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NUMERICAL SOLUTIONS FOR A BALLISTIC TRAJECTORY WITH DRAG REDUCTION PROVIDED BY A BASE BLEED UNIT

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Abstract. *The purpose of this study is to propose a simplified method of modified-point mass trajectory for base burn projectiles due to reduction of computational time. The research has showed that some forces can be neglected, which demands less inputs and it optimizes comparisons with projectiles firing tables. A Base Bleed unit reduces base drag, hence the ballistic trajectory is better when is compared to an inert projectile, even though it does not have any propulsion system. Therefore, this project offers two-fitted curves for drag and establishes connection with internal ballistics to understand how mass flow rate of burnt gases could influence in this project.*

Keywords: *external ballistics, Base Bleed, extended range, drag reduction, firing tables*

1. INTRODUCTION

For many years, flight dynamics involves equations which cost much of computational resources due to complex nature of forces and moments related to trajectory. A six degree of freedom, for example, it is a quite cumbersome method, despite the solutions produced. Adding to this fact, there are many possibilities of errors due to unpredictable phenomenons that can appear. A modified point-mass trajectory models is a more recent method, proposed by Lieske and Reiter (1966), which involves faster results because of reduction of equations needed to comprehend the motion. However, both models require multiple inputs to estimate the range of weapons and, at the same time, some characteristics of projectile are unknown.

This study deals with a supersonic regime at lift-off due to muzzle shot at sea-level. Meanwhile, there is a hot mass flow injection to increase the base pressure closer to the free air stream pressure. It reduces the base drag, then it is possible to extend the range of projectile and creates a current field of interest for many researchers. The effectiveness of base burn systems in spin-stabilized projectiles is estimated in this project. However, range extension with base flow phenomena demands a lot of caution because of simultaneous coupling of solutions for internal and external ballistics.

The core of this article is to simplify a method to predict a trajectory of a projectile. Hence, there are no models or commercially available software to be used to predict a specific propellant formulation which will deliver the desired performance - the only way is through a real test in a shooting range, which demands a costly logistics and great risks. Therefore, it is remarkably interesting to develop lab scale tests and models to predict the performance of extended range projectiles, without the need of several real shooting tests.

As base burn projectiles are complex to estimate the exactly range, so the firing table is compared with inert base and with Base Bleed unit models. Both fitted-curves used to estimate the drag are Mach functions. The validation of results are also based on PRODAS©, a commercial software of Arrow Tech, solutions. Its capacity to predict aerodynamic coefficients permits to studying effects of roll and drift more accurate.

2. BASE BLEED PRINCIPLE

A Base Bleed technology is widely applied in projectiles when is interesting to extend the range of trajectory. In this case, a hot gas is injected on the shell base and it reduces the base drag. According to Gunners *et al.* (1988), there is a optimal mass flow injection and it should happen during a third or half part of total flight. If the mass flow in the throat is supersonic, Abou-Elela *et al.* (2013) emphasizes the possibility to create a momentum which penetrates in the wake region and reduces the base drag reduction. Therefore, there is an non-dimensional factor named injection parameter which is defined with the following equation:

$$I = \frac{\rho_g r_g S_g}{\rho V A_b} \tag{1}$$

where ρ_g is the propellant density, r_g is the propellant burning rate, S_g is the instantaneous propellant burning surface, ρ is the instantaneous free air stream density, V is the instantaneous base burn projectile velocity and A_b is the base reference surface.

Observing Fig. 1 and the explication of Danberg and Nietubicz (1992), the ideal Base Bleed region is for low values of I, preferably the value of optimum parameter or just I_{lim}. As an engineering approximation, there is a linear relation between base pressure and injection parameter as follows:

$$\frac{P_b}{P_\infty} = \left(\frac{P_b}{P_\infty}\right)_{I=0} + \left[\frac{d(P_b/P_\infty)}{dI}\right]_{I=0} I \tag{2}$$

