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DETECTION OF CONTACT FAILURES IN LAMINATED COMPOSITES EMPLOYING A SINGLE DOMAIN FORMULATION, A SURROGATE RADIAL BASIS FUNCTION AND BAYESIAN INFERENCE

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Abstract. *This work deals with a heat conduction problem in a multi-layer media using a single domain formulation whose solution is obtained from the application of a hybrid methodology which combines the classical integral transform technique and the finite difference method. The single domain formulation advantage is mainly the possibility of analyzing problems with different numbers of layers and arbitrary geometries by simply defining the corresponding spatially varying coefficients. In order to overcome the intensive computational requirements, a metamodel constructed upon radial basis function is proposed. So as to illustrate the application of the methodology, the heat conduction problem within a multi-layer composite medium is considered, including the identification of a defect in the junction region, in which the adhesion of two different materials is imperfect. It is expected to identify adhesion failures by estimating a thermal conductivity with spatial variation throughout the region that contains adhesive and analyzing abrupt differences, which indicate possible contact failures.*

Keywords: *heat conduction, laminated composites, surrogate model, classical integral transform technique.*

1. INTRODUCTION

Heat conduction in a composite medium has been analyzed in various contexts, such as thermal insulation, corrosion protection and layered compounds, which offers new opportunities to adapt structures to meet different requirements in modern building materials (Grosso *et al.*, 2016; Abreu *et al.*, 2014a,b; Mascouto *et al.*, 2020). In addition, the formulation and solution of problems that allow assessing the adherence between two or more materials is of great importance in several fields, such as electronics, telecommunications, aviation, defense and oil, among others. Given the importance of non-destructive detection of adhesion failures in laminated composites and the use of infrared thermographic images for this purpose, efforts have been made to ensure that the knowledge of heat transfer is applied so that quantitative analyzes can be made possible. It is not always possible to identify adhesion or adhesion failures using only qualitative tests, since temperature gradients in contact failure regions are generally very small. In some situations, the thickness of the material and its glass transition temperatures prevent the occurrence of large gradients and cause the flaws to be identified only by thermal image (Meola and Carlomagno, 2004; Abreu *et al.*, 2014a). Thus, in Abreu *et al.* (2014b) a formulation via inverse problems was proposed in order to allow the quantitative analysis of failures by means of infrared thermography.

In Mascouto *et al.* (2020), the Classical Integral Transformation Technique (CITT) was used to solve the eigenvalue problem (Cotta, 1993; Cotta *et al.*, 2017, 2013). A hybrid solution was applied that combines the CITT with the finite difference method (FDM) (Pletcher *et al.*, 2013; Mascouto *et al.*, 2020).

In Orlande *et al.* (2008), the metamodeling technique was applied with success to interpolate the likelihood function and tested in two different problems, the first one involved the transport of tracers in soil columns and the latter, a three-dimensional heat conduction in an orthotropic media.

In this work, the physical problem involves a two-dimensional heat conduction in a laminated composite with adhesion failure and the forward solution from Mascouto *et al.* (2020) is used in order to interpolate the likelihood function using radial basis function (RBF). By means of an inverse problem with a Bayesian approach using the Markov Chain Monte Carlo (MCMC) (Kaipio and Somersalo, 2004; Christen and Fox, 2005; Orlande, 2012; Orlande *et al.*, 2014) method, it is expected to identify the thermal conductivity with spatial variation along the interface between the two layers of intrusive (synthetic) temperature measurements performed on the outer surface of the medium and to reduce the computational cost. The identification of spatial variations in thermal conductivity reveals the position where there is no adhesive joining the two layers, since the thermal conductivity of this material is much greater than that of the gas that supposedly fills in the gaps without adhesive.

2. HEAT CONDUCTION IN A LAMINATED COMPOSITE WITH ADHESION FAILURE

In this work the physical problem involves a two-dimensional heat conduction in a laminated composite with three different layers and materials according to Fig. 1, with width in x-direction L_x and height in y-direction L_y. The adhesion failure is represented by the presence of an air bubble ($L_{x_b} \times L_{y_b}$) among the adhesive, that might be caused during the manufacturing process. The height of the thermal insulation, the adhesive (and the air bubble) and the metal is, respectively, L_{y_c}, L_{y_b} and L_{y_a}. The air bubble width is L_{x_b}; the remaining distances to the left and to the right of the air bubble are, respectively, L_{x_a} and L_{x_c}.

