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**MATHEMATICAL MODELING OF CAPILLARY TUBES OF A DIRECT-EXPANSION HEAT PUMP CHARGED WITH PROPANE**

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**Abstract.** *The application of heat pumps for water heating has been a fertile field for research and development for several decades. In recent years, the Direct-Expansion Solar Assisted Heat Pump (DX-SAHP) has been extensively explored through mathematical models and experimental tests. A low-cost DX-SAHP for residential water heating was developed to operate with propane refrigerant (R290). The equipment operates in three environmental situations and a different capillary tube is required for each one. The objective of the present work is to present the mathematical modeling of the capillary tubes and to verify if the mass of refrigerant present in these devices is significant when compared with the mass of the heat exchangers of the system. The capillaries designed have an internal diameter of 0.042 inch and lengths of 2.45, 1.68 and 1.22 m, suitable for operating in low, medium and high thermal input conditions, respectively.*

**Keywords:** *Mathematical modeling, DX-SAHP, capillary tube, propane (R290).*

## 1. INTRODUCTION

The use of solar heat pumps for water heating has stood out due to the significant energy savings compared to electrical resistance and gas heaters. Thus, a R134a solar heat pump, with the purpose of heating water for residential use, was designed and built by the Refrigeration and Heating Group (GREa) of the Federal University of Minas Gerais (UFMG), resulting in master's and PhD thesis (Reis, 2012; Rodríguez, 2015; Diniz, 2017). However, the working fluid of this equipment does not meet the current criteria of environmentally friendly fluids. The design of a new equipment becomes necessary, being developed from the heat pump mentioned above.

The replacement of traditional synthetic fluids, HFCs (hydrofluorocarbons), by fluids with low environmental impact, zero ODP and low GWP, has started to be explored more intensely in recent decades by the industry and academic community, looking for better energy efficiency and environmental efficiency in cooling and heating systems and equipment. This issue was further intensified in 2016 by the Kigali Amendment (UNEP, 2016) of the Montreal protocol that defined a strict timetable for reducing the production and consumption of HFCs (Ruas, 2018). Among the current refrigeration fluids with low environmental impact, the following stand out: natural fluids, especially carbon dioxide

(CO<sub>2</sub>) due to the absence of toxicity in relation to other natural fluids. The CO<sub>2</sub> has the lowest GWP of all refrigerants, being the reference for this index. Moreover, it was one of the first refrigerant fluids applied in refrigeration. It was forgotten for decades by the advent of synthetic fluids, however, has returned to be applied on a large scale due to the restrictions of the Montreal and Kyoto protocols (Faria, 2013; Oliveira, 2013). HCs (hydrocarbons) have a similar historical context to natural fluids and are currently being widely applied by the industry and explored by researchers, especially R600a (isobutane) and R290 (propane). And finally, the Hydrofluoroolefins (HFOs), belonging to the fourth generation of fluorinated fluids, especially the R1234yf, developed for the retrofit of the R134a in the air conditioning system of new cars produced in Europe (Bobbo et al., 2014).

Vieira (2019) carried out a feasibility analysis of implementing different expansion devices in a heat pump with a solar evaporator, namely: capillary tube, needle valve, thermostatic expansion valve and electronic expansion valve. The capillary tube, although less efficient, is the device that has the lowest cost and is compatible with different fluids.

In the present study, R290 was adopted as the working fluid of the DX-SAHP due to the better environmental and thermal performance (Duarte et al., 2019), in addition to the logistical feasibility in the construction of the proposed system. In order to build a low-cost system and due the absence of a thermostatic valve compatible with R290 at the national market, capillary tubes were adopted to meet the system requirements in several environmental conditions. Therefore, the objective of this work is to present the mathematical modeling of three capillary tubes of a DX-SAHP for residential use and to evaluate the refrigerant mass demanded by these devices.