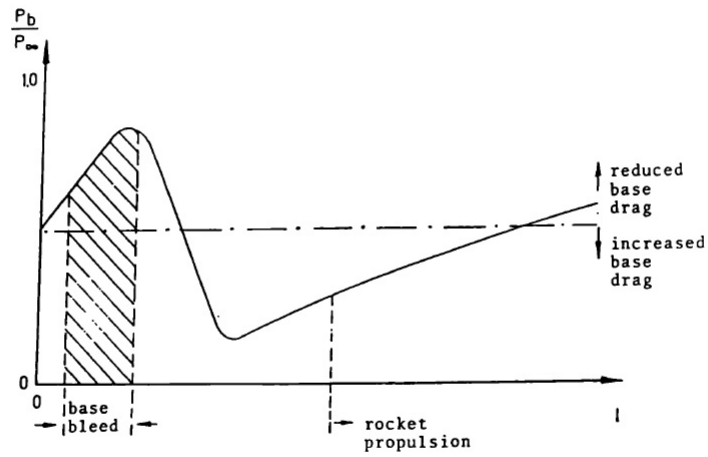


Figure 1. Injection parameter versus ratio of projectile base and free air stream pressures (Gunners *et al.*, 1988).

As the injection parameter is strongly dependent of time, it has to be predicted for each interval to estimate the force caused by base-bleed. Because of this situation, the design of base-bleed has to be reliable, otherwise it sharply affects the projectile performance. This result is also affected by ignition system, but as an introduction to base-bleed systems, igniters effects are negligible.

3. TRAJECTORY MODEL

A Modified-Point Mass Trajectory Model was proposed by Lieske and Reiter (1966), which provides definitions of yaw, drag, lift and Magnus forces. Moreover, it adds a conception named yaw of repose, an approximation used to express yawing motion for dynamic stable projectiles with trigonal symmetry. For base-burn projectiles, there is also a particular force due to drag reduction.

According to Baranowski *et al.* (2016a), the dynamic equations of translational and rotation motion can be described as:

$$m\dot{\boldsymbol{V}} = \boldsymbol{F}_W + \boldsymbol{F}_C + \boldsymbol{F}_D + \boldsymbol{F}_L + \boldsymbol{F}_M + \boldsymbol{F}_{BB} \tag{3}$$

$$I_x\dot{p} = \frac{\pi\rho d^4 V C_{spin} p}{8} \tag{4}$$

$$\boldsymbol{\alpha_e} = -\frac{8I_x p(\boldsymbol{V} \times \dot{\boldsymbol{V}})}{\rho\pi d^3(C_{m\alpha} + C_{m\alpha^3}\alpha_e^2)V^4} \tag{5}$$

Into the eq. 3, the notations of $\boldsymbol{F}_W$, $\boldsymbol{F}_C$, $\boldsymbol{F}_D$, $\boldsymbol{F}_L$, $\boldsymbol{F}_M$, $\boldsymbol{F}_{BB}$ represent vectors of gravitational, Coriolis, drag, lift, Magnus and Base Burn forces. Eqs. 4 and 5 are functions of density (ρ), projectile reference diameter d, spin of projectile p, spin damping force coefficient (C_{spin}), linear overturning moment coefficient ($C_{m\alpha}$), cubic overturning moment coefficient ($C_{m\alpha^3}$) and axial moment of inertia I_x.

3.1 Gravitational Force

Represented as $\boldsymbol{F}_W$, this force can be synthesized as:

$$\boldsymbol{F}_W = m\boldsymbol{g} \tag{6}$$

where m is mass of projectile and g is the acceleration vector due to gravity.

3.2 Coriolis Force

The principle of Coriolis is related to the Earth axis rotation and it is defined as:

$$\boldsymbol{F}_C = m\boldsymbol{\lambda} \tag{7}$$

where $\boldsymbol{\lambda}$ = -2($\boldsymbol{\omega} \times \boldsymbol{V}$) is the acceleration due to Coriolis force. Omega vector is a representation of Earth's angular velocity.

3.3 Drag Force

There are many factors that influences its result. In general, drag is described as:

$$\boldsymbol{F}_D = -\frac{\pi\rho i d^2}{8}(C_{D_0} + C_{D_{\alpha^2}}(Q_D\alpha_e)^2)V\boldsymbol{V} \tag{8}$$

where ρ is the free air stream density, i is the form factor of the projectile (for base bleed system, i is equals to 1), α_e is the yaw of repose approximation, C_{D_0} is zero-yaw rag coefficient, $C_{D_{\alpha^2}}$ is quadratic yaw drag coefficient, d is the reference diameter of the projectile, ρ is the actual free air stream density, Q_D is a yaw drag fitting factor, V is the actual total velocity magnitude with respect to the air.

