encit 2020

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

18th Brazilian Congress of Thermal Sciences and Engineering
November 16–20, 2020 (Online)

ENC-2020-0531
COMPARATIVE ANALYSIS OF DISCRETE AND CONTINUOUS SPECTRAL APPROACHES APPLIED ON HYDRODYNAMIC STABILITY

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Abstract. *Hydrodynamic instability has been used at a large amount of fluid mechanics problems. The matrix forming approach is nowadays the main methodology to solve the stability equations. In this one, the equations are spatially discretized getting a generalized eigenvalue problem. The discrete spectral approach has historically been the choice for that spatial discretization, due the optimal compromise between the highest accuracy and the amount of information to be stored. However, that positive aspect is reduced when multidimensional problems are considered. The present work proposes the use of continuous spectral formulations to test their viability in that type of problem and compare it with discrete formulations.*

Keywords: *stability analysis, matrix forming, spectral approach, GITT.*

1. INTRODUCTION

Direct numerical simulation (DNS) is not always the best way to understand numerically some problem classes in fluid mechanics. For example, the transition of fluid, the interaction between fuel/oxidant in combustion chambers of rocket engines, and so on. Hydrodynamic instability have been performed modeling the evolution of small disturbance into the flow in order to reduce the computational cost and simplify the understanding of the phenomenon, looking at the vortical structures still in their formation.

Stability equations are obtained decomposing the flow into a steady part called base flow, and an unsteady part. There are different forms to simplify the unsteady part of decomposition. At the strongest level of approximation, disturbances are assumed homogeneous in two out of the three spatial directions and the perturbations are expanded in terms of Fourier transform in one or two spatial dimensions. The matrix forming approach is a way to solve the stability equations called linearized Navier-Stokes Equations (LNSE). In this one, the equations are spatially discretized getting a generalized eigenvalue problem (Juniper *et al.*, 2014).

The spatial discretization plays a very important role in matrix storing and forming approach for solving eigenvalue problems. The choice of discretization affects directly the structure of the matrix and as a consequence in the numerical techniques employed. As a trend, spectral spatial discretization has historically been the choice for spatial discretization of the linear local stability. This is due the optimal compromise between the highest accuracy and the amount of information to be stored (Paredes *et al.*, 2013). There are in the literature a lot of suggestions to make the spatial discretization. Paredes *et al.* (2013) realized a comparison among different types of spatial discretization and showed the finite difference method is a strong competitive over other discretizations of the multidimensional eigenvalue problems. When multidimensional problems are considered the discrete spectral methods lose the advantage of low resolution and become necessary to allocate a large amount of memory (Theofilis, 2011).

A generalized integral transform technique (GITT) was employed successfully at stability problems by Sphaier and Barletta (2014) and Sphaier *et al.* (2015). They investigated the neutral stability at mixed convection in a heated horizontal and inclined porous channel, respectively.

The objective of this work is to deduce the continuous spectral formulation, through of GITT, to the Orr-Sommerfeld equations and to do a preliminary study about its advantage over the discrete ones. The Orr-Sommerfeld Equations is a fourth-order differential equation deduced from LNSE. At first, a simple case test considering local stability analysis and for future the plan is to extend this formulation for multidimensional eigenvalue problems where it is known that the discrete methodology has some disadvantages for 2D and severe issues for 3D cases.

2. METHODOLOGY

This section presents the mathematical formulation of the Orr-Sommerfeld equation. That has been used in order to evaluate the propagation of disturbances in the base flow. A temporal stability analysis is performed by using matrix forming method considering continuous and discrete spectral approaches.

2.1 Orr-Sommerfeld Equation

The incompressible equations in Cartesian coordinates with constant properties were used to derive the Orr-Sommerfeld equation. The dimensionless Navier Stokes equations are given by:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} \tag{2}$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector, p the pressure and Re the Reynolds number. For Cartesian coordinates,

$$\mathbf{u} \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{3}$$

The flow have been decomposed into the steady base flow (ϕ_b) and unsteady disturbance (ϕ_d) parts, where the disturbance is assumed to Fourier decompose into complex components typically of the form:

$$\mathbf{\Phi}_b(x, y, z) + \epsilon \mathbf{\Phi}_d(x, y, z, t) = u_b(y) + \epsilon \left(\hat{\mathbf{\Phi}}(y) e^{i(\alpha x + \beta z + -\omega t)} + cc. \right) \tag{4}$$

where $\mathbf{\Phi} = (u, v, w, p)$,$i = \sqrt{-1}$, and cc is the complex conjugate. The velocity as well as pressure disturbances u_d, v_d, w_d and p_d are described as functions of α, β and ω. The eigenfunctions $\hat{u}$, $\hat{v}$, $\hat{w}$ and $\hat{p}$ are considered as a local linear disturbance amplitudes, that is, depends just on the y direction.