## 2. METHODOLOGY

The proposed heat pump system is a vapor compression heating system, with propane as the refrigerant fluid, air at evaporator, and water at condenser, as secondary fluids. The machine is basically composed of a single-speed hermetic compressor, a coiled flooded condenser, a counter-flow coaxial condenser, a flat plate solar evaporator and three capillary tubes. In addition, it has a thermal reservoir with a storage capacity of 200 L where the condenser is installed at the bottom by immersion. The EES software (Klein and Alranrado, 2015) was used for the heating pump project and for all fluids and solids properties calculation.

### 2.1 Modeling capillary tubes

The capillary tube dimensioning of this work was done based on the methodology proposed by Gomes (2003). Figure 1 presents a schematic of the pressure variation along the capillary tube, which is fundamental for mathematical modeling. In this figure,  $P_{cond}$  is the condensation pressure,  $P_{evap}$  is the evaporation pressure and  $P_{trans}$  is the pressure at the transition between the flow regimes. A small pressure drop is observed at the entrance (point 3) and at the exit (point 4) of the capillary tube due to the change in the flow area section. The capillary tube is basically divided into an initial region and an end region, with single-phase and two-phase flow, respectively. In the single-phase region there is a pressure drop at almost constant temperature, thus the variation in thermophysical properties is insignificant. Point 4' is the transition region between the two flow regimes, where the fluid is in a saturated liquid state. It is observed that in the two-phase region the pressure reduction (also accompanied by a reduction in temperature) is considerably greater, due to the acceleration and the friction effects. In addition, the length of the capillary segment in the biphasic region is typically shorter than in the monophasic region.

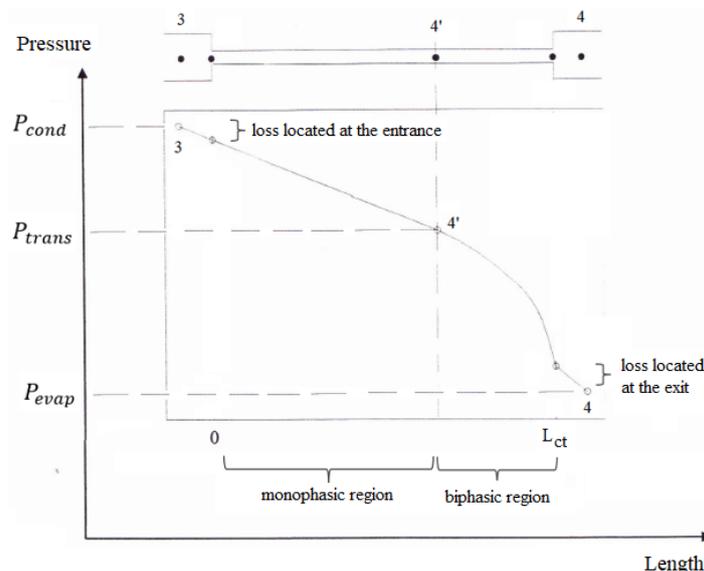


Figure 1. Pressure variation along the capillary tube. Source: adapted from Gomes (2003), p. 20.

The dimensioning methodology is also valid for fluids that operate in a transcritical state at the high pressure heat exchanger (gas cooler), such as the R744 and R170. What differs is that in the single-phase region the fluid is a gas instead of liquid, as occurs for the subcritical fluids at the high pressure heat exchanger (condenser), such as the R290, the focus of this work.

According to Collier e Thome (1996), the presure drop at the inlet region ( $\Delta P_{in}$ ) is given by Eq.(1).

$$\Delta P_{in} = 0.75 v_f G_{f_{ct}}^2 \quad (1)$$

Where  $v_f$  is the specific volume of the refrigerant and  $G_{f_{ct}}$  is the mass flux, being determined by Eq.(2), where  $\dot{m}_f$  is the refrigerant mass flow rate and  $A_{ct}$  is the capillary internal area. At the inlet of the capillary tube, the temperature of the refrigerant is considered 45 °C and the pressure is the saturation pressure at the condensing temperature. It means that the pressure drop at the condenser is not considered. The degree of subcooling is shown in the results section.

