



encit 2020



18th Brazilian Congress of Thermal Sciences and Engineering
November 16–20, 2020 (Online)

ENC-2020-0391

Evaluation of the accuracy of the SRI method in non-homogeneous conditions

Felipe Ramos Coelho

Francis Henrique Ramos França

Universidade Federal do Rio Grande do Sul, Department of Mechanical Engineering, Av. Paulo Gama, 110, Porto Alegre, RS, Brazil
felipe.coelho@ufrgs.br
frfranca@mecanica.ufrgs.br

Somesh Prasad Roy

Marquette University, Department of Mechanical Engineering, Milwaukee WI 53232, USA
somesh.roy@marquette.edu

Abstract. This study evaluates the ability of the spectrally reduced integration (SRI) method to generate benchmark solutions for the radiative transfer equation (RTE) in non-homogeneous participating gas mixtures of H_2O and CO_2 . The SRI employs a non-uniform spectral mesh that is generated from discretization schemes based on the spectral contributions of the bands. This allows the method to be significantly less computationally costly than the reference line-by-line (LBL) while still retaining good levels of accuracy. Discretization schemes based on non-homogeneous test cases are developed and compared to the LBL solution. The main goal of this comparison is to investigate whether the SRI is still able to generate benchmark levels of accuracy when the problem that is being solved is considerably different than the conditions for which the discretization schemes were developed. This is the first study that evaluates the performance of the SRI method in test cases with variable mole ratio between H_2O and CO_2 , such as those observed in oxy-fuel combustion scenarios. Results showed that the SRI presented very accurate solutions, even when the discretization schemes were based in simpler problems, with average deviations lower than 0.7% and maximum deviations lower than 0.9% with respect to the LBL.

Keywords: Radiative transfer, Spectrally reduced integration, Line-by-line, Discretization schemes, Spectral contributions of the bands.

1. INTRODUCTION

Thermal radiation is often the dominant heat transfer mechanism in combustion applications due to the high temperatures resulted from the chemical reactions (Modest, 2013). However, the presence of reaction products such as CO_2 and H_2O significantly increases the complexity of the problem, as they participate in the radiative transfer with highly irregular spectral behaviors. This issue is further aggravated by the fact that this spectral dependence is also a function of the thermodynamic state, which varies significantly in real flames. Therefore, accounting for the absorption spectra of participating gases in high resolution makes numerical solutions of the radiative transfer equation (RTE) in combustion devices too computationally costly, especially when the calculations are coupled (Modest and Haworth, 2016). Due to this limitation, this kind of problem is more often solved using approximated spectral models, which considerably simplify the spectral behavior of participating gases.

The narrow band models (Soufiani and Taine, 1997; Rivière and Soufiani, 2012; Cai and Modest, 2014), the weighted-sum-of-gray-gases (WSGG) (Bordbar *et al.*, 2014; Coelho and França, 2018), the spectral-line-based WSGG (SLW) (Pearson *et al.*, 2014; Solovjov *et al.*, 2017), and the full-spectrum k-distribution (FSK) (Zhang and Modest, 2002; Modest and Riazzi, 2005) are broadly utilized spectral models which are able to produce satisfactory accuracy in combustion scenarios. Among them, the WSGG model is the simplest and, in general, the least computationally costly. The SLW and the FSK are more accurate alternatives, but at the cost of a higher CPU time. Finally, the narrow band models perform better than all of the previous methods and are often used as more accessible benchmark solutions. Their computational cost is higher than the WSGG, the SLW, and the FSK, but still significantly lower than accounting for the highly irregular spectral behavior of participating gases.

When radiative transfer in participating media is calculated considering the absorption spectra in high resolution, it is called the line-by-line (LBL) solution. Despite being the most accurate spectral integration methodology, its high computational cost hinders its applicability in combustion scenarios, especially in multidimensional problems. To address this limitation, Ziemniczak *et al.* (2019) developed a reduction technique to smooth the LBL spectral behavior, decreasing its computational cost while maintaining satisfactory accuracy. This methodology was then employed by Rodrigues *et al.*

(2019) to perform a direct spectral integration (DSI) of the RTE in axisymmetric laminar non-premixed flames, where the computational cost of a reference LBL solution was not viable. However, despite the considerable reduction in CPU time when using the DSI, the deviations of the method are still higher than what is often desired in benchmark solutions. Therefore, Coelho *et al.* (2021) developed a more accurate methodology to reduce the computational cost of the LBL, the spectrally reduced integration (SRI), which generates non-uniform wavenumber discretization schemes based on the spectral contributions of the bands. Results showed that the most accurate schemes were able to obtain solutions with benchmark levels of accuracy 3 to 5 times faster than the reference LBL. The study from Coelho *et al.* (2021) also proposed two methodologies to address the main limitation of the method: the need of a previous solution of the RTE to evaluate the spectral contributions. These methodologies were very successful in the studied cases and illustrated that the applicability of the SRI can be generalized to problems different to those tested.

