

ENC-2020-0645

## ANALYTICAL STUDY OF EARTH-AIR HEAT EXCHANGERS ARRANGED IN A NETWORK

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**Abstract.** *The Earth-Air Heat Exchanger (EAHE) is a system used for thermal improvement of environments. It is based on the use of thermal inertia present in the subsoil to improve the thermal comfort of built environments and reduce the rates of use of conventional energy. The main objective of this work is to present a comparative analytical study of EAHE's in a network arrangement with individual EAHE's, identifying the advantages of using the networked system. An analytical model is used in the steady state of an individual EAHE, considering the enthalpy of humid air within the energy balance, assigning boundary conditions of the problem considering tropical climate. Then, an analytical modeling of the network system is used, applying the constructal theory, and an optimization of the network is performed using the Lagrange theorem. The networked EAHE system proved to be effective in achieving better levels of thermal performance compared to the single duct system.*

**Keywords:** *Earth-Air Heat Exchanger, Constructal Theory, Optimization, Network System.*

### 1. INTRODUCTION

Energy is one of the main inputs in a country's economic growth, but energy saving is one of the biggest challenges in the world today (Bordoloi et al., 2018). The growing global demand for energy and the decrease in available conventional energy sources, combined with environmental concerns, motivate the search for alternative and renewable energy sources (Brum et al., 2013).

The work by Estrada et al. (2018) presents an EAHE model based not only on sensitive heat transfer but also on latent heat exchange. A comparison is made between the climate of Brazil (Rio de Janeiro) and the south of France (Montpelier) in the relevance of such systems; the length of the duct is determined to obtain maximum underground changes, a time-dependent model combined with real meteorological data is developed. A 3D model coupled with a 1D model allows us to propose an approach to assess the cooling/heating potential of different climatic regions. In Brum et al. (2019a) an investigation is made of geometric configurations for assembling the ducts using the structural design in order to improve the thermal performance of an EAHE facility. Thermal potential, heat exchange, and annual efficiency are evaluated for eight types of EAHE installations subject to different geometric configurations. Brum et al. (2019b) have the advantage of designing an EAHE as a network from the point of view of the Constructal Law; a methodology is proposed to design a single tube EAHE when the need is defined in terms of cooling power, overall efficiency and enthalpy difference between the incoming air and the ground. A single-tube EAHE was used as a reference to design a tree-shaped network with identical fluid volume and cooling power restrictions. The present work presents an analytical study of the potential of an EAHE network system compared to single-pipe EAHE.

### 2. MATHEMATICAL MODELING PROBLEM DESCRIPTION

EAHE is a non-conventional technique that has found applications in residential buildings with air conditioning systems, greenhouses, commercial buildings, etc. (Bordoloi et al., 2018). Earth to air heat exchangers basically consists

of the underground pipes and the air system which forces the air through the pipes and eventually mixes it with the indoor air of the building or of the agricultural greenhouse. (Mihalakakou et al., 1995).

The region of interest is the horizontal part of EAHE of length  $L$  (m) and diameter  $D$  (m), as shown in Fig. 1.

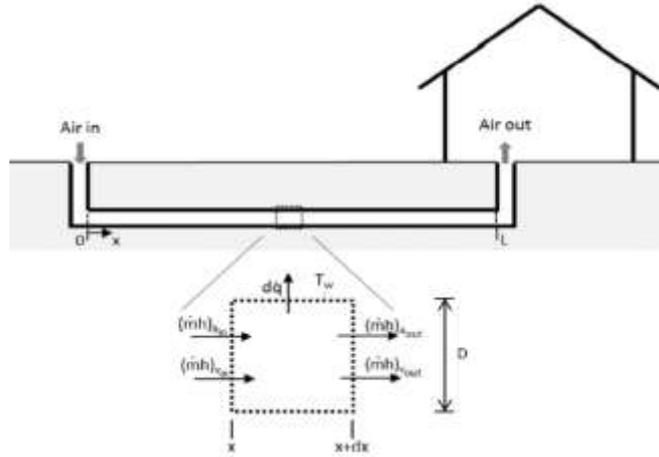


Fig 1 – EAHE system and model domain  
Available from Estrada et al. (2018)

