encit 2020

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

18th Brazilian Congress of Thermal Sciences and Engineering
November 16–20, 2020 (Online)

ENC-2020-0488
HORIZONTAL AIR-WATER TWO PHASE SLUG FLOW: EULERIAN AND LAGRANGIAN DATA ANALYSIS

Conrado Chiarello
Hedilberto A. A. Barros
Moisés A. M. Neto
Rigoberto E. M. Morales
Multiphase Flow Research Center – NUEM, Graduate Program in Mechanical and Materials Engineering – PPGEM, Federal University of Technology – Paraná, 5000 Deputado Heitor Alencar Furtado Street, Curitiba, PR, Brazil.
conrado.chiarello@gmail.com, hedilbertobarros@gmail.com, mneto@utfpr.edu.br, rmorales@utfpr.edu.br

Abstract. *Multiphase flows are recurrent phenomena both in natural environments – such as volcanic eruptions and blood flow through veins – and industrial environments – such as the nuclear and petroleum industries. Knowledge about the hydrodynamics of the flow as well as its evolution across the pipeline is essential to correctly design pipelines, equipments (such as pumps and slug catchers) and to formulate strategies to optimize the crude oil exploitation from reservoirs. A frequently reported flow pattern in oil pipelines is slug flow, reason why the present study focuses in its investigation. In order to analyze the flow, a novel methodology is proposed and applied to data retrieved in Multiphase Flow Research Center (NUEM) experimental facilities. Data was analyzed using Eulerian referential, which provides distributions for each hydrodynamic parameter in each measuring station for every experimental point, and Lagrangian referential, which follows unit cells across the pipeline in order to evaluate the change in hydrodynamic parameters of single structures. Details of the flow were investigated using the proposed methodology, which was able to provide additional and insightful information about the flow behavior – such as the oscillatory behavior of slug lengths, which is masked by slightly changes on average values.*

Keywords: *multiphase flow, horizontal slug flow, Euler-Lagrange analysis.*

1. INTRODUCTION

Multiphase phenomena are present over a wide range of applications: besides industrial applications, such as petroleum and nuclear industry, multiphase flow is also present in geophysical phenomena – such as volcanic eruptions and river flow with small rocks and sand –, and blood flow in human veins. Often, multiphase systems are approximated by a two-phase flow in order to model the phenomena. Two phases, when flowing simultaneously in the same duct, tend to distribute themselves in some particular flow settings called flow patterns. Because flow pattern classification is subjective, nomenclature may vary among authors. The present work follows the nomenclature proposed by Shoham (2006) for horizontal flow: dispersed bubble, stratified smooth, stratified wavy, slug and annular flow. Among these flow patterns, slug flow is frequently reported in oil exploitation pipelines (Fanchi and Christiansen, 2016).

Slug flow is characterized by two structures which repeat themselves intermittently along the flow: the liquid slug and the elongated bubble, which, together, form a unit cell. The liquid slug carries the liquid phase and may have some small dispersed bubbles in it. The elongated bubble, on the other hand, is a region mostly taken by the gas phase, containing a liquid film that flows around the bubble in vertical flows or underneath it in horizontal flows. Behind the elongated bubble, there is a region of high instability caused by liquid recirculation, namely, the wake region.

The non-stationary nature of slug flow substantially increases the difficulty to obtain key flow parameters, such as pressure drop and flow frequency, which are essential to design pipeline equipment, as well as to formulate strategies to optimize crude oil exploitation from natural reservoirs. In order to reproduce flow conditions found in oil exploitation, a physical model may be used, since experimental approach is not feasible, given the extreme flow conditions. The model, however, must be validated using experimental data that reproduce a flow under milder conditions.

Due to its intermittent nature, slug flow has characteristic parameters: bubble translational velocity, flow frequency, bubble and liquid void fraction and lengths. These parameters are used in order to design pipelines and equipment attached to it, such as slug catchers (Mayor *et al.*, 2008).