Based on these recent advances, this study aims to evaluate the accuracy of the SRI method in one-dimensional problems with non-homogenous gas mixtures of H₂O and CO₂. Despite the work from Coelho *et al.* (2021) also covering non-homogeneous mixtures, all the cases that were tested considered uniform mole ratio between H₂O and CO₂, which is not an accurate representation of real flames and facilitates generalizing discretization schemes to different problems. The goal is to verify if the SRI can still be generalized to distinct conditions when the mole ratio between the species is not constant along the domain. Furthermore, in order to extend the range of applicability of this methodology, spectral contributions databases based on a reference problem are generated for various values of mole ratio and an interpolation methodology is proposed to obtain intermediate values.

2. METHODOLOGY

For a gas mixture containing only CO₂ and H₂O as its participating gases, the mixture absorption coefficient κ_η is given by

$$\kappa_\eta = \kappa_{\eta,c} + \kappa_{\eta,w} \quad (1)$$

in which $\kappa_{\eta,c}$ and $\kappa_{\eta,w}$ are the absorption coefficients of the individual species CO₂ and H₂O, respectively. The absorption coefficient of each single gas is then calculated through

$$\kappa_{\eta,i}(\eta, p, T, Y_i) = N(p, T) Y_i C_{\eta,i}(\eta, p, T, Y_i) \quad (2)$$

where N is the gas molar density and $\kappa_{\eta,i}$, Y_i and $C_{\eta,i}$ are, respectively, the absorption coefficient, the mole fraction and the absorption cross-section of species i , while the index i is equal to c for CO₂ and w for H₂O. The absorption cross-sections of the individual gases are calculated through the Lorentz profile, such that

$$C_\eta = \sum_{k=1}^K C_{\eta,k} = \sum_{k=1}^K \frac{S_k}{\pi} \frac{\gamma_k}{\gamma_k^2 + (\eta - \eta_k)^2} \quad (3)$$

in which K is the total number of significant spectral lines at the wavenumber position η , $C_{\eta,k}$ is the contribution of the spectral line k to the absorption cross-section, S_k is the integrated line intensity, γ_k is the line half-width of the line profile, and η_k is the wavenumber position of the line center.

The line half-width γ_k was calculated using the high-resolution spectroscopic database HITEMP2010 (Rothman *et al.*, 2010), resulting in

$$\gamma_k = \left(\frac{T_{ref}}{T} \right)^{n_i} (p_i \gamma_{self,k} + (p - p_i) \gamma_{air,k}) \quad (4)$$

where $T_{ref} = 296$ K is the reference temperature of the HITEMP2010, n_i is the temperature dependence coefficient, p_i is the partial pressure of species i , $\gamma_{self,k}$ is the line self-broadening, and $\gamma_{air,k}$ is the broadening caused by air. The integrated line intensity S_k is then obtained through (Rothman *et al.*, 2010)

$$S_k(T) = S_k(T_{ref}) \frac{Q(T_{ref})}{Q(T)} \frac{\exp(-C_2 E_k/T)}{\exp(-C_2 E_k/T_{ref})} \frac{[1 - \exp(-C_2 \eta_k/T)]}{[1 - \exp(-C_2 \eta_k/T_{ref})]} \quad (5)$$

where Q is the total internal partition sum, E_k is the energy of the lower state, and η_k is the wavenumber location of line k . The line parameters η_k , n_i , $\gamma_{self,k}$, $\gamma_{air,k}$, $S_k(T_{ref})$, Q , and E_k are taken from the high resolution spectroscopic database HITEMP2010 (Rothman *et al.*, 2010). In this study, the absorption cross-sections C_η of H₂O and CO₂ are calculated considering a spectral discretization of $\Delta\eta = 0.0667$ cm⁻¹, total pressure of 1 atm, temperatures ranging from 300 K to 2500 K, and mole fractions $Y_c = 1$ and $Y_w = 0.01, 0.1, 0.2, 0.4, 0.6, \text{ and } 0.8$. For intermediate values of Y_w , linear interpolation was employed, while for CO₂ a simple linear relation is enough (Cassol *et al.*, 2014). This spectral discretization of $\Delta\eta = 0.0667$ cm⁻¹ is shown to be enough by Coelho *et al.* (2021) for test cases such as the ones studied here.