The physical phenomenon involves a heat flux (q'') at $y = 0$ while the opposite surface is subjected to a heat exchange by natural convection with the environment, with convection heat exchange coefficient h and with ambient temperature T_∞. The functions $f(x)$ and $g(x)$ model the thermal insulation-adhesive interface and metal-adhesive interface, respectively.

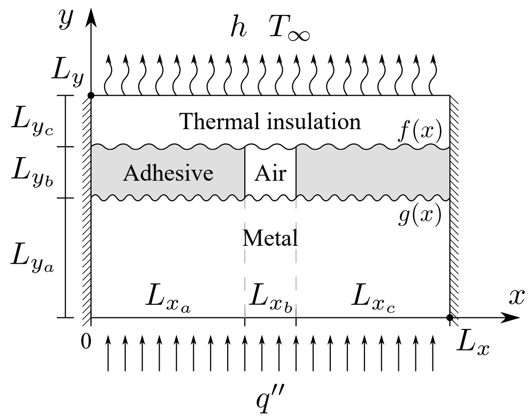


Figure 1: Schematic representation of the physical problem.

The heat conduction equation formulated in a single domain can be written as (Ozisik, 1987; Incropera *et al.*, 2014)

$$\rho(x,y)\; c_p(x,y)\; \frac{\partial T(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,y)\; \frac{\partial T(x,y,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x,y)\; \frac{\partial T(x,y,t)}{\partial y}\right) \tag{1}$$

where ρ is the density, c_p is the specific heat and k is the thermal conductivity.

The boundary and initial conditions are expressed by

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=L_x} = 0\,, \qquad \text{for } x=0\ ,\ x=L_x \text{ and } t>0 \tag{2}$$

$$-\,k(x,y)\; \left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0} = q''\,, \qquad \text{for } y=0 \text{ and } t>0 \tag{3}$$

$$k(x,y)\; \left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=L_y} + h\,T(x,y,t)\big|_{y=L_y} = h\,T_\infty\,, \qquad \text{for } y=L_y \text{ and } t>0 \tag{4}$$

$$T(x,y,t)\big|_{t=0} = T_0\,, \qquad \forall\ x\,,\ y \text{ and } t=0 \tag{5}$$

where q'' is the heat flux, h is the heat transfer coefficient and T_0 represents the initial temperature.

3. FORWARD PROBLEM SOLUTION

The CITT is a generalization of the variables separation method and can be directly applied to solve a multilayer problem, building a eigenvalue problem in a single domain, that brings together all the different sub-regions in a single set of eigenvalues (Mikhailov and Ozisik, 1984).

In this problem the filter $T(x,y,t) = T_\infty + T^*(x,y,t)$ is applied to homogenize the boundary at $y = L_y$, so the following filtered problem is obtained

$$\rho(x,y)\, c_p(x,y)\, \frac{\partial T^*(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,y)\, \frac{\partial T^*(x,y,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x,y)\, \frac{\partial T^*(x,y,t)}{\partial y}\right) \tag{6}$$

$$\left.\frac{\partial T^*(x,y,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial T^*(x,y,t)}{\partial x}\right|_{x=L_x} = 0 \tag{7}$$

$$-\, k(x,y)\, \left.\frac{\partial T^*(x,y,t)}{\partial y}\right|_{y=0} = q'' \tag{8}$$

$$k(x,y)\, \left.\frac{\partial T^*(x,y,t)}{\partial y}\right|_{y=L_y} + h\, T^*(x,y,t)\big|_{y=L_y} = 0 \tag{9}$$

$$T(x,y,t)\big|_{t=0} = T_0 - T_\infty = F(x,y) \tag{10}$$

According to Cotta *et al.* (2016); Mascouto *et al.* (2020) the solution via CITT is stated as

