18th Brazilian Congress of Thermal Sciences and Engineering
November 16-20, 2020 (Online)

ENC-2020-0393
NUMERICAL STUDY OF MASS TRANSFER BETWEEN TWO ALPHA STIRLING CYCLES

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Abstract. *The main characteristic of the Stirling cycle is external combustion, which does not depend on a specific heat source for the production of mechanical work. As a result, it has aroused interest in scientific studies on the transformation of thermal energy by renewable sources. The objective of the work is to present a numerical analysis of two outdated alpha Stirling cycles that exchange mass with each other in order to modulate the work performed. For this study, an algorithm for the thermodynamic system was written using the model proposed by Urieli that presents the cycle as being adiabatic, without loss of mass for the external environment, perfect regeneration and ideal gas. The first stage of this model is the simulation of the simple cycle and in a second moment it is necessary to solve the equations of the model of Urieli modified in the conditions of mass and energy balance, interconnecting the cycles by means of mass transfer considered instantaneous. at a given valve opening, adiabatic, isothermal and without friction losses. The results of the numerical analysis as being a simple cycle, were generated and compared with data from the literature, the cycle without mass transfer presented thermal efficiency of 60% and with mass transfer of 67%, this efficiency increase is due to the gain of energy from the hot source, which exceeds the value of work lost by the drop in pressure. There is a limit of mass transfer, so that the laws of physics are respected and the efficiency does not exceed numerically that of Carnot. In addition, it is necessary to pay attention to the fact that the valves must only be opened when there is a pressure difference between the cycles, allowing the passage of the mass from the higher pressure compartment to the lower one.*

Keywords: *Stirling Alfa, Mass transference, Urieli.*

1. INTRODUCTION

The environment is impacted in some way when we mention the production of energy for our consumption, especially when this energy comes from petroleum (JANUZZI, 2001). The Stirling engine was created in 1816 and uses a fluid confined inside, which is cooled and heated by external sources (burning biomass, solar energy, etc.), this temperature difference generates a compression and expansion of the fluid in each chamber thus moving the system's crankshaft, transforming thermal energy into mechanics. The source coming from this heating and cooling does not have a direct influence on the engine's operation and with that it has aroused interest in scientific studies on the transformation of thermal energy by renewable sources.

The present work aims to present a numerical analysis of two Stirling alpha cycles interconnected and out of phase transferring mass through a duct to each other, thus verifying the influence on parameters such as work, heat, power and efficiency. This combination causes the amount of heat that is removed from the hot source and that is not converted into work or stored in the regenerator, to be transferred to the other system. For this simulation, mathematical models of the Stirling cycles will be necessary, solved in computational software.

There are three configurations of Stirling engines according to the piston arrangement, namely: alpha, gamma and beta. The present work studies the alpha engine that requires two independent cylinders in a closed system, namely: compression cylinder (cold piston) and expansion cylinder (hot piston) (Figure 1), the arrangement is at 90º and the pistons work it is interconnected at a single point on the crankshaft. It is possible between these cylinders the presence of a regenerator that helps in the storage of heat, from the passage of fluid from the heater to the cooler.

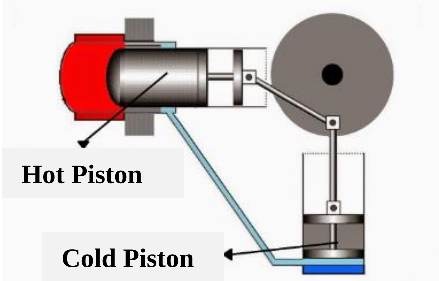

Figure 1. Alpha Stirling. Available from: http://manualdomotorstirling.blogspot.com/2014/04/como-e-um-motor-stirling-alfa.html