With the mixture absorption coefficient calculated, the spectral contributions are then generated from a previous LBL solution of the RTE. Since the scattering effect is negligible in gas mixtures composed of only CO₂ and H₂O, the RTE is given by (Modest, 2013; Howell *et al.*, 2016)

$$\frac{dI_\eta}{ds} = -\kappa_\eta I_\eta + \kappa_\eta I_{b\eta} \quad (6)$$

in which I_η is the spectral radiation intensity, $I_{b\eta}$ is the blackbody spectral radiation intensity, and s is the path length along a line of sight. Similar to the analysis performed by Coelho *et al.* (2021), all the test cases studied here consist of 1D mixtures of H₂O and CO₂ at atmospheric total pressure and surrounded by black walls. For such configuration, the solution of Eq. (6) for the spectral radiative heat flux and source term, when using the discrete ordinates method (DOM) for angular discretization, results in

$$q_{r,\eta}(\eta, x) = \sum_{l=1}^{n_d} [2\pi\mu_l\omega_l(I_{\eta,l}^+(x) - I_{\eta,l}^-(x))] \quad (7)$$

$$S_{r,\eta}(\eta, x) = \sum_{l=1}^{n_d} [2\pi\kappa_\eta\omega_l(I_{\eta,l}^+(x) - I_{\eta,l}^-(x))] - 4\pi\kappa_\eta I_{b\eta} \quad (8)$$

where ω_l is the quadrature weight for direction l , n_d is the total number of directions, μ_l are the direction cosines, and $I_{\eta,l}^+$ and $I_{\eta,l}^-$ are, respectively, the spectral radiation intensity for $\mu_l > 0$ and $\mu_l < 0$. The solution of Eqs. (7) and (8) are performed considering a spectral discretization of $\Delta\eta = 0.0667 \text{ cm}^{-1}$, spatial discretization of 200 uniform elements, and directional discretization for 30 total directions in the DOM in all the test cases studied here. According to Coelho *et al.* (2021), the spectral contributions of the bands to the radiative heat flux and source term are then calculated as

$$\zeta_{q_r}(\eta) = \frac{1}{L} \int_0^L |q_{r,\eta}(\eta, x)| dx \quad (9)$$

$$\zeta_{S_r}(\eta) = \frac{1}{L} \int_0^L |S_{r,\eta}(\eta, x)| dx \quad (10)$$

in which ζ_{q_r} and ζ_{S_r} are the spectral contributions of each spectral interval $\Delta\eta$ for the radiative heat flux and source term, respectively. The parameters ζ_{q_r} and ζ_{S_r} are interpreted as the importance of the intervals $\Delta\eta$ to the radiative heat flux and source term, respectively. To allow a direct comparison between spectral contributions based on different quantities or problems, having them in dimensionless form is more convenient, such that (Coelho *et al.*, 2021)

$$\zeta_{q_r}^*(\eta) = \frac{\zeta_{q_r}(\eta)}{\zeta_{q_r,max}} \quad (11)$$

$$\zeta_{S_r}^*(\eta) = \frac{\zeta_{S_r}(\eta)}{\zeta_{S_r,max}} \quad (12)$$

where $\zeta_{q_r,max}$ and $\zeta_{S_r,max}$ are the maximum values in the entire spectrum, respectively.

In their current formulation, $\zeta_{q_r}^*$ and $\zeta_{S_r}^*$ present a highly irregular spectral behavior which is inherited from the absorption spectra of CO₂ and H₂O. In fact, the oscillations are so accentuated that defining a less refined discretization scheme based on them does not seem viable (Coelho *et al.*, 2021). To address this, a spectral reduction is employed on the $\zeta_{q_r}^*$ and $\zeta_{S_r}^*$, resulting in

$$\zeta_{q_r,red}^* = \frac{1}{\Delta\eta_{red}} \int_{\Delta\eta_{red}} \zeta_{q_r}^* d\eta \quad (13)$$

$$\zeta_{S_r,red}^* = \frac{1}{\Delta\eta_{red}} \int_{\Delta\eta_{red}} \zeta_{S_r}^* d\eta \quad (14)$$

where n_{red} is the reduction sample and $\zeta_{q_r,red}^*$ and $\zeta_{S_r,red}^*$ are the reduced spectral contributions to the radiative heat flux and source term, respectively. In this study, the same value $n_{red} = 100$ as in the work from Coelho *et al.* (2021) is considered. The SRI method uses the magnitude of the parameters $\zeta_{q_r,red}^*$ and $\zeta_{S_r,red}^*$ to generate non-uniform spectral discretization schemes, employing higher resolutions on the more important regions to radiative transfer and lower resolutions on the less important ones. Figure 1 shows an example of how two discretization schemes divide the absorption spectrum based on the values of $\zeta_{q_r,red}^*$, (Coelho *et al.*, 2021).