$$T(x,y,t) = \sum_{i=1}^{\infty} \tilde{\psi}_i(x,y)\, e^{-\mu_i^2\, t} \left(\bar{f}_i + \int_0^t e^{\mu_i^2\, t'}\, \bar{g}_i(t')\, \mathrm{d}t'\right) \tag{11}$$

where

$$\bar{f}_i = \int_0^{L_x} \int_0^{L_y} \rho(x,y)\, c_p(x,y)\, \tilde{\psi}_i(x)\, F(x,y)\, \mathrm{d}y\, \mathrm{d}x \tag{12}$$

and

$$\bar{g}_i(t) = \int_0^{L_x} \frac{q''\, \left(k(x,0)\, \frac{\partial \tilde{\psi}_i(x,y)}{\partial x}\big|_{y=0} + \tilde{\psi}_i(x,0)\right)}{h}\, \mathrm{d}x \tag{13}$$

in which $\tilde{\psi}_i(x,y)$ are the normalized eigenfunctions associated to each eigenvalue μ_i obtained from the Sturm-Liouville generalized problem.

4. INVERSE PROBLEM - BAYESIAN TECHNIQUE

In this work, we seek to estimate the vector $\mathbf{P}$, which represents the discrete values of the thermal conductivity (k) in the region where, theoretically, there should be only adhesive. Once the thermal conductivity of the air is different from the adhesive's, it must be clear whether there is a adhesion failure.

The Bayesian Inference is a method of statistical inference that essentially consists of using all the information available *a priori* in order to reduce uncertainty in decision-making problems. The term Bayesian is often used to describe the so-called *statistical inversion approach*, which is based on the following principles (Kaipio and Somersalo, 2004; Orlande *et al.*, 2008; Mascouto *et al.*, 2020):

1. All variables included in the model are modelled as random variables;
2. The randomness describes the degree of information concerning their realizations;
3. The degree of information concerning these values is coded in probability distributions;
4. The solution of the inverse problem is the posterior probability distribution.

The Bayes' theorem, given by Eq. (14) is used to combine new information with all previous information in order to form the basis of statistical processes.

$$\pi_{post}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi_{pri}(\mathbf{P})\pi(\mathbf{Y}|\mathbf{P})}{\pi(\mathbf{Y})} \tag{14}$$

where $\pi_{post}(\mathbf{P})$ is the posterior probability density, that is, the conditional probability of the parameters $\mathbf{P}$ given the measurements $\mathbf{Y}$; $\pi_{pri}(\mathbf{P})$ is the prior density, in other words, the coded information about the parameters prior to the measurements; $\pi(\mathbf{Y}|\mathbf{P})$ is the likelihoon function, which expresses the likelihood of different measurement outcomes $\mathbf{Y}$

with $\mathbf{P}$ given; and $\pi(\mathbf{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

The vector of parameters is written as

$$\mathbf{P} = \{P_1, P_2, \ldots, P_N\} \tag{15}$$

and the available measurements' as

$$\mathbf{Y} = \{Y_1, Y_2, \ldots, Y_I\} \tag{16}$$

where N is the number of parameters and I is the number of measurements (Kaipio and Somersalo, 2004; Ozisik and Orlande, 2000; Ozisik, 1987). The vector $\mathbf{P} = \{P_1, P_2, \ldots, P_N\}$ represents the value of k in discrete nodes along the x direction at $y = 0.051$ (adhesion region medium point), or else, P_1 is equivalent to the value of k at $x = 0$ and P_N represents the value of k at $x = L_x$.

Considering the measurement errors as Gaussian random variables, with zero mean and known covariance matrix $\mathbf{W}$, and also additives and independent of the parameters $\mathbf{P}$, the likelihood function can be represented by

$$\pi(\mathbf{Y}|\mathbf{P}) = (2\pi)^{-I/2}|\mathbf{W}|^{-1/2} exp\left[-\frac{1}{2}(\mathbf{Y} - \mathbf{T}(\mathbf{P}))^T \mathbf{W}^{-1}(\mathbf{Y} - \mathbf{T}(\mathbf{P}))\right] \tag{17}$$

at which, $\mathbf{T}$ is the vector of estimated variables, obtained from the solution of the forward model with an estimate for the parameters $\mathbf{P}$.