Initially, the mass flow rate (kg/s) of the humid air is determined, shown in Eq. (1), where  $\dot{m}_a$  and  $\dot{m}_v$  are the air mass flow rate and the vapor mass flow rate, respectively.

$$\dot{m}_{ha} = \dot{m}_a + \dot{m}_v \quad (1)$$

The incoming energy is given by the variation of the humid air mass by the total enthalpy  $h$  (kJ / kg), presented by:

$$[(\dot{m}h)_{a,out} - (\dot{m}h)_{a,in}] + [(\dot{m}h)_{v,out} - (\dot{m}h)_{v,in}] = -d\dot{q} \quad (2)$$

According to the work of Estrada et al. (2018), the dimensionless enthalpy is determined by the following equation.

$$\tilde{h} = \frac{h - h_w}{h_{in} - h_w} = \exp\left(-\frac{h_c p}{\dot{m}_a C_{p,ha}} x\right) \quad (3)$$

The enthalpy of humid air can be determined by:

$$h_{ha} = (h_{in} - h_w) e^{-\frac{h_c p}{C_{p,ha} \dot{m}_a} x} + h_w \quad (4)$$

where  $h_{in}$  and  $h_w$  are the enthalpy of the humid air at the inlet and the wall of the duct, respectively,  $h_c$  is the convective heat transfer coefficient (W/kg.K),  $C_{p,ha}$  is the heat capacity of humid air (J/kg.K) and  $p$  is the perimeter of the pipe (m). The analytical expression for specific humidity is given by:

$$w = (w_{in} - w_w) e^{-\frac{h_{cv} p}{C_{p,ha} \dot{m}_a} x} + w_w \quad (5)$$

where,  $w_{in}$  and  $w_w$  are the humidity at the entrance and in the duct wall, respectively (g/kg). The temperature at the outlet of the duct is then given by the following equation:

$$T = \frac{h_{ha} - w L_v}{C_{p,a} + w C_{p,v}} \quad (6)$$

where  $L_v$  is the latent heat of vaporization (J/kg),  $C_{p,a}$  and  $C_{p,v}$  are the specific heat of the air and specific heat of the vapor, respectively (J/kg.K).

## 2.1. Analytical model verification

The application of computational modeling occurs to verify the analytical model based on the work of Estrada et al. (2018), through a comparative study between the tropical climate considering the latent and sensitive heat exchanges, and later for a study between the efficiency of the single-tube EAHE and networked EAHE's systems.

They are shown in Figs. 2 the evolution of total enthalpy along the duct considering the conditions of a tropical climate, where h1 represents the total enthalpy presented in Estrada et al. (2018) and h2 is the total enthalpy determined using the analytical model of this work.

The boundary conditions applied to verify the analytical model of this work are the same applied in Estrada et al. (2018), the air temperatures at the entrance are 27 ° C for the tropical climate (hot and humid), the relative humidity of the air at the entrance is 85% while the soil temperature considered is 23 ° C. The flow speed is 3.3 m / s.

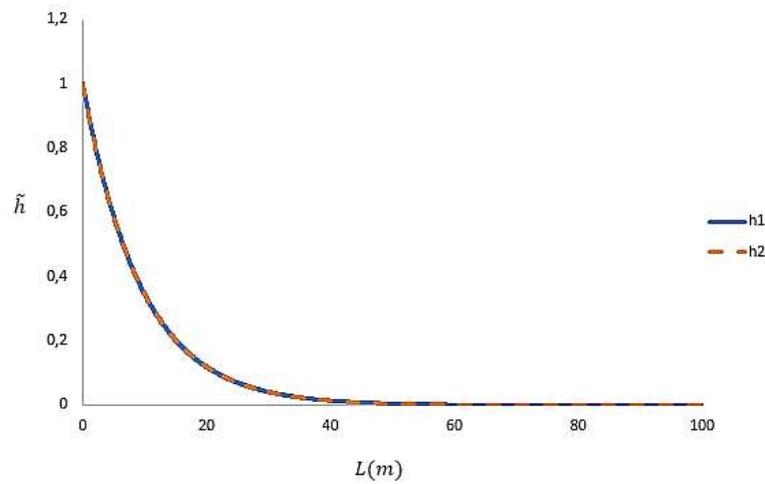


Figure 2 - Evolution of enthalpy in a tropical climate zone.