In this study, the spectral contributions $\zeta_{q_r,red}^*$ and $\zeta_{S_r,red}^*$ of the analyzed test cases are obtained through two approaches: one where the contributions are based on the LBL solution of that test case itself, which is the standard methodology of the SRI presented by Coelho *et al.* (2021), and one where the contributions are based on a reference problem

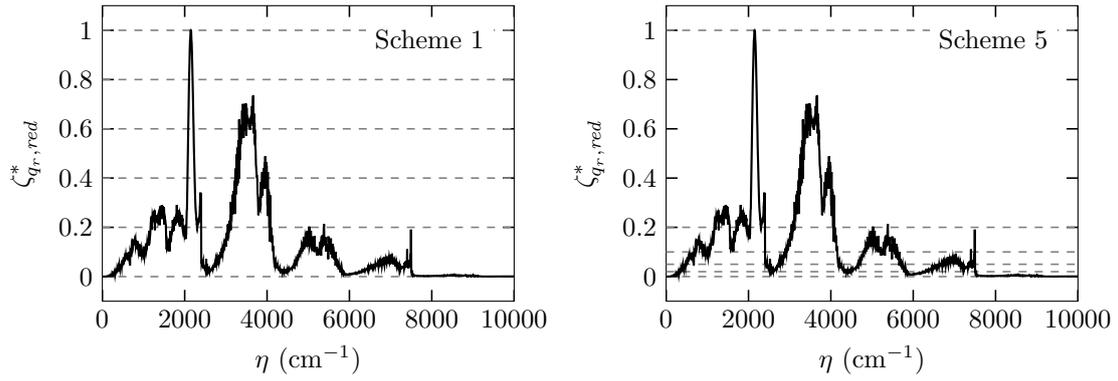


Figure 1. Example of how two discretization schemes, Scheme 1 and Scheme 5, divide the spectrum according to the spectral contributions of the bands to the radiative heat flux (Coelho *et al.*, 2021).

which is sufficiently similar to the one that is being solved. This second approach is also first proposed in the work from Coelho *et al.* (2021) and was very successful for the test cases evaluated there. The high performance of the methodology was mainly attributed to the similarity between the test cases, which all presented the same fixed mole ratio $M_r = Y_w/Y_c$ along the domain. Moreover, the work from Coelho *et al.* (2021) also recommends that at least the regions of most important to the radiative transfer (in the problem that is being solved) should present M_r value close to the one of the case from which the spectral contributions were generated. Following this suggestion, the present study verifies whether the generalization of the spectral contributions from a fixed M_r reference problem to another with variable M_r along the domain is possible. To guarantee a satisfactory degree of similarity between the cases, the M_r from the reference case is chosen as the same as the one from the highest temperature region of the problem that is being solved, which, for the test cases studied in this work, is considered as the most important region to radiative transfer.

The baseline problem used to generate the reference spectral contributions, Case 1, considers a non-homogeneous mixture with fixed M_r along the domain, with temperature and mole fraction distributions given by

$$T = 300 \text{ K} + 1500 \text{ K} \sin^2(\pi x) \quad (15)$$

$$Y_w = 2Y_c = 0.2 \sin^2(\pi x) \quad (16)$$

This case is also employed by Coelho *et al.* (2021) and represents an approximation of the thermodynamic conditions verified in stoichiometric combustion of methane in air. Nonetheless, in the way that it is presented in Eq. (16), SRI discretization schemes based on Case 1 should only be generalized to other test cases employing mole ratios close to 2 near their highest temperature region. To widen the applicability range of this reference case, variations with different M_r are also considered, resulting in a database of spectral contributions for various M_r values. These variations are obtained by fixing the mole fraction of H_2O while changing the distribution of CO_2 to achieve values of M_r between 0.25 and 2, such that

$$Y_c = \frac{Y_w}{M_r} = \frac{0.2 \sin^2(\pi x)}{M_r} \quad (17)$$

In this work, the database is generated for $M_r = 0.25, 0.5, 0.75, 1, 1.5,$ and 2 . To obtain spectral contributions for intermediate values of M_r , a linear interpolation methodology is employed. A wider range of M_r and lower intervals between them might be needed depending on the problem being solved, but, for the purpose of this study, the presented values are satisfactory.