Some authors have studied mathematical models numerically used for Stirling engines, Santos (2015) compare in their work the adiabatic and non-adiabatic models in a Stirling engine, in addition to improving the modeling by inserting the loss through transport and imperfect regeneration, it comes to the conclusion that the adiabatic model is quite didactic to demonstrate the laws of thermodynamics. Lima et al. (2016) also compares in his work the adiabatic and non-adiabatic models considering heat loss through imperfect regeneration and transport, the models were simulated on an existing engine in the literature and demonstrated that the loss modeling has greater similarity with the data of the existing engine. Medina (2012) demonstrated the modeling of a Stirling engine, in order to optimize power and efficiency to supply energy in the isolated regions of Brazil from biomass, the ideal adiabatic model was selected and subsequently corrected with losses of internal and external heat. Andretta (2017) proposes a new two-temperature model based on Schmidt's theory, this model showed results similar to those developed by Schmidt. Drumond (2017) uses the models by Schmidt and Urieli for the modeling of a rotary Stirling engine, this comparison showed that both models bring satisfactory results, but compared to experimental data, they end up differentiating themselves by not considering losses in thermal processes. Iockheck (2015) developed in his work a concept of thermal machine, which operates with two thermodynamic cycles with gas in a closed circuit, transferring mass between them. This transfer occurs when the expansion of one cycle is completed to end the compression of the second cycle. According to the author, this transfer is able to increase the pressure and volume curves, thus increasing the graph area and the work done. In Iockheck's work, there is no modeling of the cycle according to the models proposed for the Stirling cycle.

As presented by Iockheck (2015) it is possible to combine thermodynamic cycles that work simultaneously, making each of the cycles refer to each other In view of this, the present work studies the instantaneous transfer of mass between two cycles. The process is described as: fluid expands at high temperature and starts the transfer process, part of the mass is transferred to the second unit of power. In the second power unit, the end of the compression process is taking place and the transfer to the heating process is initiated, with which already heated fluid transferred from the first cycle enters the system to the expansion chamber with the addition of thermal energy from the first cycle.

2. MATHEMATICAL MODEL

2.1 Stirling cycle

For the present work, the adiabatic model presented in the work of Drumond (2017) was used, which is based on the model of Urieli (1984). For the beginning of the modeling, the model must be divided into five control volumes, which are the compression space, expansion space, heater, cooler, and regenerator (figure 2) and make the considerations presented below:

a) Constant working mass;
b) Constant pressure in the system (there is no pressure loss);
c) The fluid is treated as an ideal gas;
d) Volume of expansion and compression are considered adiabatic;
e) Hot and cold compartments are treated as isotherms;
f) Formulation of the model governed by the angular variation of the combustion chamber crankshaft.

The consideration that all model information is governed by the angular variation of the combustion chamber crankshaft, brings an advantage in relation to the study of engines that relate the speed that is not constant in the system.

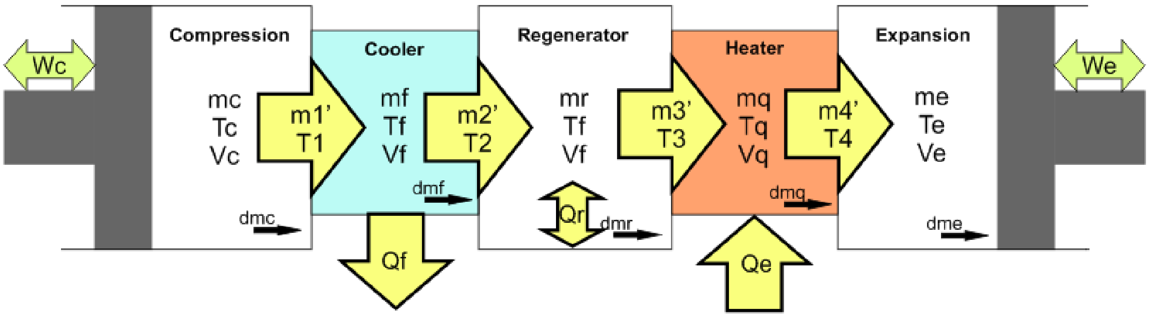

Figure 2. Engine compartments

The equations used in the Urieli (1984) model are shown in table 1.