The Metropolis-Hastings algorithm used was used in this work and its implementation starts with the selection of a movement distribution $q(\mathbf{P}^*, \mathbf{P}^{(t-1)})$ which generates a new candidate $\mathbf{P}^*$ given the current state $\mathbf{P}^{(t-1)}$ of the Markov chain. Once the distribution is selected, the Metropolis-Hastings algorithm is implemented following the steps (Kaipio and Somersalo, 2004; Ozisik and Orlande, 2000; Ozisik, 1987):

1. Sample a Candidate Point $\mathbf{P}^*$ from a jumping distribuition $q(\mathbf{P}^*, \mathbf{P}^{(t-1)})$;
2. Calculate
$$\beta = min\left[1, \frac{\pi(\mathbf{P}^*|\mathbf{Y})\; q(\mathbf{P}^{(t-1)}, \mathbf{P}^*)}{\pi(\mathbf{P}^{(t-1)}|\mathbf{Y})\; q(\mathbf{P}^*, \mathbf{P}^{(t-1)})}\right]; \tag{18}$$
3. Generate a random value U with uniform distribution on $(0, 1)$;
4. If $U \leq \beta$, define $\mathbf{P}^t = \mathbf{P}^*$; Otherwise, define $\mathbf{P}^t = \mathbf{P}^{(t-1)}$;
5. Return to step 1 in order to generate the sequence $[\mathbf{P}^1, \mathbf{P}^2, \mathbf{P}^3, ..., \mathbf{P}^n]$.

5. Radial Basis Function - Interpolating The Likelihood Function

In order to overcome the intensive computational cost required by the iterative calculation of the likelihood function, a metamodel is constructed upon radial basis functions. The RBF model used in this work has the form (Orlande *et al.*, 2008; Colaço *et al.*, 2007)

$$f_{rbf}(\mathbf{x}) = \sum_{i=1}^{N_{rbf}} \mathbf{p}_i \phi(|\mathbf{x} - \mathbf{x}_i|) = \sum_{j=1}^{N} \mathbf{p}_i \sqrt{(\mathbf{x} - \mathbf{x}_j)^2 + c_j^2} \tag{19}$$

where N_{rbf} is the number of points used in the interpolation; $\mathbf{p}_i$ are the weights; ϕ is a group of N_{rbf} multi-quadratic radial base functions; $\mathbf{x}$ is the input vector of parameters of the interpolated function $f_{rbf}(x)$; $\mathbf{x}_i$ represents the central values of the associated function; and c_j is a shape parameter, which represents the RBF centers.

The problem is solved for the unknown $\mathbf{p}_i$ from a system of N linear equations (Haykin, 2008)

$$\begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,N_{rbf}} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,N_{rbf}} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N_{rbf},1} & \phi_{N_{rbf},2} & \cdots & \phi_{N_{rbf},N_{rbf}} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{N_{rbf}} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{N_{rbf}} \end{bmatrix} \tag{20}$$

where

$$\phi_{i,j} = \phi(|\mathbf{x}_i - \mathbf{x}_j|)\,, i, j = 1, 2, 3, \cdots, N_{rbf} \tag{21}$$

$$\mathbf{p}_i = \{p_1, p_2, \cdots, p_{N_{rbf}}\} \tag{22}$$

$$\mathbf{f}_i = \{f_1, f_2, \cdots, f_{N_{rbf}}\} \tag{23}$$

$\mathbf{p}_i$ are the estimated weights for each $\mathbf{x}_j$ and $\mathbf{f}_i$ is the likelihood function (Eq.(17)) output calculated with estimated temperatures obtained via CITT with the input $\mathbf{x}_j$.