3. RESULTS

Since test cases with constant mole ratio, such as Case 1, were already extensively studied by Coelho *et al.* (2021), this study focuses its analysis on SRI solutions of non-homogeneous media with variable mole ratio instead. Therefore, the next test cases, Cases 2 to 4, consider variable M_r along the whole domain. These three new problems employ the same temperature profile, given by

$$T = 500 \text{ K} + 1900 \text{ K} \sin^2(\pi x) \quad (18)$$

This distribution aims to represent oxy-fuel conditions, which involves higher temperatures than those observed in air-fuel combustion scenarios, such as the ones in Eq. (15). However, results from Coelho *et al.* (2021) show that this difference in temperature profiles should not invalidate the use of Case 1 as a reference case, as all problems consider a hot medium bounded by cold walls.

The mole fraction profiles from Cases 2 to 4 are also based in oxy-fuel scenarios. Case 2 represents combustion of methane in an atmosphere rich in CO₂, usually referred as dry flue gas recirculation (dry-FGR) (Kangwanpongpan *et al.*, 2012), such that

$$Y_w = 0.2 \sin^2(\pi x) \quad (19)$$

$$Y_c = 1 - Y_w \quad (20)$$

The mole ratio M_r from this distribution ranges from 0 to 0.25, with its maximum value located in the same position as the maximum temperature from Eq. (18). Thus, to employ the SRI discretization schemes based on Case 1 to solve Case 2, the reference spectral contributions for $M_r = 0.25$ should be used. For Case 3, the mole fraction profiles are chosen as

$$Y_w = \frac{1}{3} + \frac{1}{3} \sin^2(\pi x) \quad (21)$$

$$Y_c = 1 - Y_w \quad (22)$$

which represent an atmosphere rich in both H₂O and CO₂. In the middle of the domain, where Y_w is maximum and $M_r = 2$, the conditions are similar to typical wet flue gas recirculation (wet-FGR) applications (Kangwanpongpan *et al.*, 2012). Despite that, Case 3 is a hypothetical profile which aims to generate a wider range of M_r variation, from 0.5 to 2. Once more, the maximum temperature occurs in the same position as the maximum M_r , so the generalization of the SRI discretization schemes from Case 1 to solve Case 3 should be based on the spectral contributions for $M_r = 2$. Case 4 is very similar to Case 2, also representing dry-FGR conditions, but now with slightly higher Y_w concentration, given by

$$Y_w = 0.25 \sin^2(\pi x) \quad (23)$$

$$Y_c = 1 - Y_w \quad (24)$$

In Case 4, the highest M_r also occurs in the same position of the highest temperature, but now its value of $M_r = 0.33$ is in between the two lower values from the database, $M_r = 0.25$ and 0.5. Thus, in order to utilize the reference Case 1 to solve Case 4, linear interpolation between the spectral contributions of the database is necessary. A graphical representations of the temperature and mole fraction profiles from Cases 2 to 4 is illustrated in Fig. 2.

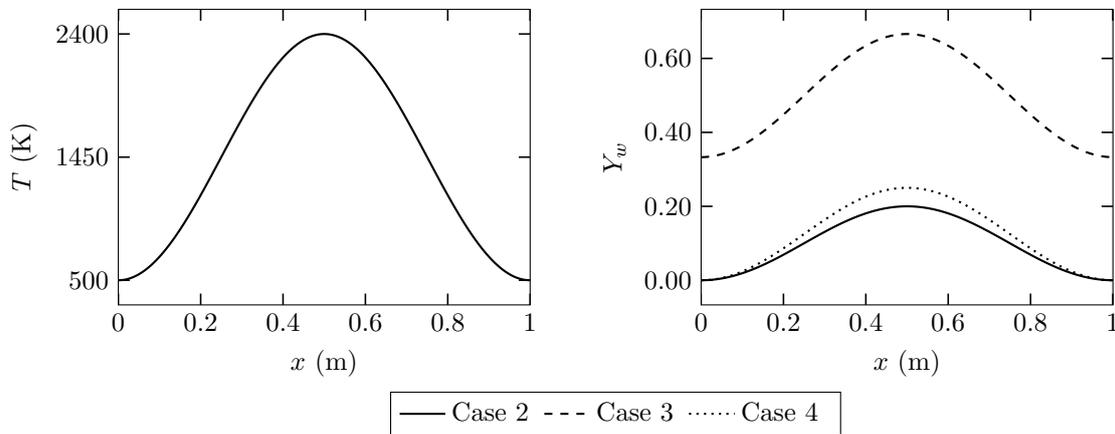


Figure 2. Temperature and mole fraction profiles considered in Cases 2 to 4. The temperature distributions from Cases 2 to 4 coincide along the whole domain.