Table 1. Equations of the Urieli model

$Ve=Vpe/2\,[1-\cos(\theta)]+Vme$ $Vc=Vpc/2\,[1-\cos(\theta-\beta)]+Vmc$ *Where:* *Ve = Expansion Volume* *Vc = Compression Volume* *θ = Lag angle* *Vme = Dead volume expansion* *Vmc = Dead compression volume*	(1) (2)	Volume Equations
$dP=(-kP(dVc/T1+dVe/T4))/(Vc/T1+k(Vf/Tf+Vr/Tr+Vq/Tq)+Ve/T4)$ $P=mtR/((Vc/Tc+Vf/Tf+Vr/Tr+Vq/Tq+Ve/Te))$ *Where:* *dP = Accumulated pressure* *P = Pressure* *k = cp / cv* *dVc = Compression Volume* *θ = Lag angle* *Vme = Dead volume expansion* *Vmc = Dead compression volume* *Tr = Temperature Regenerator* *Tc = Compression Temperature* *Te = Expansion Temperature* *Tf = Cold compartment temperature* *Tq = Hot compartment temperature* *mt = Total mass*	(3) (4)	Pressure Equations
$mq=PVq/(RTq)$ $mr=PVr/(RTr)$ $mf=PVf/(RTf)$ $me=PVe/(RTe)$ $mc=PVc/(RTc)$ *Where:* *R - Universal gas constant*	(5) (6) (7) (8) (9)	Mass Equations
$dmq=mqdP/P$ $dmr=mrdP/P$ $dmf=mfdP/P$ $dmc=((PdVc+Vc\,dP/k))/(RT1)$ $dme=((PdVe+Ve\,dP/k))/(RT4)$	(10) (11) (12) (13) (14)	Accumulated Mass Equations

Where: *mr = mass Regenerator* *mc = mass Compression* *me = mass Expansion* *mf = cold compartment mass* *mq = hot compartment mass*		
$m1' = -dmc$ $m2' = m1' - dmf$ $m3' = m2' - dmr$ $m4' = m3' - dmq$	(15) (16) (17) (18)	Mass Flow Equations
If $m1' > 0$ $T1 = Tc$ else $T1 = Tf$ If $m2' > 0$ $T2 = Tf$ else $T2 = Tr$ If $m3' > 0$ $T3 = Tr$ else $T3 = Tq$ If $m4' > 0$ $T4 = Tq$ else $T4 = Te$	(19) (20) (21) (22)	Temperature Conditions
$dTc = Tc(dP/P + dVc/Vc - dmc/mc)$ $dTe = Te(dP/P + dVe/Ve - dme/me)$	(23) (24)	Temperature Equations
$dQf = VfdP\ cv/R - cp(T1m1' - T2m2')$ $dQr = VrdP\ cv/R - cp(T2m2' - T3m3')$ $dQq = VrdP\ cv/R - cp(T3m3' - T4m4')$ $dWc = PdVc$ $dWe = PdVe$	(25) (26) (27) (28) (29)	Energy and Work

The present work modeled a Stirling cycle following the model proposed by Urieli (1984), because it is a model that, despite simplifications, resembles the real cycle. With the results obtained in the first stage of the modeling and the comparison with data obtained in the literature, it is possible to follow the numerical modeling of the next stage of the work, where the modeling will be with the interconnection of the two Stirling cycles by a mass transfer unit (CTM) (Figure 3).