Applying this decompositions, equation (4) into equation (1) to (2) and collecting the terms of $O(\epsilon)$ yields the LNSE :

$$\mathcal{L}(\hat{u}) + \frac{\partial u_b}{\partial y} \hat{v} + i\alpha \hat{p} = 0 \tag{5}$$

$$\mathcal{L}(\hat{v}) + \frac{\partial \hat{p}}{\partial y} = 0 \tag{6}$$

$$\frac{\partial w_b}{\partial y} \hat{v} + \mathcal{L}(\hat{w}) - i\beta \hat{p} = 0 \tag{7}$$

$$i\alpha \hat{u} + \frac{\partial \hat{v}}{\partial y} + \frac{\partial \hat{w}}{\partial z} = 0 \tag{8}$$

where $\mathcal{L}$ is the operator $\mathcal{L}(.) = i\alpha u_b(.) + i\beta w_b(.) - i\omega(.) - \frac{1}{Re}\left(\frac{\partial^2(.)}{\partial y^2} - \beta^2(.) - \alpha^2(.)\right)$. With some mathematical manipulations we can to write the equations (5 - 8) as one fourth-order differential equation.

$$\frac{i}{Re}\frac{d^4\hat{v}}{dy^4} + \left(\alpha u_b - 2k^2 \frac{i}{Re}\right)\frac{d^2\hat{v}}{dy^2} + \left(-\alpha k^2 u_b - \alpha \frac{d^2 u_b}{dy^2} + \frac{i}{Re}k^4\right)\hat{v} + k^2 \omega \hat{v} - \omega \frac{d^2\hat{v}}{dy^2} = 0 \tag{9}$$

where $k^2 = (\alpha^2 + \beta^2)$. Using the matrix forming method the equation (9) becomes a generalized eigenvalue problem $\mathbf{A}\hat{\phi} = \omega \mathbf{B}\hat{\phi}$, where $\mathbf{A}$ and $\mathbf{B}$ are given by:

$$\mathbf{A} = \left(\frac{i}{Re}\frac{d^4(.)}{dy^4} + \left(\alpha u_b - 2k^2\frac{i}{Re}\right)\frac{d^2(.)}{dy^2} + \left(-\alpha k^2 u_b - \alpha\frac{d^2 u_b}{dy^2} + \frac{i}{Re}k^4\right)(.)\right) \mathbf{B} = \left(\omega\frac{d^2(.)}{dy^2} - k^2\omega(.)\right). \tag{10}$$

Additional considerations must be made about the operators $\frac{d^4(.)}{dy^4}$ and $\frac{d^2(.)}{dy^2}$ in order to determine the generalized eigenvalue problem. In a regular basis discrete finite difference and spectral collection have been used in this one. Here only Gauss-Lobatto discrete spectral approximation is considered and compared to continuous spectral formulations.

2.2 Discrete Spectral Method

Spectral methods are a part of the most general weighted residual methods and it assumes that differential equation solution can be approximated with a truncated series expansion. We may expand a function u with an infinite sequence of orthogonal function as

$$u = \sum_{k=-\infty}^{\infty} \hat{u}_k \phi_k \tag{11}$$

where $\hat{u}_k$ is the expansion coefficient and ϕ_k is the orthogonal function. The orthogonal functions are usually trigonometric polynomials, Chebyshev polynomials, and Legendre polynomials (Canuto *et al.*, 2012).