$$G_{f_{ct}} = \frac{\dot{m}_f}{A_{ct}} \quad (2)$$

The mass flow rate is determined by an energy balance at the evaporator using Eq.(3), according to the thermal input available at the heat exchanger.

$$\dot{Q}_{evap} = \dot{m}_f (h_{f_4} - h_{f_1}) \quad (3)$$

Where  $h_{f_4}$  and  $h_{f_1}$  are, respectively, the specific enthalpy of the refrigerant at the evaporator inlet and outlet. It is considered that the process is isenthalpic and there is no pressure drop at the evaporator. In addition, the evaporation temperature and the superheat degree are shown in the results section.

Also according to Collier e Thome (1996), the pressure drop at the capillary outlet ( $\Delta P_{out}$ ) is given by Eq.(4).

$$\Delta P_{out} = G_{f_{ct}}^2 v_{f_l} \frac{A_{ct}}{A_{tub}} \left(1 - \frac{A_{ct}}{A_{tub}}\right) \left(1 + x_{in} \frac{v_{f_l} - v_{f_v}}{v_{f_l}}\right) \quad (4)$$

Where  $v_{f_l}$  and  $v_{f_v}$  are the specific volume of the refrigerant at saturated liquid and vapor states, respectively, at the evaporator temperature,  $A_{tub}$  is the internal section area of the pipe that connects the capillary tube to the evaporator (1/4 inch), e  $x_{in}$  is the quality at the evaporator inlet. The capillary tube length of the single-phase region ( $L_{mon}$ ) is determined, according to Collier e Thome (1996), Eq.(5), and  $\Delta P_{mon}$  is calculate by Eq.(6).

$$\Delta P_{mon} = f_{mon} \frac{L_{mon} v_f G_{f_{ct}}^2}{d_{ct}} \quad (5)$$

Where  $f_{mon}$  is the single-phase friction factor, which according to Incropera et al. (2007) is given by Eq.(6) for laminar flow and by Eq.(7) for turbulent flow, both considering smooth tube. The capillary tube internal diameter is  $d_{ct}$  and  $\Delta P_{mon}$  is the single-phase pressure drop. Finally,  $Re_{f_{ct}}$  is the Reynolds number, given by Eq.(8), where  $\mu_f$  is the dynamic viscosity of the refrigerant.

$$f_{mon} = \frac{64}{Re_{f_{ct}}} \quad (6)$$

$$f_{mon} = (0.79 \ln Re_{f_{ct}} - 1.64)^{-2} \quad (7)$$

$$Re_{f_{ct}} = \frac{G_{f_{ct}} d_{ct}}{\mu_f} \quad (8)$$

The capillary tube length of the two-phase region  $L_{bip}$  is determined by Eq.(9) and Eq.(10), similarly to the single-phase one, although with the modification proposed by Lockhart and Martinelli (1949). The authors considered linear variation of the quality along the length of the capillary tube.

$$\Delta P_{bip} = f_{bip} \frac{L_{bip} v_f [G_{f_{ct}}(1-x)]^2}{d_{ct}} \phi_{lo}^2 \quad (9)$$

$$\Delta P_{bip} = (P_{trans} - P_{evap}) - \Delta P_{out} \quad (10)$$

Where  $f_{bip}$  is the friction factor (turbulent regime) at the two-phase region, given by an equation similar to Eq.(7), but with  $Re_{f_{ct}}$  calculated by Eq.(11), where  $\mu_{f_l}$  is the dynamic viscosity of the refrigerant in the saturated liquid state and  $x$  is the vapor quality. Besides,  $\Delta P_{bip}$  is the pressure drop in two-phase region of the capillary tube. The factor  $\phi_{lo}^2$  corrects the pressure drop due to the presence of vapor fluid in the flow and is given by Eq.(12).