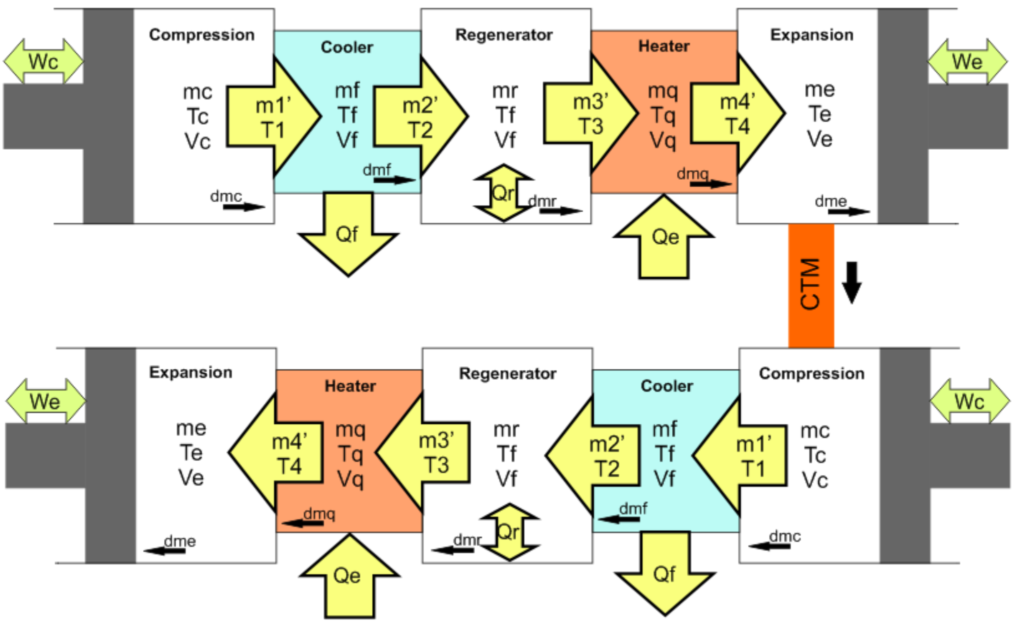

Figure 3. Control Volumes with mass transference

For modeling the transfer of mass from one cycle to the next, it is necessary to modify the equations of the simple cycle by adding the term of the transferred mass, with the following considerations:

a) Instant mass transfer without related losses;

b) Transfer respects the flow direction;
c) Isothermal transfer.
The affected equations are shown in table 2.

Table 2. Modified equations

dP=(-kP(dVc/T1+dVe/T4))/(Vc/T1+k(Vf/Tf+Vr/Tr+Vq/Tq+k(R/P)mes)+Ve/T4 *P=(mt-mes)*R/((Vc/Tc+Vf/Tf+Vr/Tr+Vq/Tq+Ve/Te))* *Where:* *mes = transferred mass*	(30) (31)
me=PVe/(RTe)-mes *mc=PVc/(RTc)+mes*	(32) (33)
dmc=((PdVc+Vc dP/k))/RT1+ dms *dme=((PdVe+Ve dP/k))/(RT4)+dms* *dms=mesdP/P*	(34) (35) (36)
m1'=dms-dmc *m4'=dme-dms*	(37) (38)

3. RESULTS

The first results obtained are consistent with the computational modeling of a Stirling cycle. For this, the Python programming language developed in the Spyder environment was used to model the simple cycle, without connection with mass transfer. The parameters used for modeling and later comparison with the literature were taken from Garcia's work (2018). Table 03 shows the parameters in this work.

Table 3. Parameters for computational modeling

Working Fluid	Air
Heating Temperature	1023K
Cooling Temperature	293K
Total mass	1,3000 x 10^{-3}kg
Regenerating volume	4,7262 x 10^{-4} m^3
Volume cooler	4,4450 x 10^{-5} m^3
Volume heater	1,3334 x 10^{-4} m^3
Dead volume exp.	1,5330 x 10^{-4} m^3
Dead volume comp.	1,5330 x 10^{-4} m^3
Swept volume exp.	5,2460 x 10^{-4} m^3
Swept volume comp.	5,2460 x 10^{-4} m^3
Initial pressure	15 bar
Mass transferred	0.2 g

The first results obtained are consistent with the computational modeling of the simple cycle, without and with mass transfer (Figures 4, 5 and 6). The parameters used for the modeling and the results compared with the literature were taken from Garcia's work (2018). In the first data analyzed with the mass transfer inserted, it is possible to observe the decline in the mass value at an angle 120º, when the transfer valve was opened in the code, this angle was chosen because we have a conditional on the pressure function, and this will be explain on the next step. This opening takes place at the same time that the compression work reverses the direction, that is to say that the piston is pushing the fluid in the opposite direction of the first cycle, thereby also pushing the fluid into the second cycle. This transfer should also interfere with the accumulated heat of the first and second cycles.

The amount of mass transferred must be transferred so that the mass calculation of the expansion is not negative, this means that, physically, it is not possible to transfer any amount of mass, as this would result in a non-real phenomenon, which would be and the inexistence of mass in the expansion cylinder, for this reason the code is limited to the transfer function of 0.2g

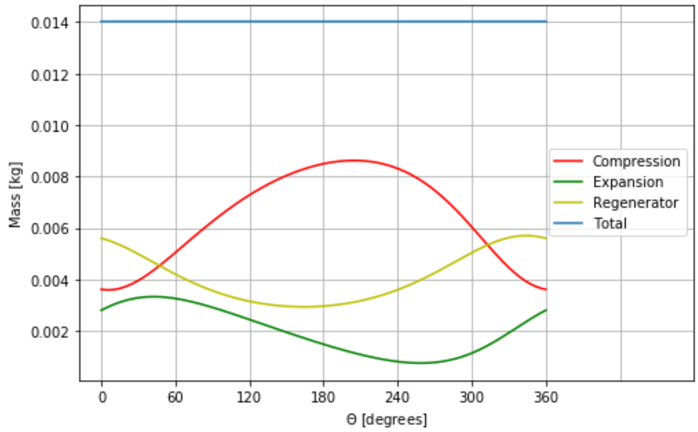

Figure 4. Mass Graph (simple cycle)