Trigonometric functions are most used when the problem has periodic boundary conditions and its expansion is the knowled Fourier series, otherwise, we may use orthogonal polynomials over the interval $[-1, 1]$, in particular, the Chebyshev polynomials allow good approximations (Boyd, 2001; Hesthaven *et al.*, 2007; Canuto *et al.*, 2012). To expand any function u as in equation (11) in terms of orthogonal polynomial space with a truncated series of order N is performed

$$u_N(x) = \sum_{k=0}^{N} \hat{u}_k p_k(x) \qquad \text{with } \hat{u}_k = \frac{(u, p_k)_w}{\|p_k\|_w^2} = \frac{1}{\|p_k\|_w^2} \int_{-1}^{1} u(x) p_k(x) w(x) dx \tag{12}$$

where w is a weight function, p_k is a polynomial. Such that, u_N is an orthogonal projection of u in the polynomial space and the norm $\|u - u_N\|_w \to 0$ whenever $N \to \infty$ (Mendonça and Medeiros, 2008; Canuto *et al.*, 2012; Boyd, 2001). The Chebyshev polynomials $T_k(x)$ are orthogonal polynomials in the interval $[-1, 1]$ and they are the eigenfunctions of Sturm-Liouville problem

$$(\sqrt{1-x^2} T_k'(x))' + \frac{k^2}{\sqrt{1-x^2}} T_k(x) = 0 \qquad \text{where } T_k(x) = \cos k\theta \quad \text{and} \quad \theta = arc \cos x. \tag{13}$$

Therefore an expansion of $u(x)$ in Chebyshev space may be written as

$$u(x) = \sum_{k=0}^{\infty} \hat{u}_k T_k(x) \qquad \hat{u}_k = \frac{2}{\pi c_k} \int_{-1}^{1} u(x) T_k(x) w(x) dx \qquad \text{where } c_k = \begin{cases} 2 & \text{if } k = 0 \\ 1 & \text{if } k \geq 1 \end{cases}. \tag{14}$$

With the Chebyshev-Gauss-Lobatto discrete formula $x_j = cos \frac{\pi j}{N}$, a truncated series and a Lagrange polynomial interpolation of degree N we may write

$$u_N(x) = \sum_{k=0}^{N} h_k u(x_j) \tag{15}$$

where h_j is the Lagrange function defined by

$$h_j(x) = \frac{(-1)j + 1(1-x^2) T_N'(x)}{\bar{c}_j N^2 (x - x_j)}. \tag{16}$$

This expression can be derived and find the derivative operator in Chebyshev space

$$(\partial_n u)(x_j) = \sum_{k=0}^{N} h_k'(x_j) u(x_k) \tag{17}$$

where $h_j'(x_i)$ is a operator named Chebyshev pseudo-spectral matrix and is given

$$h_j'(x_i) = (D)_{ij} = \begin{cases} \frac{\bar{c}_i}{\bar{c}_j} \frac{(-1)^{i+j}}{x_i - x_j} & i \neq j \\ \frac{-x_j}{2(1-x_j^2)} & 1 \leq i = j \leq N-1 \\ \frac{2N^2+1}{6} & i = j = 0 \\ -\frac{2N^2+1}{6} & i = j = N. \end{cases} \tag{18}$$

and the derivate of U is calculated as $U' = DU$.

2.3 Continuous Spectral Method

The continuous spectral method is a Generalized Integral Transform Technique. To apply this approach in a differential equation we need orthogonal basis functions and the Sturm-Liouville eigenfunctions provide an orthogonal basis for a series solution (Chalhub *et al.*, 2013; Hahn and Özisik, 2012). An auxiliary eigenvalue problem may be used to generate a set of orthogonal functions, in this work, the eigenvalue problem is given by

$$\frac{d^4 z_m(y)}{dy^4} = \lambda_m^4 z_m(y) \tag{19}$$

with homogeneous boundary conditions $z_m(-1) = z_m(1) = z_m'(-1) = z_m'(1) = 0$. This eigenvalue problem is similar to the Chebybshev eigenvalue problem at equation (13) with polar transformation $x = cos(\theta)$, and the normalized eigenfunction can be stablished by