$$Re_{f_{ct}} = G_{f_{ct}}(1-x) \frac{d_{ct}}{\mu_{f_l}} \quad (11)$$

$$\phi_{lo}^2 = 1 + \frac{C}{\chi} + \frac{1}{\chi^2} \quad (12)$$

Where  $\chi$  is the Lockhart e Martinelli (1949) parameter, given by Eq.(13).  $\rho_{f_l}$  e  $\rho_{f_v}$  are, respectively, the densities of the refrigerant in saturated liquid state and saturated vapor, and  $\mu_{f_v}$  is the dynamic viscosity of the refrigerant in saturated vapor state.

$$\chi = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_{f_l}}{\rho_{f_v}}\right)^{0.5} \left(\frac{\mu_{f_l}}{\mu_{f_v}}\right)^{0.1} \quad (13)$$

In Eq. (12), C is a constant with a value equal to: 20 when the flow in the liquid and vapor phases are turbulent; 12 when the flow in the liquid phase is laminar and in the vapor phase is turbulent; 10 when the flow in liquid phase is turbulent and in vapor phase is laminar; and 5 when the flow in both liquid and vapor phases are laminar. To determined the type of flow, Eq.(11) is used for the liquid phase and Eq.(14) for the vapor phase.

$$Re_{f_{ct}} = G_{f_{ct}} x \frac{d_{ct}}{\mu_{f_v}} \quad (14)$$

Considering that the quality of the refrigerant fluid starts from zero at the transition region and grows until the value corresponding to the quality at the evaporator inlet, the quality at the two-phase region is divided into N parts and the parameters  $f_{bip}$  and  $\phi_{lo}^2$  are calculated N times, being their average values considered. The same procedure is performed for the thermophysical properties. Finally, the length of the capillary tube  $L_{ct}$  is given by Eq.(15).

$$L_{ct} = L_{mon} + L_{bip} \quad (15)$$

## 2.2 Determination of the refrigerant fluid mass

The calculation of the total fluid mass at the heat exchangers is divided into two regions: single-phase and two-phase. The mass at the single-phase regions of the system ( $m_{mon}$ ) is determined by Eq.(16).

$$m_{mon} = \sum_{i=1}^N \rho_{mon_i} \forall_i \quad (16)$$

Where  $\rho_{mon_i}$  and  $\forall_i$  are the specific mass at the single-phase region and the refrigerant volume at each tube division, respectively. The index i represents the i-th variable. Eq.(17) was used to determine the mass at the two-phase regions of the system ( $m_{bip}$ ). It was considered N=200 divisions for all two-phase regions.

$$m_{bip} = \sum_{i=1}^N [\alpha_{vazio_i} \rho_{f_v} + (1 - \alpha_{vazio_i}) \rho_{f_l}] \forall_i \quad (17)$$

Where  $\alpha_{vazio_i}$  is the void fraction of the refrigerant at each tube division and is determined by Rouhani e Axelsson (1970) correlation, modified by Steiner (1993), and given by Eq.(18) and Eq.(19).

$$\alpha_{vazio} = \frac{x}{\rho_{f_v}} \left[ C_o \left( \frac{x}{\rho_{f_v}} + \frac{1-x}{\rho_{f_l}} \right) + \left( \frac{1.18(1-x)[g\sigma(\rho_{f_l}-\rho_{f_v})]^{0.25}}{G_f \rho_{f_l}^{0.5}} \right) \right]^{-1} \quad (18)$$

$$C_o = 1 + 0,12(1-x) \quad (19)$$

Where  $C_o$  is a correlation parameter,  $\sigma$  is the surface tension of the refrigerant and  $g$  is gravity.

## 3. RESULTS AND DISCUSSIONS

The capillary tube is recommended for the system to work with the minimum variation of thermal load at the heat exchangers (intrinsically related to the compressor inlet and outlet pressures), especially for the solar evaporator of the heat pump of this study. The most impacting consequence of this fact is the compromise of the thermal performance of the system when it operates under different conditions than the design point, as studied by Costa (2014).