The results are given in the non-dimensional form defined in Eq. (3). It is possible to notice, from Fig. 2 that the model presented in this work agrees with the reference model presented in Estrada et al. (2018) because the curves are superimposed. It is worth remembering that, in the case analyzed, the total enthalpy is calculated keeping the latent and sensitive heat exchanges.

## 3. NETWORK MODELING

According to Bejan and Zane (2012), for a finite flow system to persist over time (to live), its configuration must evolve to facilitate access to the currents that flow through it.

The modeling presented in this section shows how the Constructal theory is applied in the construction of an EAHE system in networks.

Following the work of Estrada et al. (2018) and Brum et al. (2019) the evolution of the enthalpy of humid air ( $h_{ha}$ ) is given by Eq. (4). Humid wall conditions are assumed in this study, being more realistic in relation to dry walls in terms of thermal exchanges.

The cooling power  $\dot{Q}$  is given by:

$$\dot{Q} = (1 - \eta)\Delta h\dot{m}_a \quad (7)$$

According to Brum et al. (2019b), the dimensionless enthalpy at the outlet of the duct is denoted  $\eta$ . The term  $(x)$  depends on the length of the tube.

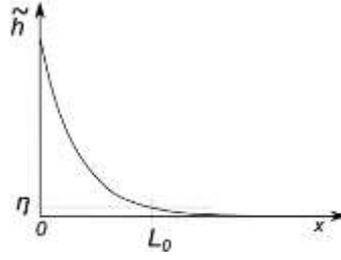


Figure 3 - Dimensionless enthalpy at the outlet of a pipe  
Available from Brum et al. (2019b)

In Fig. 3,  $L_0$  is the length of the duct where  $\tilde{h}$  remains constant, so it is possible to define  $\eta$  by:

$$\eta = \exp\left(-\frac{h_{cv}\pi d_0 L_0}{\dot{m}C_p}\right) \quad (8)$$

where,  $d_0$  the diameter of the duct (m),  $\dot{m}$  is the mass flow at the inlet of the duct.

Isolating the minimum mass flow  $\dot{m}$ , we obtain:

$$\dot{m} = -\frac{h_{cv}\pi d_0 L_0}{\ln(\eta)C_p} \quad (9)$$

The thermal performance coefficient  $\varepsilon$  can be determined by:

$$\varepsilon = \frac{(1 - \eta)\Delta h\rho}{\Delta P} \quad (10)$$

The pressure in the duct is given by:

$$\Delta P = \frac{8 \dot{m}^2 L_0 f}{\rho \pi^2 d_0^5} \quad (11)$$

The diameter can then be determined by:

$$d_0 = \left(\frac{8 h_{cv}^2 L_0^3 f \varepsilon}{\ln^2(\eta) C_p^2 \rho^2 (1 - \eta) \Delta h}\right)^{\frac{1}{3}} \quad (12)$$

where,  $f$  is the friction factor.

According to Brum et al. (2019b) the study of the dimensioning of the network compared to individual installations, the geometries for each type of configuration used to serve  $2^{n-1}$  buildings with  $n \in \mathbb{N}$ , where,  $n$  is called the construction level. It is assumed that each building occupies the same surface area of the ground  $A_0 = (2L_0)^2$  therefore,  $m$  buildings occupy an area of  $2^{(n-1)} \cdot A_0$ .

Figure 4 shows the model considering a single-pipe EAHE system.

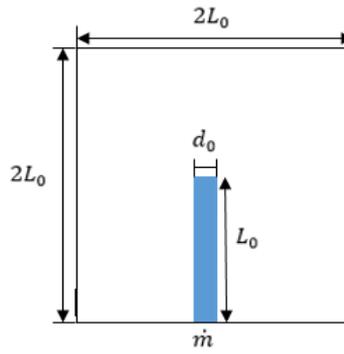


Figure 4 - EAHE single duct system.