6. RESULTS AND DISCUSSION

During the inverse problem, the forward problem needs to be computed once for each vector of candidates k proposed, which increase the computational cost. In order to reduce it, the likelihood function was interpolated via RBF. Its input is a vector of the parameters k in the adhesion region and its output is the value of the likelihood calculated using three different time instants ($t = 5,000s$, $t = 10,000s$, $t = 20,000s$). The forward problem solution from Mascouto *et al.* (2020) was used only to obtain a trained RBF that interpolates the likelihood function, and then, use it in the inverse problem. For such, the space x and the parameter $k(x)$ were discretized into 21 cells and only the temperature in the surface (at $y = L_y$) was considered. The experimental data (presented in Fig. 2)) was simulated from the forward problem with a noise from a normal distribution with mean 0 and standard deviation 0.05. The number of points used in the interpolation via RBF (Eq. (19)) was $N = 1000$ and the shape factor chosen was $c = 0.05$.

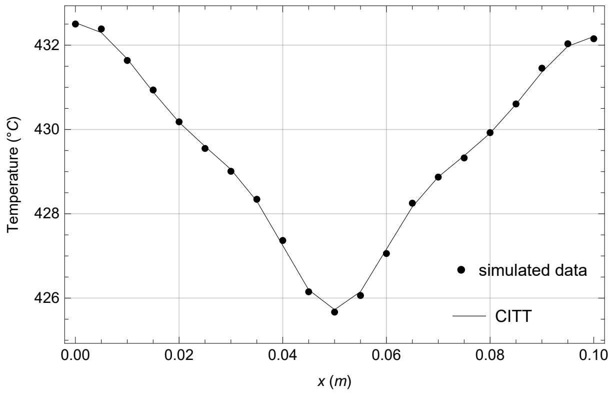


Figure 2: Representation of the experimental data at $t = 20,000s$.

After the interpolation was done, the computation of the likelihood became approximately 5 times faster (RBF: $0.14s$ vs. CITT: $0.70s$).

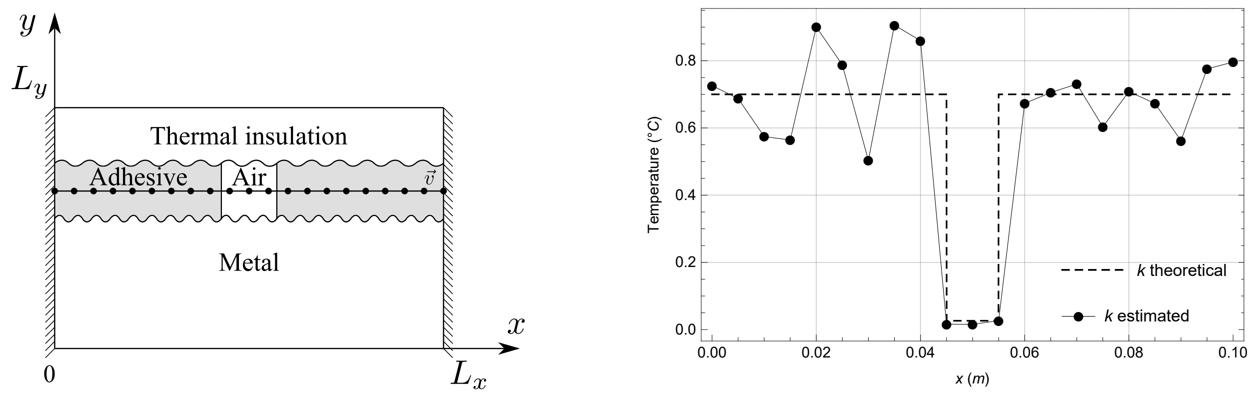
The Tab. 1 shows the parameters involved in the present problem (Incropera *et al.*, 2014).

Table 1: Parameters used in the forward problem.

