\left(\beta_{j+1}^E\int_{z_k}^{z_k+h_k}\int_0^{li}\left[N'^{(2)}_u\right]^T\left[N'^{(2)}_u\right]dxdz + \beta_{j+1}^G\int_{z_k}^{z_k+h_k}\int_0^{li}\left[N^{(2)}_\beta\right]^T\left[N^{(2)}_\beta\right]dxdz\right) \tag{16}$$

The contribution of each element is summed. In this step, it is important to notice that the finite elements of the frame may have different orientations. In this case, the DOF are transformed from the local coordinate system of the element to the global coordinate system of the structure. Fig. 1 illustrates a finite element whose local coordinate system is rotated in an angle ϕ. The transform matrix is shown in Eq. (17). Global mass and stiffness matrices are assembled as described in Eqs. (18)-(20), where N represents the number of elements.

$$[T] = \begin{bmatrix} \cos\phi & sen\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ -sen\phi & \cos\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\phi & sen\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & -sen\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

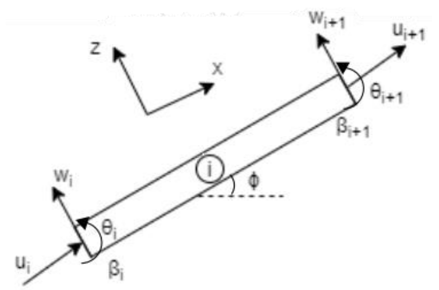

Figure 1. Finite element and local coordinate system representation.

$$[M] = \bigcup_{e=1}^{N}[T]^T[M_e][T] \tag{18}$$

$$\left[K^{(e)}\right] = \bigcup_{e=1}^{N}[T]^T\left[K_e^{(e)}\right][T] \tag{19}$$

$$\left[K^{**(v)}\right]=\bigcup_{e=1}^{N}[T]^T\left[K_e^{**(v)}\right][T] \tag{20}$$

The system equation of motion is found using Lagrange's equation, which results in Eq. (21).

$$[M]\{\ddot{q}(t)\}+\left[K^{(e)}\right]\{q(t)\}+\sum_{j=0}^{N_l}\left[K^{**(v)}\right]_j\{q(t-j\Delta t)\}=\{f(t)\} \tag{21}$$

Depending on the number of DOF of the model, global matrices may have hundreds of thousands or even millions of lines, this makes computational analysis of Eq. (21) difficult or even impractical in ordinary computers. One way to simplify this model and still maintain the accuracy of the responses is to use a model reduction method. The principle of practically every model reduction technique is to multiply mass and stiffness matrices by a basis containing a small number of vectors when compared to the total number of DOFs. The fundamental difference is how to construct this basis.

In this work the proposed basis is the system dynamical response itself. The complete system is subjected to an impulse force, the displacement vectors for a small period of time are stored and orthonormalized. The resulting vectors will compose the base $[T_r]$, as shown in Eq. (22), where NR is the order of the reduction basis. Equations (23)-(26) describe reduced mass and stiffness matrices and the external force vector.

$$[T_r]=orth\left([\{q_1\} \quad \{q_2\} \quad ... \quad \{q_{NR}\}]\right) \tag{22}$$

$$[M_r]=[T_r]^T[M][T_r] \tag{23}$$

$$\left[K_r^{(e)}\right]=[T_r]^T\left[K^{(e)}\right][T_r] \tag{24}$$

$$\left[K_r^{**(v)}\right]=[T_r]^T\left[K^{**(v)}\right][T_r] \tag{25}$$

$$\{f_r\}=[T_r]^T\{f\} \tag{26}$$

After this procedure, the system equation is reduced to Eq. (27) and every matrix has order NR, which is very smaller than the original order size.

$$[M_r]\{\ddot{q}_r(t)\}+\left[K_r^{(e)}\right]\{q_r(t)\}+\sum_{j=0}^{N_l}\left[K_r^{**(v)}\right]_j\{q_r(t-j\Delta t)\}=\{f_r(t)\} \tag{27}$$

This equation is then solved using Newmark's constant average acceleration method.

3. SIMULATIONS

Consider the bi-dimensional sandwich frame shown in Fig. 2. Table 1 shows the mechanical and geometrical properties of this structure. The frame is subjected to an external loading at point P (Fig. 2). Horizontal and vertical members are discretized in 30 and 45 finite elements. The time interval for the transient analysis was divided into 1 ms steps and 500 points were used as the memory length. The viscoelastic layer is considered to be at 27 ºC, and, according to Nunes (2020), the properties of ISD-112 at this temperature are shown in Table 2.