The volume of the duct can be determined by the following equation.

$$V_t = \frac{\pi}{4} d_0^2 L_0 \quad (13)$$

For the case of the system with the first bifurcation level, where exits for two buildings are considered, the installation area of the network system is equal to the sum of the areas of two EAHE's unitary systems. Fig. 3.3 illustrates the scheme.

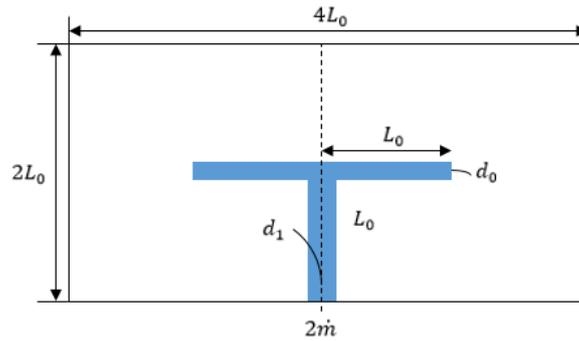


Figure 5 - EAHE network system with two air exits.

Analyzing the EAHE system with two air outlets, double the mass flow is needed, in relation to a single duct, so the mass flow for the two-outlet network is  $2\dot{m}$ . The volume of can be given by the following equation:

$$V_t = 2 \frac{\pi \cdot d_0^2}{4} L_0 + \frac{\pi \cdot d_1^2}{4} L_0 \quad (14)$$

The pressure in the ducts for the case of the network with two outlets can be determined by:

$$\Delta P = \frac{8 \dot{m}^2 L_0}{\rho_{ha} \pi^2} \left[ \left( \frac{4f_1}{d_1^5} \right) + \left( \frac{f_0}{d_0^5} \right) \right] \quad (15)$$

The dimensionless enthalpy at the outlet of the duct is determined by:

$$\tilde{h}_{sai} = \exp \left( - \frac{\pi \cdot h_{cv} \cdot L_0}{C_{p,ha} \dot{m}_a} \left( \frac{d_1}{2} + d_0 \right) \right) \quad (16)$$

To optimize the diameters, the Lagrange multiplier method was used (STEWART, 1998). Assuming two restrictions, pressure and volume, the aggregate function is  $\beta = \Delta P(d_n) + \lambda V(d_n)$  where  $\lambda$  is the Lagrange multiplier. Its minimum is given by  $\partial\beta/\partial d_n = 0$ , so a value is assumed such that:

$$\beta = (d_0, d_1, \dots, d_{n-1}, \lambda) = P(d_0, d_1, \dots, d_{n-1}) + \lambda V_t(d_0, d_1, \dots, d_{n-1}) \quad (17)$$

Applying Lagrange multipliers for  $n = 2$ :

$$\beta = (d_0, d_1, \lambda) = P(d_0, d_1) + \lambda V_t(d_0, d_1) \quad (18)$$

The function  $\beta$  can be given by the equation below.

$$\beta = \frac{L_0}{4d_0^5} + \frac{L_0}{d_1^5} + \lambda(2d_0^2L_0 + d_1^2L_0) \quad (19)$$

Following the work of Brum et al. (2019b), the ratio between the diameters for a networked system with two air outlets is represented by:

$$\frac{d_1}{d_0} = 2^{3/7} \quad (20)$$

Then, the ratio between the diameters of the networked system ducts can be determined by the following equation:

$$\frac{d_{N+1}}{d_N} = R_d \quad (21)$$

The efficiency of the networked system can be determined by the following equation:

$$\tilde{\varepsilon} = \frac{1}{\Delta\bar{P}} \quad (22)$$

#### 4. RESULTS AND DISCUSSIONS

The work presented in this article shows the comparison of the single-pipe EAHE system with a networked system.

Figure 6 shows the pressure of an EAHE network system with two houses. We can notice an increase in the pressure of the ducts as we increase the branching, however the network pressure is less than the sum of the pressures in the EAHE of two houses without network arrangement. . If we multiply the pressure of  $n = 1$ , which represents a house, by the number of outlets in the network represented by  $n = 2$ , which are four air outlets, we obtain a value for the pressure much higher than the EAHE system in the network.