Bubble translational velocity is a parameter of paramount importance, as it is often a key closure relationship in mechanistic models. Nicklin *et al.* (1961) found it to be a composition of the mixture velocity and the bubble drift velocity for vertical flows – which was confirmed by Bendiksen (1984) to hold to horizontal and inclined tubes as well. Dumitrescu

(1943) concluded that drift velocity Froude number is 0.351 for inertia-dominant vertical flows, while Benjamin (1968) observed a value of 0.54 for horizontal flows. Furthermore, Moissis and Griffith (1962) noticed that, whenever two bubbles are sufficiently close to each other, the leading bubble influences the trailing bubble velocity due to the so called wake effect – an acceleration of the trailing bubble caused by a distortion in the velocity profile ahead of it due to the wake on the tail of the leading bubble. Collins *et al.* (1978) provide theoretical support to the fact that the bubble velocity is directly affected by the velocity in front of its nose.

Slug length is another key parameter in multiphase flows, being critical to designing slug catchers, which do not depend on average slug length, but must consider the maximum liquid slug length instead. Statistical analysis, therefore, becomes an invaluable tool to investigate such phenomenon. Brill *et al.* (1981) reported that slug lengths follow a log-normal distribution, which is widely confirmed by many authors. Correlations for slug length, though, are quite unusual, as they have limited prediction accuracy; instead, numerical models are preferred, such as the slug tracking algorithm, first proposed by Barnea and Taitel (1993) and further improved by several authors, including Taitel and Barnea (2000), Al-Safran *et al.* (2004) and Rodrigues *et al.* (2009).

Two-phase horizontal slug flow is extensively researched in literature. Yet, it is not fully understood and there are still points open to discussion and further investigation. While experimental studies tend to focus on Eulerian referential, as the data analyzed is usually separated by measuring stations, numerical models usually require information from a Lagrangian referential, e.g. the behavior of unit cells across different stations. Furthermore, insightful information about flow phenomena may be obtained from Lagrangian referential analysis, such as the changes of individual unit cells' hydrodynamic parameters, which could be used in order to better understand the involved phenomena – often obfuscated by average values.

The present work aims to provide a novel methodology to analyze data in order to supply the required information of the flow. Data reported in literature does not often exceed one thousand diameters of pipeline length; on the other hand, data used in the present work goes over 1400D, allowing a more embracing description of the flow. Another major concern of this work is to provide accurate data from modern measuring systems, as most of the reference works in literature used equipment which would be far outdated nowadays. As many variables under different experimental conditions are evaluated, results will be reported for slug lengths, since that is the most common unknown parameter sought by numerical computations.

2. EXPERIMENTAL PROCEDURE

2.1 Experimental Apparatus

The data used in this work was retrieved by Barros *et al.* (2019) using a 35 m long and 0,026 m internal diameter experimental circuit at Multiphase Flow Research Center (NUEM) experimental facilities. The circuit is divided in two main sections: the test section – made of acrylic in order to visualize the flow – and the return pipeline, which aims to recirculate the liquid used in the study. A schematic representation of the experimental apparatus is shown in Fig. 1.

The experimental loop has five measuring stations – positioned at 390D, 538D, 615D, 807D and 1154D – equipped with two-wire sensors and pressure transducers in order to register the void fraction time series and to correct gas superficial velocity due to gas expansion. Both fluids are mixed using a parallel plate device in order to promote stratified flow at the tube entrance, as stratified flow tends to quickly develop to slug flow. Details of the experimental procedure and uncertainties are found in Barros *et al.* (2019).

Superficial liquid and gas velocities are chosen respecting operational limitations of the experimental apparatus, both for the lower (accuracy) and upper (maximum flow) limits, ensuring that slug flow conditions are found for each experimental point analyzed according to the Taitel and Dukler (1976) flow map. Gas superficial velocities range from 0.3 to 2.5 m/s and liquid superficial velocities range from 0.3 to 3.0 m/s.