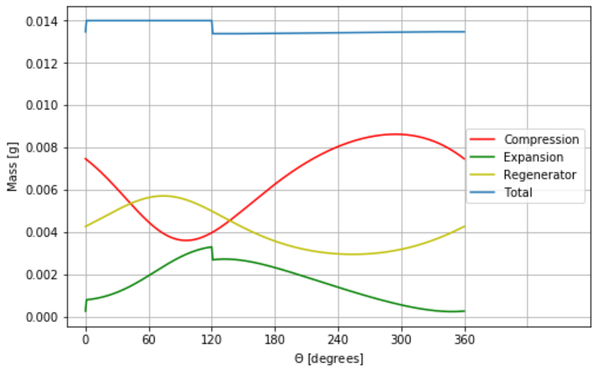

Figure 5. Mass Graph cycle 01 (with mass transfer)

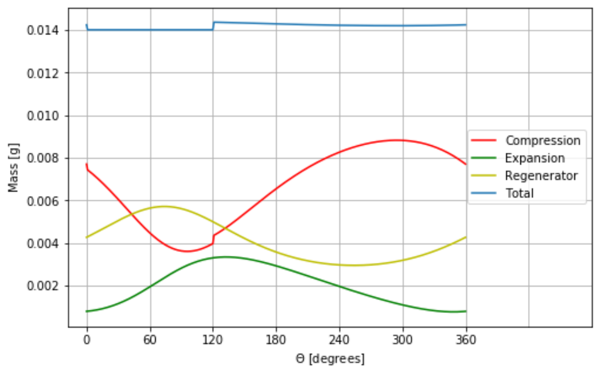

Figure 6. Mass Graph cycle 02 (with mass transfer)

For the mass transfer to take place, it is necessary to open the valve, this opening is conditioned to the pressure variable, since for the mass transfer, it is necessary that the expansion compartment of cycle 01 has greater pressure than the compression compartment of the cycle 02, this pressure difference is what made it possible for the mass to pass from one compartment to another. The selected angle was 120º, in figure 08 it is possible to notice that the two cycles are out of phase, and with that, there is a loss of pressure from cycle 01 to cycle 02, due to the mass transfer. The pressure gain must revert to the operation of the cycle, with that in 360º the cycle restarts. Figure 7 represents the cycle without transference of mass, it's possible to see that is a continuous curve.

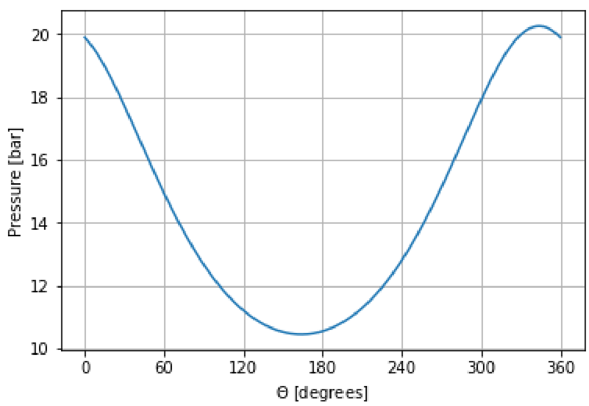

Figure 7. Pressure Graph (simple cycle)

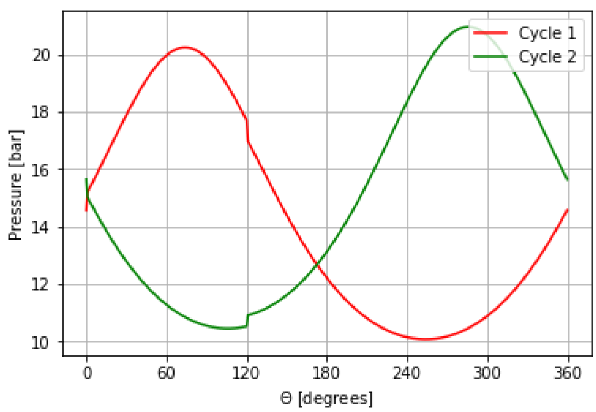

Figure 8. Pressure Graph (with transfer)