$$z_m(y) = \frac{\csc(\lambda)\text{sech}^4(\lambda)e^{\lambda(-y)}\left(-e^{3\lambda} + e^{\lambda+2\lambda y} + \cos(2\lambda)\left(e^{\lambda} - e^{\lambda(2y+3)}\right) + \left(e^{4\lambda} - 1\right)e^{\lambda y}\cos(\lambda(y+1))\right)}{4(\tanh(\lambda)+1)^2(\cot(\lambda)\tanh(\lambda)-1)(\sqrt{N_m})} + \frac{\csc(\lambda)\text{sech}^4(\lambda)e^{\lambda(-y)}\left(e^{\lambda}\sin(2\lambda) + \sin(2\lambda)e^{\lambda(2y+3)} - \left(e^{4\lambda}+1\right)e^{\lambda y}\sin(\lambda(y+1)) - 2e^{\lambda(y+2)}\sin(\lambda - \lambda y)\right)}{4(\tanh(\lambda)+1)^2(\cot(\lambda)\tanh(\lambda)-1)(\sqrt{N_m})} \tag{20}$$

where the norm is

$$N_m = \int_{-1}^{1} z_m(y)^2 dy, \tag{21}$$

and λ_m are the eigenvalues. We can calculate theses eigenvalues making the boundary conditions to be satisfied when $y = 1$

$$z_m'(1) = \sin(\lambda_m) + \cos(\lambda_m)\tanh(\lambda_m) = 0. \tag{22}$$

Therefore, each λ_m is calculated finding the roots of equation (22). With these eigenfunctions and eigenvalues for the auxiliary eigenvalue problem, we may define the inverse/transform pair

$$\hat{v}(y) = \sum_{m=1}^{\infty} \bar{v}_m z_m(y) \quad \text{and} \tag{23}$$

$$\bar{v}_m = \int_{-1}^{1} \hat{v}(y) z_m(y) dy, \tag{24}$$

and to introduce the inverse/transform pair into Orr-Sommerfeld equation (9). With this integral transformation, we can re-write the system with compact form (Alves *et al.*, 2002; Brandao *et al.*, 2014),

$$\sum_{n=1}^{N} \mathbf{A}_{m,n}\bar{v}_n = \omega \sum_{n=1}^{N} \mathbf{B}_{m,n}\bar{v}_n \tag{25}$$

where

$$\mathbf{A}_{m,n} = \frac{i}{Re}A_{m,n}^{(0)} + \alpha A_{m,n}^{(1)} - \frac{k^2 i}{Re}A_{m,n}^{(2)} - \alpha A_{m,n}^{(3)} - \alpha A_{m,n}^{(4)} + \frac{i}{Re}k^4 A_{m,n}^{(5)} \tag{26}$$

and

$$\mathbf{B}_{m,n} = B_{m,n}^{(0)} + k^2 B_{m,n}^{(1)}. \tag{27}$$

The integral transform coefficients are established by

$$A_{k,l}^{(0)} = \int_{-1}^{1} z_m(y) z_n^{''''}(y) dy, \tag{28}$$

$$A_{k,l}^{(1)} = \int_{-1}^{1} z_m(y) u_b(y) z_n^{''}(y) dy, \tag{29}$$

$$A^{(2)}_{k,l} = \int_{-1}^{1} z_m(y) z_n^{''}(y) dy, \tag{30}$$

$$A^{(3)}_{k,l} = \int_{-1}^{1} z_m(y) z_n(y) dy, \tag{31}$$

$$A^{(4)}_{k,l} = \int_{-1}^{1} u_b(y) z_m(y) z_n(y) dy, \tag{32}$$

$$A^{(5)}_{k,l} = \int_{-1}^{1} \frac{d^2 u_b}{dy^2} z_m(y) z_n(y) dy, \tag{33}$$

$$B^{(0)}_{k,l} = \int_{-1}^{1} z_m(y) z_n^{''}(y) dy, \tag{34}$$

$$B^{(1)}_{k,l} = \int_{-1}^{1} z_m(y) z_n(y) dy. \tag{35}$$

and the equation (25) can be solved as a generalized eigenvalue problem.

3. NUMERICAL RESULTS

The Poiseiulle problem has been chosen because this one has a domain in $y \in [-1, 1]$ with analytical solution $u_b(y) = 1 - y^2$ and does not need mapping to use the spectral Chebyshevy derivative. The generalized eigenvalue problem has been solved considering the Arnoldi method, where the matrix $\mathbf{A}$ is decomposed to obtain the Hessenberg matrix. This matrix has part of the eigenvalues of $\mathbf{A}$ called Ritz values. The spectrum of the Poiseiulle eigenvalues is presented in figure 1. To validate the discrete approach that result was compared with the solution by Kirchner (2000). For the continuous approach, we compare the solution by Orszag (1971) and the results can be seen in table 1.