Table 1. Mechanical and geometrical properties.

	1st layer	**2nd layer**	**3rd layer**
Material	Aluminum	ISD-112	Aluminum
Vertical members length	1 m	1 m	1 m
Horizontal members length	1.5 m	1.5 m	1.5 m

Width	5 mm	5 mm	5 mm
Thickness	5 mm	1 mm	1 mm
Young's modulus	70.3 GPa	-	70.3 GPa
Poisson's ratio	0.345	0.5	0.345
Density	2690 kg/m³	1600 kg/m³	2690 kg/m³

Table 2. ISD damping properties at 27 ºC.

G_0 [Pa]	G_∞ [Pa s$^\alpha$]	a_G [s$^\alpha$]	α [-]
423,632.8	30,177.8	0.00022	0.6766

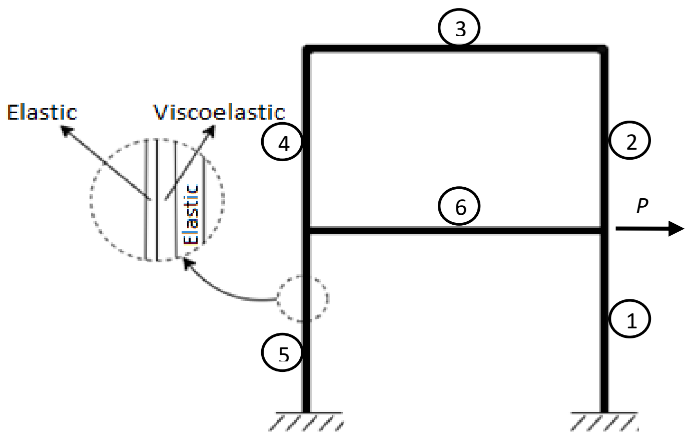

Figure 2. Bi-dimensional sandwich frame.

First, in order to show the efficiency of the viscoelastic material in mitigating vibration in the bi-dimensional frame and the recurrence-based fractional derivative model to describe the dissipative effects of the viscoelastic layer, Fig. 3 shows a frequency domain comparison for both treated and untreated bi-dimensional frames. It is possible to notice that the viscoelastically treated system has lower amplitudes in almost every frequency in the analyzed spectrum, including the resonance peaks.

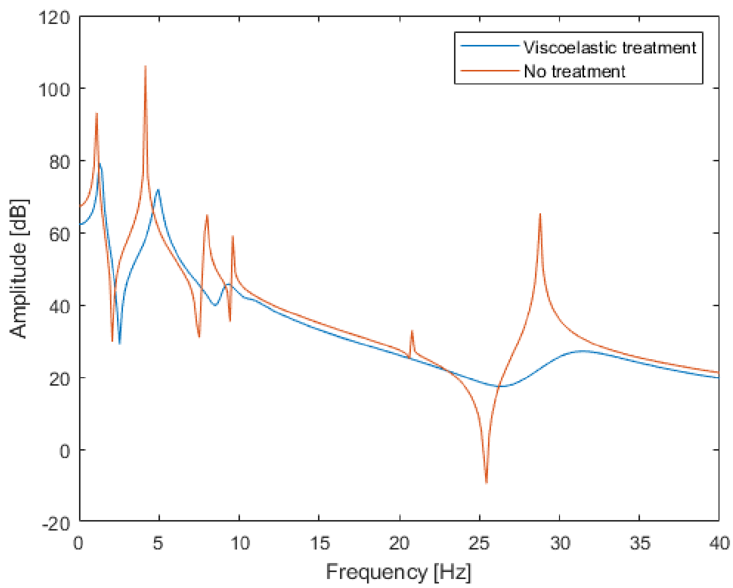

Figure 3. Bi-dimensional frame FRF.

In terms of model reduction, the reduction technique was applied to the time domain analysis. In this case, a basis containing 60 vectors was used. Fig. 4 shows the comparison for both complete and reduced response to a unit triangular impulse force with the peak at 2 ms, and Fig. 5 shows the same comparison for a unit 4.15 Hz harmonic force. It can be seen that there is a great agreement between the responses in both cases, which shows the accuracy of the reduced model. In terms of efficiency, time for both simulations was measured and is shown in Table 3, which demonstrates that there was a computational time gain of about 95% in the reduced model analysis. For more complex systems, with a larger number of DOF, it is expected that this reduction may be even greater.