There are two types of situations in which a DX-SAHP can be used. The first refers for the system operating with low radiation, inside the laboratory, out of the influence of wind. The second situation refers to when the system operates with high radiation, in an open environment exposed to the action of the wind.

Therefore, a capillary tube was designed and implemented to be used when the system operates with low radiation and another to be applied when the system operates with high radiation. Solenoid valves control the activation of capillary tubes according to the boiling temperature by means of a thermostat. A third capillary tube was designed and installed to an average operating condition of the system. Figure 2 shows the arrangement of the capillary tubes.

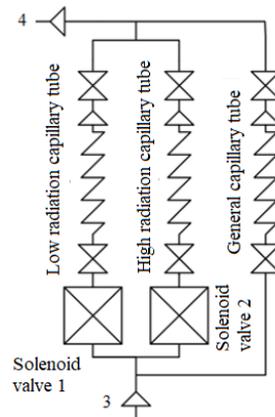


Figure 2. Arrangement of capillaries tubes of the system.

The local market offers the following capillary tube diameters at low cost: 0.031 in, 0.036 in, 0.042 in and 0.050 in. For all diameters, the available length is 3 m, which was the limit adopted during the design. It was decided to start the simulations with the largest diameter, aiming to reduce the difficulty of welding the capillary tube at the system piping, and with the largest capillary length, which contributes to dissolving the numerical difference between the ideal project demand and the experimental demand. However, when the length of 3 m was exceeded, the diameter was reduced to obtain a reduction in the capillary tube length, making it possible to be manufactured.

Table 1 presents the simulation results for the design of the three capillaries, based on the workbench of Diniz (2017), assuming the hypothesis that R290 has operation points similar to R134a.

Table 1. Parameters for the design of capillary tubes.

| Capillary Tube       | Cond. Temp. (°C) | Evap. Temp. (°C) | S.H and S.C. Degree (°C) | Refrig. Capacity (W) | Capillary internal diameter (in) | R290 mass (g) | Capillary length (m) |
|----------------------|------------------|------------------|--------------------------|----------------------|----------------------------------|---------------|----------------------|
| Low thermal input    | 50               | -2               | 7                        | 550                  | 0.042                            | 0.87          | 2.45                 |
| Medium thermal input | 52               | 5                | 7                        | 700                  | 0.042                            | 0.59          | 1.68                 |
| High thermal input   | 54               | 11               | 7                        | 850                  | 0.042<br>0.050                   | 0.42<br>1.4   | 1.22<br>2.90         |

It was observed a refrigerant mass of less than 1g in two capillary tubes and less than 2g in the third one. Therefore, the hypothesis of negligible mass at the capillaries is reasonable. It was noted that the designed coaxial condenser, immersion condenser, and solar evaporator require, respectively, 42, 40 and 35 g of refrigerant for system operation. The capillary tube recommended for low radiation operation has 2.45 m and a diameter of 0.042 in. For the high radiation operation, there are two possibilities: length of 1.22 m and diameter of 0.042 in or length of 2.90 m and diameter of 0.050 in. For the capillary tube that operates in an intermediate situation is required a length of 1.68 m and a diameter of 0.042 in.

#### 4. CONCLUSIONS

This study adopted R290 as the refrigerant flowing through capillary tubes in a DX-SAHP. For this, a survey of national thermostatic expansion valve manufacturers was done. As this device was not found for sale in Brazil, it was decided to use a capillary tube in the system in order to eliminate the costs of importing the valve. Furthermore, the adoption of the capillary tube as the expansion device adheres to the concept of a low-cost domestic application system.

Three capillary tubes were designed to serve the system at three environmental situations, being automatically controlled by solenoid valves. The devices have an internal diameter of 0.042 inch and lengths of 2.45, 1.68 and 1.22 m, to operate in low, medium and high thermal input conditions, respectively.

It was concluded that this work provides a useful tool for modeling capillary tubes applied to water heating heat pumps.

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