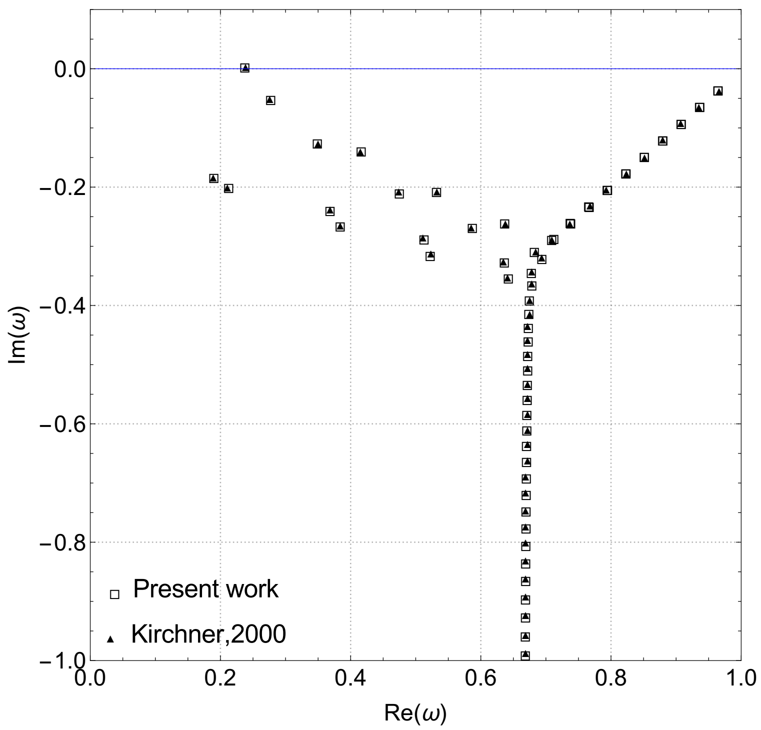

Figure 1. Poiseiulle spectrum for discrete formulation, $\alpha = 1$, $\beta = 0$ and $Re = 10^4$ and N=200.

Table 1. Frequency for continuous formulation, N=200, $\beta = 0$ and $Re = 10^4$.

$\alpha = 1$	Present work	Orszag (1971)
ω_r	0.237522	0.237526
ω_i	0.003739	0.003739

The series convergence was performed for three different modes, and these results may be seen in table 2. The results were compared with the Orszag (1971) solution. No changes in results can be seen when $N > 150$ and the error is smaller for stable modes and for unstable mode, the error scale was 10^{-6}.

Table 2. Eigenvalues convergence for continuous formulation, $\alpha = 1$, $\beta = 0$ and $Re = 10^4$.

N	ω_1	ω_2	ω_3
25	$0.23107151 + 0.01204925i$	$0.96491277 - 0.03514614i$	$0.27268915 - 0.01544196i$
50	$0.23752273 + 0.00373974i$	$0.96464251 - 0.03518658i$	$0.27719906 - 0.05090181i$
75	$0.23748016 + 0.00372472i$	$0.96464251 - 0.03518658i$	$0.27711006 - 0.05089659i$
100	$0.23751156 + 0.00373656i$	$0.96464251 - 0.03518658i$	$0.27718002 - 0.05089936i$
125	$0.23752033 + 0.00373889i$	$0.96464251 - 0.03518658i$	$0.27719411 - 0.05090138i$
150	$0.23752273 + 0.00373974i$	$0.96464250 - 0.03518658i$	$0.27719906 - 0.05090181i$
175	$0.23752273 + 0.00373974i$	$0.96464250 - 0.03518658i$	$0.27719906 - 0.05090181i$
200	$0.23752273 + 0.00373974i$	$0.96464250 - 0.03518658i$	$0.27719906 - 0.05090181i$

The Poiseiulle spectrum is showed in figure 2, in this one the discrete solution is a converged solution, as figure 1. The discrete spatial discretization was carried out with 200 points in the domain and will be used to compare with the continuous formulation. The most unstable modes have a good convergence for $N = 50$, in continuous approach, and when we increase N more modes were converged. For values of $N \geq 200$, the series is not able to converge other eigenvalues of family S of Poiseiulle spectrum. This limitation maybe to have relation with the orthogonal eigenfunction and other orthogonal eigenfunctions must be used to confirm this hypothesis.