In order to evaluate the performance of generalizing the discretization schemes based on Case 1 to solve Cases 2 to 4, the SRI solutions based on Case 1 were compared to the benchmark LBL solution and the SRI solutions based on the test cases that are being solved themselves. For instance, when analyzing Case 2, the solutions from the LBL, the SRI based on Case 2, and the SRI based on Case 1 were compared. The discretization scheme chosen for the SRI calculations is the Scheme 5 (S5) from Coelho *et al.* (2021), which was the most accurate scheme developed in that study and, thus, should be more reliable when generalizing it to different problems with variable mole ratio.

The results comparing the LBL and the SRI solutions of radiative heat flux and source term for Cases 2 to 4 is presented in Fig. 3. To differentiate the schemes based on the cases that are being solved to those based on Case 1, the SRI solution of S5 based on Case 1 is named as S5_{C1}. According to the figure, both SRI solutions from all the cases were very accurate. The curves are so close to each other that it is hard to define which of the SRI solutions perform better in each case. The high performance from the SRI S5 was expected since the study from Coelho *et al.* (2021) already showed that S5 was able to obtain benchmark levels of accuracy when it was based on the problem that was being solved. However,

the low deviations from the SRI $S5_{C1}$ observed here in all test cases was quite surprising. These results illustrated that discretization schemes based on simpler fixed mole ratio problems could be generalized to variable mole ratio ones. In fact, even in Case 3, which presents a wide variation of M_r along the domain and considerably different mole fraction distributions, the generalization based on Case 1 was successful. It can also be concluded that the interpolation of the spectral contribution database on the mole was an effective approach.