3.4 Lift Force

This force, which is well explained by McCoy (2012), acts perpendicularly to the trajectory, tending to adjust the projectile in the direction of nose point. Due to strong relation with total yaw angle, lift is zero if this angle is zero. For long range artillery, aerodynamic lift is the responsible of drift.

$$\boldsymbol{F_L} = \frac{\pi\rho d^2 f_L}{8}(C_{L_\alpha} + C_{L_{\alpha^3}}\alpha_e^2)V^2\boldsymbol{\alpha_e} \tag{9}$$

where $C_{L\alpha}$ is the total lift coefficient, f_L is the lift factor, $\boldsymbol{\alpha_e}$ is the yaw of repose vector. The coefficient $C_{L\alpha}$ often behaves in non-linear because of dependency of a quadratic term of yaw of repose. However, this is beyond of the scope of this article.

3.5 Magnus Force

The Magnus force is produced by difference of pressure on opposite sides of a spinning body. In that case, the viscosity of air interacts with spinning surface, whereas the magnitude of its force is much lower than drag and gravitational magnitudes.

$$\boldsymbol{F_M} = -\frac{\pi\rho d^3 Q_M p C_{mag-f}}{8}(\boldsymbol{\alpha_e} \times \boldsymbol{V}) \tag{10}$$

where C_{mag-f} is the total Magnus force coefficient, Q_M is the Magnus fitting factor and p is the axial spin rate of the projectile.

3.6 Base Burn Force

Although the effect of Base Bleed is not considered a thrust force, the injection of hot mass flow in the base extend the range acting in the longitudinal axis direction. Base burn force can be stated as follows:

$$\boldsymbol{F_{BB}} = +\frac{\pi \rho i_{BB} d^2}{8} f(I) C_{X_{BB}} V^2 \left(\frac{\boldsymbol{V} cos(\alpha_e)}{V} + \boldsymbol{\alpha_e} \right) \tag{11}$$

$$f(I) = \frac{I}{I_0} \tag{12}$$

where $C_{X_{BB}}$ is the total base burn force coefficient, i_{BB} is the fitting factor to adjust the drag reduction as a function of quadrant elevation, $f(I)$ is the flow rate function of base burning rate compared to optimum injection parameter I_0.

4. INPUTS

In such model a rigid body is treated as a point mass, which reduces the inputs, then the computational cost is lighter than a common rigid body dynamic simulations. Table 1 emphasizes minimum data, likewise Balon and Komenda (2006) and with support of PRODAS to calculate axial moment of inertia and other important, but not essential informations. This model predicted the trajectory of a 4.5-inches projectile without fins.

Table 1. Projectile characteristics.

Description	Value	Unit
Projectile reference diameter	0.112738	m
Projectile boat-tail diameter	0.10690	m
Projectile mass without propellant	19.8190	kg
Propellant mass	0.5853	kg
Propellant initial temperature	288.15	K
Axial moment of inertia at the beginning	0.03759	kg·m^{-2}
Axial moment of inertia at the propellant burnout	0.03710	kg·m^{-2}
Center of gravity at the beginning from nose	0.407632	m
Center of gravity at the propellant burnout from nose	0.4022700	m

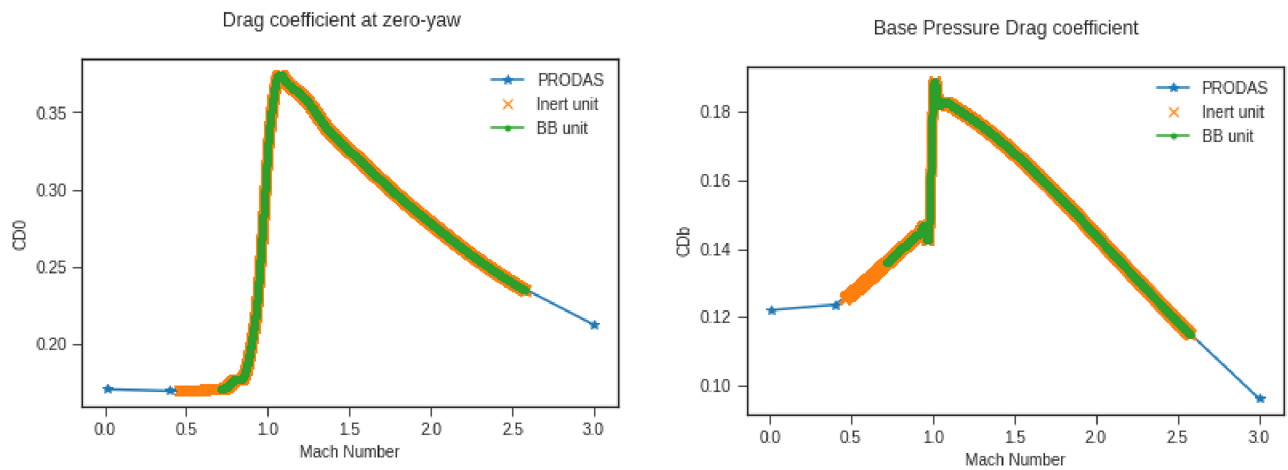
The muzzle was centered in O_{XYZ} in Earth axis and set to launch at 800 and 1200 mils with the same velocity. As referred in Table 2, Drag and Magnus factor are constants expressed in US Model of modified-point mass trajectory model proposed by Lieske and Reiter (1966). As there is a great influence of height, atmosphere conditions are also represented.