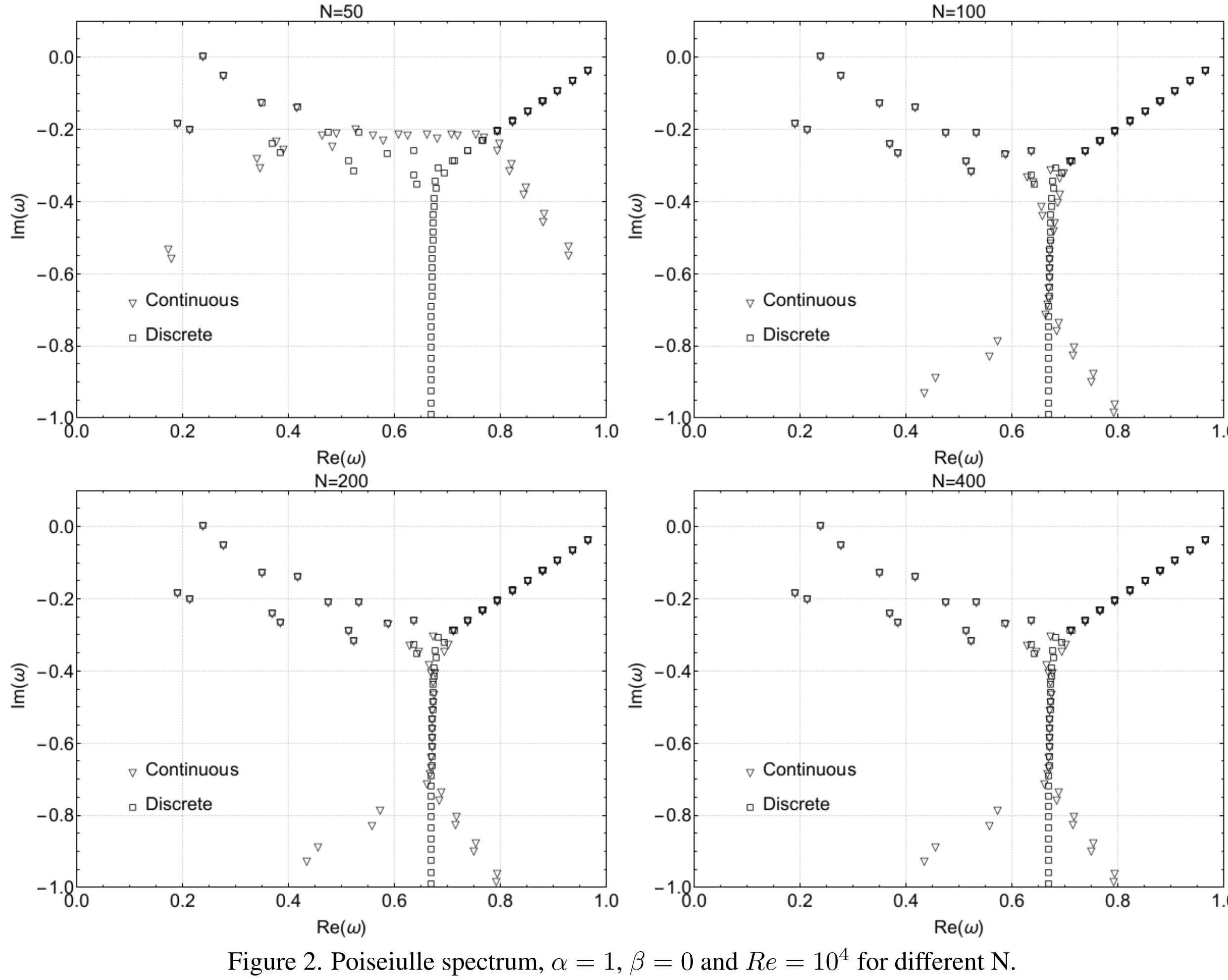

Figure 2. Poiseiulle spectrum, $\alpha = 1$, $\beta = 0$ and $Re = 10^4$ for different N.