2.2 Data Analysis

The time series retrieved is given in terms of voltage. The conversion to liquid height depends on the calibration of the sensor, which is made measuring single-phase flow voltage. It is assumed that voltage registered in two-phase flow varies linearly between two extremes – liquid and gas single phase flow –, as shown in Eq. 1.

$$h_L = \frac{U_{liq} - U}{U_{liq} - U_{gas}}, \tag{1}$$

in which U is the sensor voltage registered and h_L is the liquid height.

Conversion between liquid height and void fraction is given by Eq. 2, after Taitel and Dukler (1976).

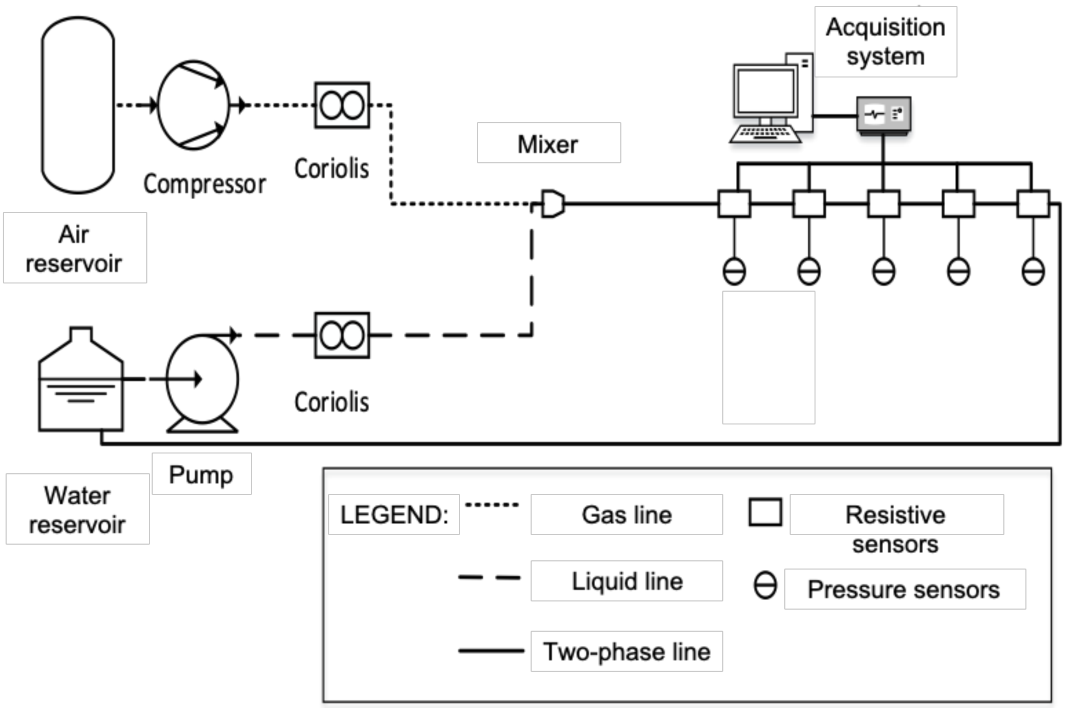

Figure 1. Schematic representation of the experimental circuit.

$$R_G = \frac{1}{\pi}\left[cos^{-1}\left(\frac{2h_L}{D} - 1\right) - \left(\frac{2h_L}{D} - 1\right)\sqrt{1 - \left(\frac{2h_L}{D} - 1\right)^2}\right], \quad (2)$$

where R_G is the void fraction and D is the tube diameter.

With void fraction time series, the analysis may proceed. The algorithm is organized in common routines for both approaches – unit cell detection and hydrodynamic parameters evaluation – and one specific routine for each method.

2.2.1 Unit Cell Detection

The first step towards flow analysis is to detect unit cells and to determine whether these detections are true or false positives. The detection itself is divided in six steps: coarse void fraction cutting factor, fine void fraction cutting factor, derivative sign change, minimum average void fraction, minimum length and detection fix.