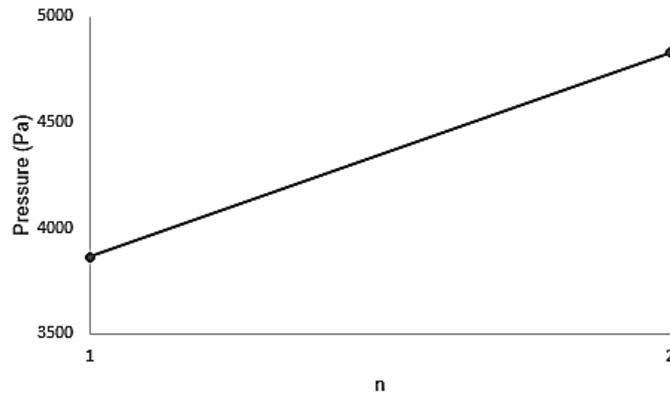


Figure 6 - Increased pressure in the network for different levels of bifurcation of the EAHE network

Figure 7 shows the dimensionless enthalpy at the outlet of the duct for one and two houses in a network, we observed a considerable decrease, which means that there was a greater exchange of heat in the network EAHE.

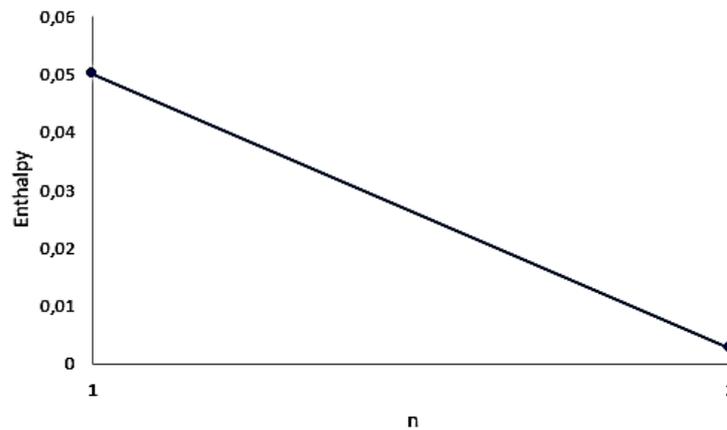


Figure 7 - Evolution of enthalpy for different levels of bifurcation

Figure 8 shows the result of the non-dimensional thermal efficiency of the EAHE system in network compared to a single duct presented by Eq. (22), indicating that the methodology of the network design can be applied in the study of the EAHE system.

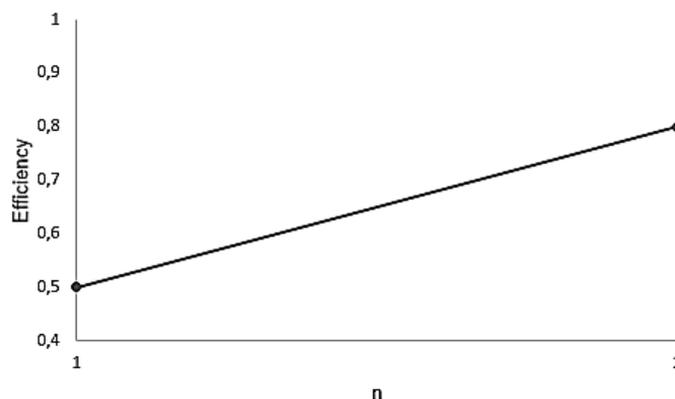


Figure 8 - Evolution of efficiency for different levels of bifurcation

## 5. PRELIMINARY CONCLUSIONS

The main conclusion of the analysis carried out in this article is that the use of a networked EAHE system is an effective way to achieve better levels of performance compared to the single-duct EAHE when being spoken at a district

scale. The networked EAHE proved to be better than the single duct design, even considering the humidity along the duct. Improvements were made in terms of thermal performance, which provides conventional energy savings.

Future work should consider a network with a greater number of bifurcations and climatic data for network analysis.

## 6. ACKNOWLEDGMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001”.

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