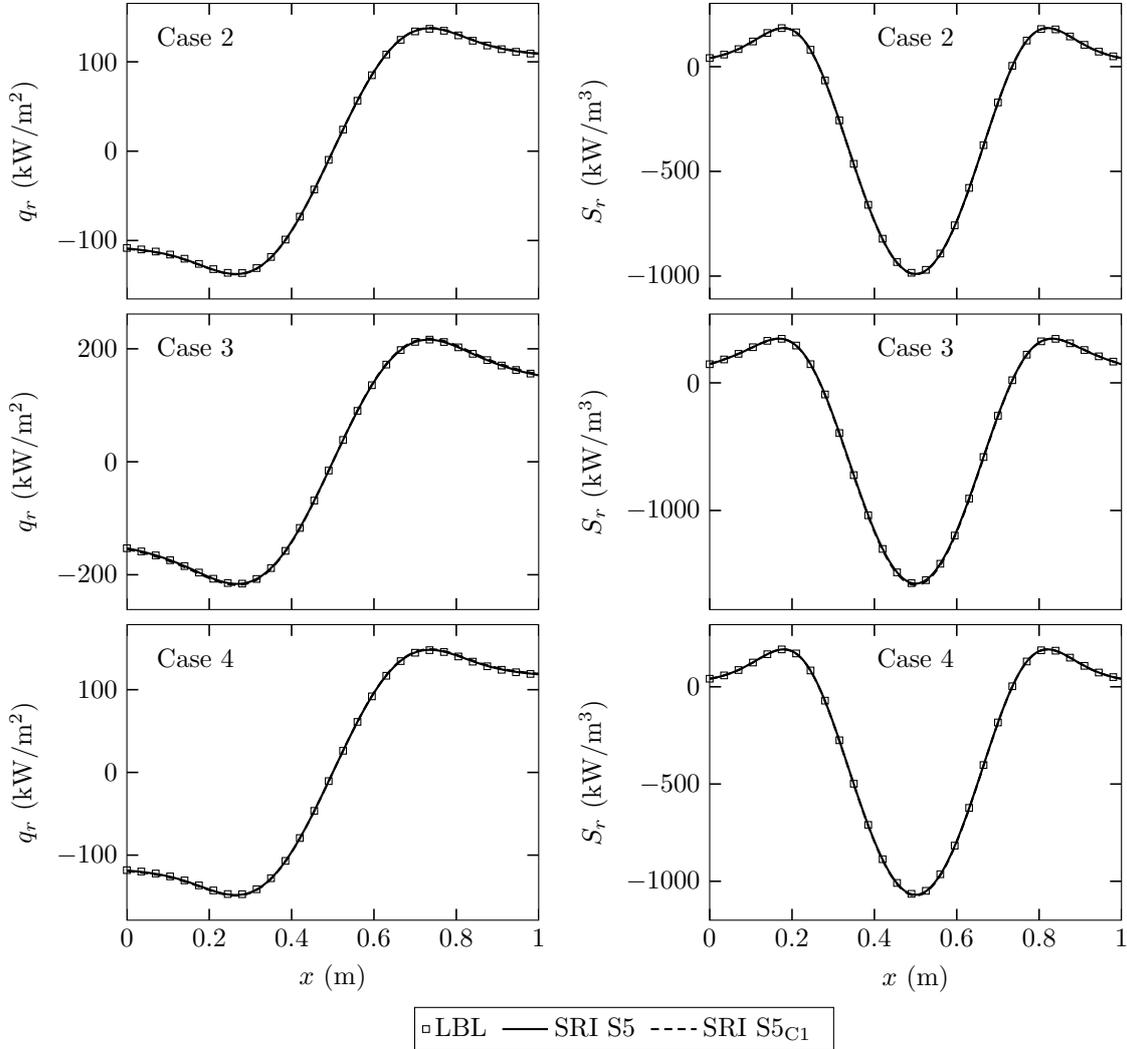


Figure 3. Radiative heat flux and source term solutions from the LBL and the SRI using schemes S5 and $S5_{C1}$ for Cases 2 to 4.

To better differentiate the accuracy between the solutions, Tab. 1 presents the average and maximum deviations between the SRI solutions, S5 and $S5_{C1}$, and the LBL for Cases 2 to 4. This table shows that the solutions from $S5_{C1}$ were less accurate than the solutions from S5 for all test cases, which was the expected outcome. However, it is important to notice that, despite being more accurate, the schemes S5 also result in a higher number of spectral intervals, which is directly proportional to their computational cost. In fact, this might be one of the main factors contributing to the better performance of the schemes S5, as the study from Coelho *et al.* (2021) shows that the number of spectral intervals is highly related to the scheme's accuracy. This is especially apparent in S5 solution of Case 3, which presents extremely low deviations, but at the cost of lower solution speed. Despite the differences evidenced by Tab. 1, the solutions from $S5_{C1}$ still present benchmark levels of accuracy, with average deviations lower than 0.7% and maximum deviations lower than 0.9%.

4. CONCLUSIONS

This study evaluated the accuracy of the SRI method in non-homogeneous gas media with variable mole ratio between H_2O and CO_2 along the domain, such as those observed in oxy-fuel scenarios. In order to do that, two approaches were used: one where the SRI discretization schemes were based on the problem that is being solved and one where the schemes are generalized from a similar reference problem. The second method has the advantage of not requiring a previous LBL

Table 1. Number of spectral intervals and average and maximum deviations of the SRI S5 and S5_{C1} solutions for the radiative heat flux and source term with respect to the LBL for Cases 2 to 4.

Test case	Scheme	$n_{p,t}$	$\delta_{q_r,avg}$ (%)	$\delta_{S_r,avg}$ (%)	$\delta_{q_r,max}$ (%)	$\delta_{S_r,max}$ (%)
Case 2	S5	43 715	0.34	0.13	0.45	0.27
	S5 _{C1}	39 667	0.55	0.21	0.74	0.47
Case 3	S5	69 547	0.05	0.03	0.07	0.06
	S5 _{C1}	39 179	0.67	0.26	0.87	0.62
Case 4	S5	47 950	0.23	0.09	0.32	0.18
	S5 _{C1}	39 211	0.62	0.23	0.81	0.55

solution of the problem, which would be the ideal scenario, and was successful when evaluated for constant mole ratio test cases (Coelho *et al.*, 2021). However, real combustion scenarios involve variable mole ratio distributions along the domain, which makes the evaluation of this methodology in such conditions necessary. Furthermore, to increase the applicability of this methodology, databases which are able to generalize discretization schemes to a wide range of mole ratios were developed. Results from this study showed that the SRI presented very accurate solutions, even when the discretization schemes were based in simpler reference problems, with average deviations lower than 0.7% and maximum deviations lower than 0.9% with respect to the LBL. It was concluded that the database developed here was a good alternative to generalize the SRI discretization schemes to the non-homogeneous test cases proposed in this study.

5. ACKNOWLEDGEMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. Author FHRF also thanks CNPq (Brazil) for research grants 302686/2017-7.