Another comparative data is the pressure x volume curve (Figures 7 and 8). The work can be calculated by the graph area, as well as by the sum of the work on the compression and expansion cylinder. It is possible to observe the increase and the pressure drop with the mass transfer. The same behavior occurs for both cycles 1 and 2, but there is a lag between them, so that the mass transfer occurs at the end of the expansion of cycle 01 to the beginning of compression of cycle 02.

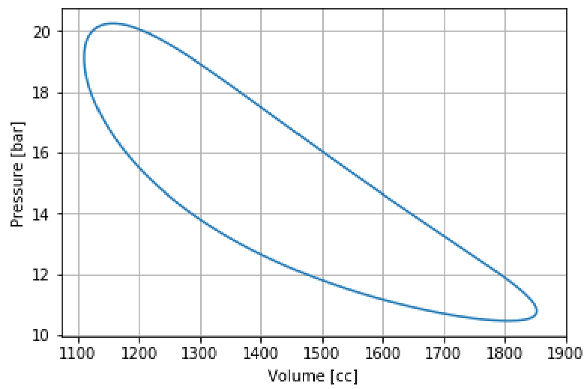

Figure 7. Pressure x Volume (simple cycle)

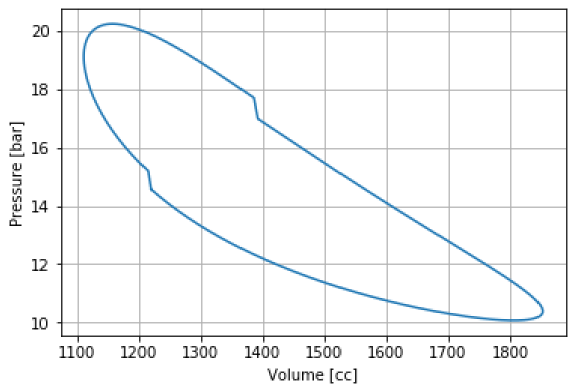

Figure 8. Pressure x Volume (with mass transfer)

The energy expenditure (Q_Q) occurs in the expansion chamber due to the heating of the fluid, the system receives heat already in the cooling chamber, there is heat loss (Q_f) by the system. The result for Garcia (2018) and the simple cycle is very similar, as well as the efficiency (ηt), a very similar value of 60% was obtained for both codes. The differences found in the results (table 1) are due to the difference in the arrangement of the volume equation used by Garcia (2018).

When results with cycle transferring mass are obtained, some differences are found. The work value (W_t) suffers a 15% decline, this is due to the fact that the work depends on the pressure and the volume, in the numerical value of the volume there is no difference between the cycles, however the pressure suffers an oscillation between the cycles. As a result, the indicated power of both cycles also undergoes an oscillation of 15%, since the calculation depends on the work performed at a given rotation.

The energy expenditure decreased, since the calculation of the heat required by the hot source depends on the pressure and the transferred masses. As part of the heat is transferred to the other cycle, being used by the hot source, the heat rejected by the cold source is less, since the cycle used less heat in the process. When calculating thermal efficiency (ηt) the value obtained is higher (67%), as the difference between energy expenditure and mechanical work reached a lower value, due to the drop in pressure. All the results can be observed in the table 4.

Table 4. Results obtained.

Variable	GARCIA (2018)	Simple Cycle	Cycle with transfer
W_t	273,6 J	279,5 J	236.7 J
Q_q	449,8 J	484,3 J	351.3 J
Q_f	-176,2 J	- 152,3 J	-52.6 J
η_t	0,60	0,58	0,67

4. CONCLUSION

The Carnot efficiency for this cycle is 71%, therefore the efficiency calculated as 60% in the simple cycle is lower, this is due to the fact that the cycle is calculated through the angle of the crankshaft, with this calculation adiabatic processes or isotherms of the Carnot cycle, taking the cycle a little closer to the real cycle. However, the data obtained are established only according to the Urieli model, and therefore did not take into account other considerations that can be made, such as losses in the regenerator, mass transfer abroad, heat loss, etc., these hypotheses they are not modeled in the present work and can considerably modify the efficiency of the cycle. The mass transfer reflects the result obtained with efficiency, since the difference in the mass calculation, will interfere in the calculation of the energy that the cycle receives from the hot source, thus increasing efficiency.