Parameter	Unit	Value
k_{steel}		13.4
$k_{adhesive}$	$\left[\frac{W}{mK}\right]$	0.7
k_{air}		0.0263
$k_{insulation}$		1.171
$(\rho \cdot c_p)_{steel}$		3.86
$(\rho \cdot c_p)_{adhesive}$	$\left[\frac{J}{m^3K}\right]$	1.75
$(\rho \cdot c_p)_{air}$		1.17
$(\rho \cdot c_p)_{insulation}$		2.65
$L_{x_a} = L_{x_c}$		0.045
L_{x_b}	$[m]$	0.01
L_{y_a}		0.05
L_{y_b}		0.001
L_{y_c}		0.002

Knowing, by analyzing the simulated data, that there may be an adhesion failure, in the region between 0.04 and 0.06, the initial Markov chain employed was a normal distribution with mean 0.7 and standard deviation 0.1 for $x < 0.045$ and $x > 0.055$; and normal distribution with mean 0.027 and standard deviation 0.01 for $0.045 \leq x \leq 0.055$. Also a gaussian piori with means equal to the theoretical values of $k(x)$ and standard deviation 0.5 was applied.

The Fig. 3a represents the vector of parameters k discretized in 21 points in function of the space x and the Fig. 3b shows an arithmetic mean of the last 20,000 out of 80,000 Markov chains obtained. In the latter, it shows a comparison of the theoretical values of k and the ones estimated in the inverse problem. The percentage error (calculate by $|estimated - theoretical| / theoretical \times 100$) is shown in the Fig. 4, where the maximum error was about 40% at $x = 0.055m$.



(a) Vector of parameters estimated in the inverse problem.

(b) Percentage error of the temperature profile in the x direction with $y = L_y$ and $t = 20,000\,s$.

Figure 3: Representation of the vector of parameters $k(x)$.

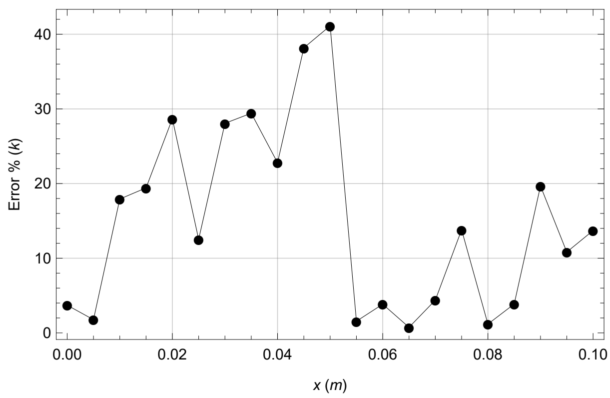
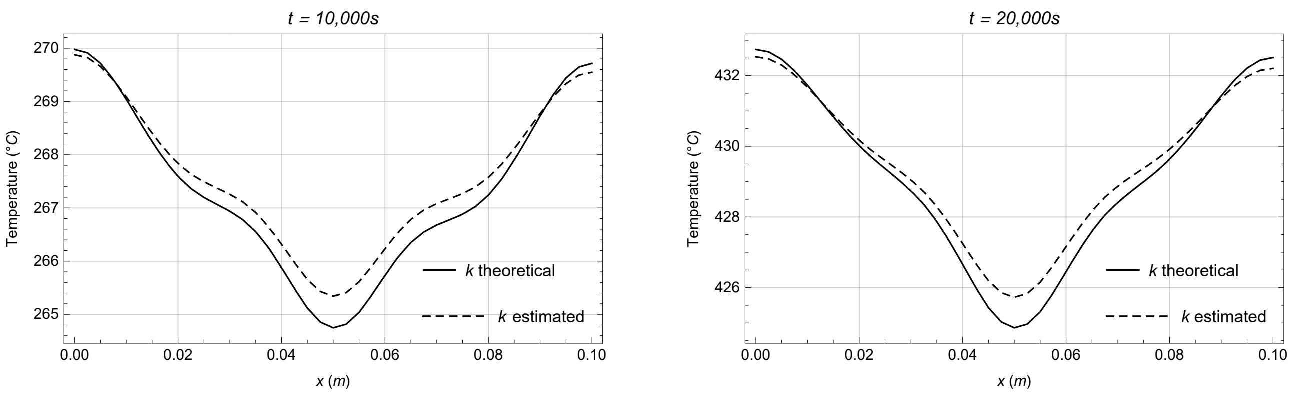


Figure 4: Percentage error between the theoretical k and the k estimated in the inverse problem.