Table 2. Initial ballistic characteristics.

Description	Value	Unit
Muzzle velocity	878	m·s^{-1}
Quadrant elevation	$\pi/4, \pi/3$	radians
Latitude	$-23\pi/180$	radians
Azimuth	0	radians
Twist rate at muzzle	25	calibers per revolution
Drag factor	1.2	-
Magnus factor	1.2	-
Temperature at sea-level	288.15	K
Temperature gradient	6.5×10^{-3}	K·m^{-1}
Air pressure at sea-level	1.01325×10^5	Pa
Air density at sea-level	1.225	kg·m^{-2}
Air specific heat ratio	1.4	-
Gravity at the sea-level	9.80665	m·s^{-2}
Earth radius	6.371×10^6	m
Earth angular velocity	7.292×10^{-5}	m·s^{-1}

5. RESULTS

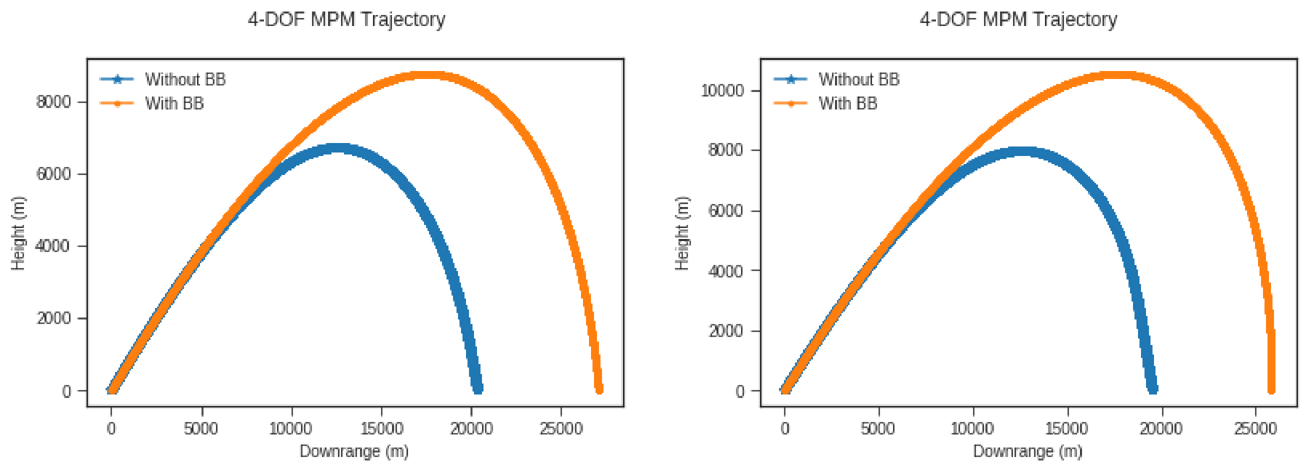
These results were obtained with some conditions predetermined to focus the studies in trajectory model, then internal ballistics of that munition was considered with constant mass flux ($\dot{m} = 0.02kg/s$) and ideal injection parameter was set to 5.0×10^{-3}. In this model, wind velocity was neglected, therefore windage jump is equal to zero. For aerodynamic coefficients, they were calculated using PRODAS software and interpolated using cubic method as a function of Mach Number, such as represented in Figure 2. In that specific model, $C_{m\alpha^3}$ and $C_{L\alpha^3}$ are zero, then overturning moment coefficient and lift force coefficient are linear functions of yaw of repose. Although it is important to treat the ignition delay of gas generator, the firing tests used high-speed cameras, then the interval of time is neglected.