6. REFERENCES

- Bordbar, M.H., Weçel, G. and Hyppänen, T., 2014. “A line by line based weighted sum of gray gases model for inhomogeneous CO₂–H₂O mixture in oxy-fired combustion”. *Combustion and Flame*, Vol. 161, No. 9, pp. 2435–2445.
- Cai, J. and Modest, M.F., 2014. “Improved full-spectrum k-distribution implementation for inhomogeneous media using a narrow-band database”. *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 141, pp. 65–72.
- Cassol, F., Brittes, R., França, F.H. and Ezekoye, O.A., 2014. “Application of the weighted-sum-of-gray-gases model for media composed of arbitrary concentrations of H₂O, CO₂ and soot”. *International Journal of Heat and Mass Transfer*, Vol. 79, pp. 796–806. doi:10.1016/j.ijheatmasstransfer.2014.08.032.
- Coelho, F.R. and França, F.H., 2018. “WSGG correlations based on HITEMP2010 for gas mixtures of H₂O and CO₂ in high total pressure conditions”. *International Journal of Heat and Mass Transfer*, Vol. 127, pp. 105–114.
- Coelho, F.R., Ziemniczak, A., Roy, S.P. and França, F.H., 2021. “A new line-by-line methodology based on the spectral contributions of the bands”. *International Journal of Heat and Mass Transfer*, Vol. 164, p. 120423.
- Howell, J.R., Mengüç, M.P. and Siegel, R., 2016. *Thermal Radiation Heat Transfer*. CRC press, 6th edition.
- Kangwanpongpan, T., França, F.H., da Silva, R.C., Schneider, P.S. and Krautz, H.J., 2012. “New correlations for the weighted-sum-of-gray-gases model in oxy-fuel conditions based on HITEMP 2010 database”. *International Journal of Heat and Mass Transfer*, Vol. 55, No. 25, pp. 7419–7433.
- Modest, M.F., 2013. *Radiative Heat Transfer*. Academic Press.
- Modest, M.F. and Haworth, D.C., 2016. *Radiative Heat Transfer in Turbulent Combustion Systems: Theory and Applications*. Springer.
- Modest, M.F. and Riazzi, R.J., 2005. “Assembly of full-spectrum k-distributions from a narrow-band database effects of mixing gases, gases and nongray absorbing particles, and mixtures with nongray scatterers in nongray enclosures”. *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 90, No. 2, pp. 169–189.
- Pearson, J.T., Webb, B.W., Solovjov, V.P. and Ma, J., 2014. “Effect of total pressure on the absorption line blackbody distribution function and radiative transfer in H₂O, CO₂, and CO”. *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 143, pp. 100–110.
- Rivière, P. and Soufiani, A., 2012. “Updated band model parameters for H₂O, CO₂, CH₄ and CO radiation at high temperature”. *International Journal of Heat and Mass Transfer*, Vol. 55, No. 13-14, pp. 3349–3358.
- Rodrigues, L.G.P., Machado, I.M., Ziemniczak, A., Pereira, F.M., Pagot, P.R. and França, F.H.R., 2019. “Comparisons between numerical simulations and experimental measurements of radiative heat flux for a series of CH₄/N₂ diluted laminar non-premixed flames”. *Combustion Science and Technology*, pp. 1–22.
- Rothman, L., Gordon, I., Barber, R., Dothe, H., Gamache, R., Goldman, A., Perevalov, V., Tashkun, S. and Tennyson, J.,

2010. "HITEMP, the high-temperature molecular spectroscopic database". *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 111, No. 15, pp. 2139–2150.
- Solovjov, V.P., Andre, F., Lemonnier, D. and Webb, B.W., 2017. "The rank correlated SLW model of gas radiation in non-uniform media". *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 197, pp. 26–44.
- Soufiani, A. and Taine, J., 1997. "High temperature gas radiative property parameters of statistical narrow-band model for H₂O, CO₂ and CO, and correlated-k model for H₂O and CO₂". *International Journal of Heat and Mass Transfer*, Vol. 40, No. 4, pp. 987–991.
- Zhang, H. and Modest, M.F., 2002. "A multi-scale full-spectrum correlated-k distribution for radiative heat transfer in inhomogeneous gas mixtures". *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 73, No. 2-5, pp. 349–360.
- Ziemniczak, A., Coelho, F.R., Pereira, F.M., Pagot, P.R. and França, F.H.R., 2019. "Evaluation of the discretization in the spectral resolution for the solution of the line-by-line method in problems with participating gases". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 41, No. 9.

7. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.