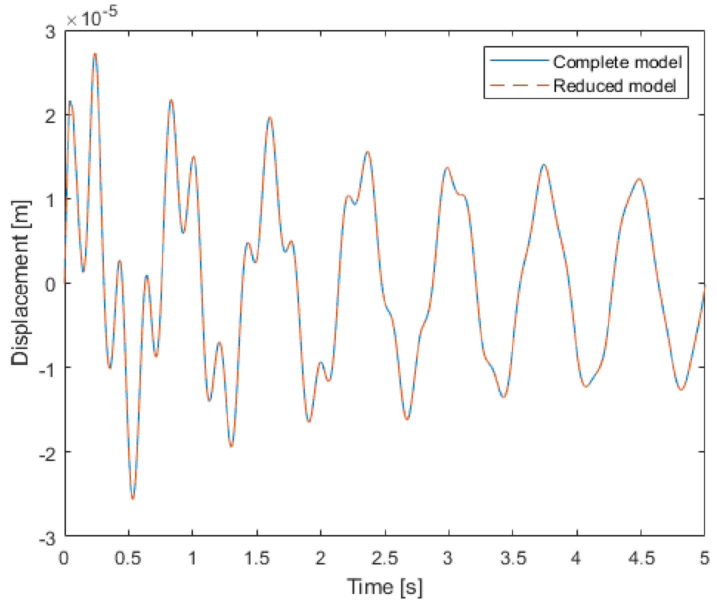

Figure 4. Bi-dimensional frame time domain response to an impulse force.

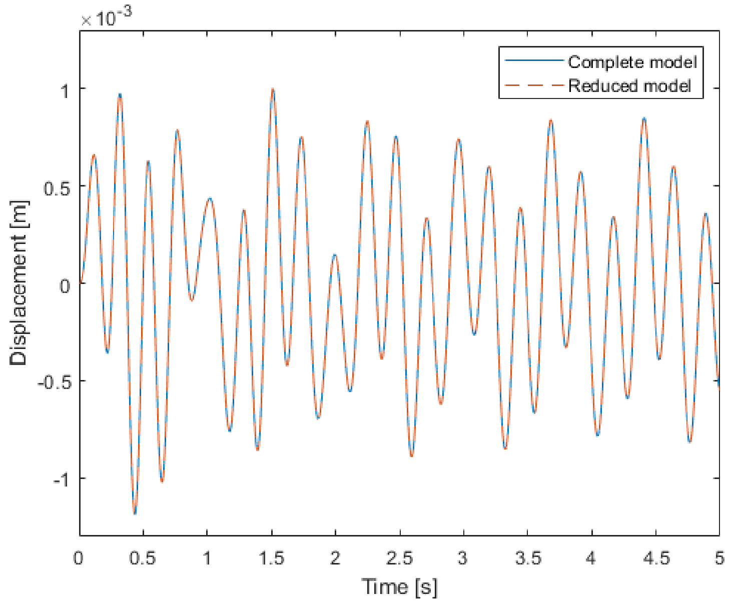

Figure 5. Bi-dimensional frame time domain response to a harmonic force.

Table 3. Computational time associated with the complete and reduced models.

	Matrices size	**Time – Impulse force**	**Time – Harmonic force**
Complete model	832 x 832	433.36 s	426.33 s
Reduced model	60 x 60	20.569 s	27.875 s

Computer simulations were performed in the Matlab programming environment on a single computer with the following configuration: 4 GB of RAM; 500 GB of 50 HDD storage; Intel Core i5-3317U processor CPU @ 1.70 GHz; Windows 10 Home Single Language 64 bits.

4. CONCLUSIONS

This work proposed the use of a more efficient fractional derivative model and a new reduction technique in the analysis of a viscoelastically treated bi-dimensional frame. From the obtained results, it is possible to note the efficiency of the viscoelastic treatment on vibration damping. The simulations performed in this work also show that this model reduction method satisfactorily predicts the transient responses of the viscoelastic systems, presenting a 95% efficiency in terms of reducing the computational cost, what makes it a promising tool to be used in complex dynamic systems analyses.

As future works, the authors suggest a similar analysis for more complex structures of industrial interest, the use of recurrence FDM and the reduction basis on viscoelastic systems considering the presence of parametric uncertainties and the analysis of the application of this model reduction technique on structures built of other materials such as piezoelectric and laminate composites.

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