(a) (b)

Figure 2. Total zero-yaw drag coefficient (a) and base pressure drag coefficient (b)

The methodology to prove the efficiency of base bleed is comparing achieved downranges. The difference between the curves of Figure 3 obtained with or without base burn technology. Using an elevation of 800 mils, Base Bleed technology increased the downrange more than 30 percent, which is described in Tables 3 and 4. Those simulations used an approximation of spherical Earth, then the latitude of Rio de Janeiro was considered in this context. Both cases used the muzzle velocity equals to 878 meters per second.



(a) (b)

Figure 3. Ballistic trajectories with (a) QE = 771 mils (40 degrees) (b) QE = 800 mils (45 degrees).

Table 3. Ballistic results at angle of departure = $40°$ with v = 878 m/s.

Description	Inert Base	Base Bleed	Range Increase with BB
Downrange (m)	20359.59	27100.26	6740.69 (30.61%)
Apogee (m)	6712.68	8767.27	2054.59 (33.11%)

Table 4. Ballistic results at angle of departure = $45°$ with v = 878 m/s.

Description	Inert Base	Base Bleed	Range Increase with BB
Downrange (m)	19524.98	25852.30	6327.32 (32.41%)
Apogee (m)	7966.75	10536.75	2569.76 (32.26%)

6. CONCLUSIONS

The Modified Point Mass Trajectory Model (MPMTM) provides a four degrees-of-freedom flight dynamic simulation for spin stabilized projectiles. Adding the Base Bleed principle inside this method proves a satisfactory increase of downrange and apogee, even projectile became heavier. However, this model deals with an ideal condition of constant mass flow and an assumption of injection parameter without obtaining a regression fitting curve of propellant burning rate. Adding to this fact, a interpolation of PRODAS software of base pressure was used rather than a complete model of internal ballistics explained by Gunners *et al.* (1988) due to leakage of inputs.

As every numerical integration system, there is a slight error that could have been predicted. Meteorological conditions could also affected the results, so an assumption of no-wind firing is one of possibilities of real tests. A possibility to create a explicit method to predict the trajectory is a proper suggestion to improve results because it consumes less time, as denoted by Baranowski *et al.* (2016b), unlike the standard MPMTM model, an implicit system of equations related to yaw of repose. On the other hand, it is for further projects with respect to Base Bleed projectiles.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Abou-Elela, H.A., Ibrahim, A.Z., Mahmoud, O.K. and Abdel-Hamid, O.E., 2013. “Ballistic analysis of a projectile provided with base bleed unit”. *15th International Conference on Aerospace Sciences & Aviation Technology*, Vol. 15, No. 116, pp. 1–22.

Balon, R. and Komenda, J., 2006. “Analysis of the 155-mm erfb/bb projectile trajectory”. *Advances in MT*, , No. 1, pp. 91–114.

Baranowski, L., Gadomski, B., Majewski, P. and Szymonik, J., 2016a. “Explicit "ballistic m-model": A refinement of the implicit "modified point mass trajectory model"”. *Bulletin of the Polish Academy of Sciences Technical Sciences*, Vol. 64. doi:10.1515/bpasts-2016-0010.

Baranowski, L., Gadomski, B., Szymonik, J. and Majewski, P., 2016b. “Comparison of explicit and implicit forms of the modified point mass trajectory model”. *Journal of Theoretical and Applied Mechanics*, Vol. 54, pp. 1183–1195. doi:10.15632/jtam-pl.54.4.1183.

Danberg, J.E. and Nietubicz, C.J., 1992. “Predicted flight performance of base-bleed projectiles”. *Journal of Spacecraft and Rockets*, Vol. 29, No. 3, pp. 366–372. doi:10.2514/3.26360. URL `https://doi.org/10.2514/3.26360`.

Gunners, N.E., Andersson, K. and Hellgren, R., 1988. “Base bleed systems for gun projectiles”. *1st International Symposium on Special Topics in Chemical Propulsion*, Vol. 109, No. 16, pp. 537–562.

Lieske, R. and Reiter, M., 1966. “Equations of motion for a modified point mass trajectory”. *Ballistic Research Laboratories*, Vol. 1, No. 1, p. 24.

McCoy, R., 2012. *Modern Exterior Ballistics: The Launch and Flight Dynamics of Symmetric Projectiles*. Schiffer Publishing, Atglen, United States of America, 2nd edition.

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