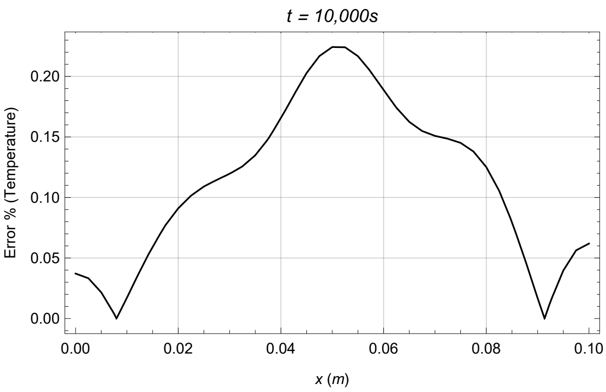
The forward problem was calculated once again, this time using the values of the thermal conductivity $k(x)$ obtained in the inverse problem (Fig. 3b). The Fig. 5 shows the temperature profile at $y = L_y$ in two different instants, $t = 10,000s$ (Fig. 5a) and $t = 20,000s$ (Fig. 5b). The differences between the curves are due to model error as result of the difficulty of the RBF in representing precisely the likelihood function. The respective percentage error is represented in the Fig. 6, where the maximum errors were also at $x = 0.055m$ at both instants.



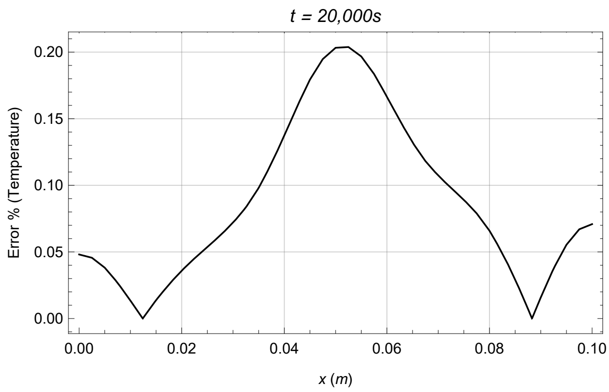
(a) Temperature profile in the x direction with $y = L_y$ and $t = 10,000\,s$.

(b) Temperature profile in the x direction with $y = L_y$ and $t = 20,000\,s$.

Figure 5: Comparison of the temperature calculated using the parameters k estimated in the Inverse Problem; and the theoretical $k(x)$.



(a) Percentage error of the temperature profile in the x direction with $y = L_y$ and $t = 10,000\,s$.



(b) Percentage error of the temperature profile in the x direction with $y = L_y$ and $t = 20,000\,s$.

Figure 6: Percentage error between the temperature calculated with the parameters k estimated in the Inverse Problem; and the theoretical $k(x)$.

7. CONCLUSION

In this work, the likelihood function (Eq. (17)) was interpolated using radial basis functions (RBF) in order to reduce the high computational cost during the iterative process in the inverse problem. The objective was to estimate a vector of parameters of the thermal conductivity k along a line in the adhesion region (Fig. 3a) in the inverse problem and, thus, identify whether there is a contact failure since the thermal conductivity of the air is much smaller than the adhesive's. The likelihood interpolated function performed about 5 times faster in comparison with the likelihood calculated with the CITT method.

The considerable percentage error of 40% in the estimated parameters k shows that there is a model error in the interpolated likelihood, which can't be represent precisely by the RBF and misleads the inverse problem and candidates that don't represent accurately the solution are accepted.

The next steps for this work consist in improving the methodology adopted. It is expected to employ a proper orthogonal decomposition (POD) integrated with a trained radial basis function (RBF) in order to develop a more precise interpolated function (Ostrowski *et al.*, 2008; Liang *et al.*, 2020; Rogers *et al.*, 2010). Plus the application of a regularization technique in the inverse problem is also desirable, such as total variation decomposition (TVD) technique (Mota *et al.*, 2010; Orlande *et al.*, 2014).

8. ACKNOWLEDGEMENTS

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