In order to determine both coarse and fine void fraction cutting factors, the void fraction histogram is analyzed. Two-phase slug flow exhibits a bimodal histogram, where the first peak characterizes low void fraction flow, i.e. liquid slugs, and the second peak characterizes high void fraction flow, i.e. elongated bubbles. The histogram is divided in two and the fine cutting factor is defined as the peak of the first half of the histogram. The coarse cutting factor is defined as the average between the fine cutting factor and the half of the histogram. The coarse void fraction cutting factor is applied in order to differentiate bubbles and slugs. Bubbles are defined as the data above the coarse cutting factor and liquid slugs as the data below it. After applying the coarse cutting factor, the algorithm searches for the first correspondence before (for bubble nose) or after (for bubble tail) which is equal or lower than the fine cutting factor, improving the accuracy of the detection.

The next step in unit cell detection is to analyze the void fraction time series derivative. Whenever a bubble nose touches the sensor, void fraction tends to increase substantially up from a lower value. Similarly, whenever the bubble tail last touches the sensor, void fraction is expected to decrease substantially. The algorithm, hence, searches for the first detection before (for bubble nose) or after (for bubble tail) where the derivative change its sign or is equal to zero. This step is the last effort to improve detections.

After detecting unit cells, additional procedures are applied in order to eliminate false positives, namely, fake bubbles. The first additional procedure searches for bubbles with low average void fraction. An intermediate value between the two histogram peaks – the void fraction corresponding to the half of the histogram – is chosen, as a less restrictive value in order to delete as few real bubbles as possible. Bubbles with average void fraction equal or less than this value are considered fake bubbles and summarily deleted, being, therefore, incorporated to the adjacent slugs.

Shoham (2006) reports that very small bubbles (L < 1.2D) tend to break into smaller bubbles along the flow. Since the bubble length is calculated using the velocity, it is not possible, at this time, to determine the true length of the structures. One approximation used in order to evaluate lengths is to consider that every bubble in the flow has the velocity as proposed by Bendiksen (1984). Therefore, Lengths are approximated and bubbles with length equal or less than one tube diameter are considered false positives and deleted.

One last step to unit cell detection is to find incongruousness. The algorithm searches for bubble nose detections which happened at the same time or even after its tail. These are considered bad detections and are summarily deleted. Also, it is checked whether a bubble nose is registered at the same time or before its leading bubble tail. These detections are deleted in order to unify both bubbles.

2.2.2 Hydrodynamic Parameters Evaluation

In order to minimize distortions caused by the oscillatory movement of bubbles across the pipeline – as reported by Mao and Dukler (1989) –, mass conservation based on Taitel and Barnea (1990) model is used to evaluate bubble translational velocity. Assuming that liquid slugs travel at mixture velocity, bubble translational velocity may be calculated as per Eq. 3.

$$V_{TB} = \frac{N_U[J_L - J(1 - R_{GS})]}{R_{GS}N_F - \int_0^{N_F} R_{GB} dN}, \tag{3}$$

where V_{TB} is the bubble translational velocity, N_U is the unit cell length in number of acquisitions, N_F is the length of the film region in number of acquisitions, R_{GS} is the void fraction in the slug length region and R_{GB} is the void fraction in the elongated bubble region.

Similar methodology is applied by Fossa *et al.* (2003), whose compared the phase continuity calculation with measurements from video frames. A maximum deviation of about 15% was observed, which is a satisfactory accuracy to the present work, given the limitations of the other methods. Since mass conservation must be achieved in order to use the retrieved data to validate numerical models, the continuity method is applied to velocity calculations in this work.