The continuous formulation is capable of recovering, with good accuracy, the family A and the Family P of Poiseiulle spectrum, however, the family S was not recovered completely (Mack, 1976). These results indicate that the continuous formulation may be used in non modal analysis. The Chebyshev eigenfunctions are good eigenfunctions to perform a continuous approach and in a future work, these functions will be used and compare with a discrete version.

The figure3 shows the absolute error with respect to the CPU time. Here, the CPU time refers to the time to calculate the eigenvalues of generalized eigenvalue problem through matrix $\mathbf{A}$ and $\mathbf{B}$. From here, all results were calculated with the parameters $Re = 10^4$, $\alpha = 1$, $\beta = 0$. Initially, the continuous approach has some vantage, however, the discrete approach has a bigger convergence rate. The continuous formulations has demonstrated some difficult to converge the eigenvalues results, especially $Re(\omega)$, with the basis used.

That bigger convergence rate allows to the discrete approch to produce more accurate results, with a lesser time when compare to continuous. The accurate of continuous solutions depends substantially of orthogonal basis used in the integral transformations, and better basis can be founded with different auxiliary eigenvalues problems as equations (19).

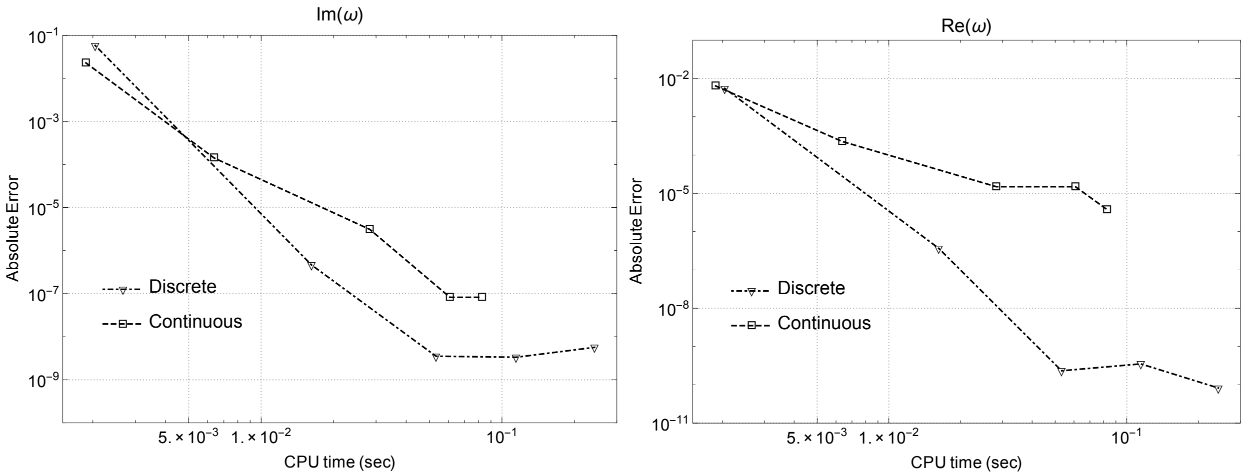

Figure 3. Left, absolute error for $Im(\omega)$ and right, absolute error for $Re(\omega)$, with $\alpha = 1$, $\beta = 0$ and $Re = 10^4$.

4. CONCLUSIONS

In this paper, two different spectral approaches were presented and all numerical results were discussed. The machine precision in the continuous formulation was smaller when compared with the discrete formulation and may be better choice when are accept a higher tolerance because it will be needed smaller CPU time.

When the integrals have analytical solutions, the continuous approach may have some advantage, but if the integrals have no analytical solutions, the continuous formulation may get worse, because it must numerically integration, increasing the CPU time.

Another point is the base flow, to calculate the integral transform coefficients with a different base flow will we must calculate all integrals again while in discrete formulation a new base flow can be used with no increase the CPU time. In future work, the time used to calculate the integrals must be included in a CPU time and furthermore, a Chebyshev continuous expansion will be carried out and a new comparative analysis will be performed.

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