There is a limit of the mass transfer value, the present work used the value of 0.2 g, if the value is increased without estimation, the mass value of the expansion compartment can be negative in the software, but this phenomenon is not allowed physically. In addition, the mass value between 0.21 and 0.3 g causes the calculated efficiency value to exceed that of Carnot, a result that does not obey the physical laws. It is possible to notice that the indicated power of the cycle decreases, since the transferred mass changes the pressure calculation, consequently the mechanical work is less.

For mass transfer in the actual cycle to occur, it is necessary that this transfer occurs from the highest pressure compartment to the lowest pressure compartment. Therefore, the transfer angle must be selected to take this physical fact into account.

The main objective of this work was the development of a code that informs whether the mass transfer between two Stirling cycles at the end of the expansion contributes to factors such as: efficiency, indicated power, pressure, etc. With the results already obtained, it is possible to understand that the mass transferred between the cycles decreases the energy expenditure of the hot source of both power units, thus the thermal efficiency is higher, but the insertion of the losses of this process was not considered (friction, heat loss in the duct, etc.) in the simulation of the cycle so that the result approaches the real cycle. With that, it is still open the study of the insertion of these losses, to evaluate if this increase in the efficiency of the cycle is really viable.

5. REFERENCES

Cordeiro, J. S.; O Uso do Ciclo Stirling no Aproveitamento de Fontes Térmicas, 2002.Disponível em: <https://www.researchgate.net/profile/Ronaldo_Hoffmann/publication/267861904_O_USO_DO_CICLO_STIRLING_NO_APROVEITAMENTO_DE_FONTES_TERMICAS/links/5526b2450cf2e486ae408f59/O-USO-DO-CICLO-STIRLING-NO-APROVEITAMENTO-DE-FONTES-TERMICAS.pdf> Acesso em: 03 abr 2019.

Drumond, Carlo Cesar. Simulação Numérica de um Motor Stirling Rotativo. 2017. 70 f. Dissertação (Mestrado) - Curso de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2017. Disponível em: <https://www.maxwell.vrac.puc-rio.br/30089/30089.PDF>. Acessoem: 03 março 2019.

García, Miguel Torres et al. Mechanical Analysis of Genoa 03 Stirling Engine. Journal of Sustainable Development of Energy, Water and Environment Systems. Spain, p. 521-533. dez. 2018.

Iockheck, Marno. Desenvolvimento de máquina térmica de ciclo diferencial. 2015. 97 f. Dissertação (Mestrado) - Curso de Engenharia Mecânica, Pontifícia Universidade Católica do Paraná, Curitiba, 2015.

Januzzi, Gilberto De Martino. Energia e Meio Ambiente. 2001. Entrevista Revista Com Ciência. Disponível em: <http://www.comciencia.br/reportagens/energiaeletrica/energia12.htm>. Acesso em: 04 abr. 2019.

Keveney, Matt. Two Cylinder Stirling Engine. AnimatedEngines. 2011. Disponível em: http://www.animatedengines.com/vstirling.html. Acesso em: 22 abr. 2020.

Lima, Paulo Henrique de Freitas; SANTOS, Ana Carolina Ávila; CUNHA, Fábio Alfaia da; BRASIL, Augusto César de M. Análise de um motor stirling através de um modelo adiabático e de um não adiabático. In: CILAMCE, XVII., 2016, Brasília. Proceedings [...] . Brasília: Suzana Moreira Ávila, 2016. p. 1-20.

Medina, Juan Ricardo Vidal. Análise teórica do motor stirling amazon visando à otimização do desempenho. 2012. 217 f. Tese (Doutorado) - Curso de Engenharia Mecânica, Universidade Federal de Itajubá, Itajubá, 2012.

Urieli, Israel. Stirling Cycle Machine Analysis. Disponível em: <https://www.ohio.edu/mechanical/stirling/>. Acesso em: 05 jul. 2019.

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