The velocity retrieved for single bubbles is assumed to be the same for the whole unit cell. Fabre (2003) supports that this is a valid assumption, as bubble and slug velocities are narrowly distributed around an average. This assumption will affect bubble and liquid slug length only, but has no effect in frequency, average void fraction or intermittence factor. The calculation of subsequent parameters is straightforward, as demonstrated in Eq. 4-7.

$$f_U = \frac{f_s}{N_U}, \tag{4}$$

$$L_k = \frac{V_{TB} N_k}{f_s}, \tag{5}$$

$$\beta = \frac{N_F}{N_U}, \tag{6}$$

$$R_{Gk} = \frac{1}{n}\sum_{i=1}^{n} \bar{R}_{Gk,i}, \tag{7}$$

where f_s is the sensor acquisition frequency, f_U is the unit-cell frequency – not to be confounded with flow frequency, as demonstrated by Conte *et al.* (2017) –, L is the length of the structure, N is the length in number of acquisitions, k stands for the phase – either liquid or gas –, F stands for film (elongated bubble) region, U stands for the whole unit-cell, R_{Gk} is the void fraction in phase k, $\bar{R}_{Gk,i}$ is the i-th structure average void fraction and n is the number of structures.

2.2.3 Eulerian referential analysis

After retrieving every hydrodynamic parameter, the first step to the Eulerian analysis is to fit statistical distributions using the results. Even though there is enough literature background regarding statistical distributions for specific parameters, it is up to the analyst to choose the distribution which better fits the data retrieved. The chi-square method is a quantitative criterion to evaluate, given the significance level, whether the statistical distribution fits well to results.

The next step in Eulerian analysis is to evaluate average values of the hydrodynamic parameters, as well as their standard deviations. Retrieving these values allows the search for correlations of these variables as a function of given parameters, such as superficial velocities or mixture velocity, for example. Engineering correlations of this kind may be used in field operations as well as in numerical simulations. Once more, it is up to the analyst to choose the correct parameters in order to obtain useful engineering correlations.

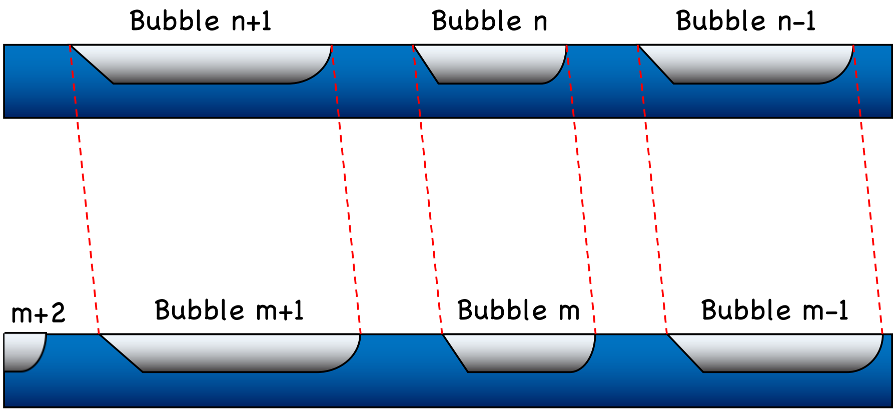

Figure 2. Nomenclature used by the matching algorithm.

2.2.4 Lagrangian referential analysis

The goal of Lagrangian analysis is to follow unit cells along the flow in order to analyze changes in their hydrodynamic parameters, rather than analyzing average values and their standard deviations. The first step to this approach, therefore, is to track the same unit cell across different stations. Bubble and liquid slugs may not keep their characteristic parameters, such as length and void fraction, along the pipe. As bubbles may coalesce, liquid slugs may experiment drastic reduction in their size. Fortunately, most of the structures do not deform significantly across different stations and the signals are similar enough to be aligned.

In order to align different signals, the cross-correlation is used to retrieve their average delay. Since all the stations start to gather acquisitions at the same time, it is necessary to trim the signals: some unit cells in the first station would not be registered in the last station as some in the last measuring station would have passed without being registered in the first station. The final acquisitions of the first measuring stations and the initial acquisitions of the last measuring stations are, hence, trimmed out of the analysis.

In order to identify unit cells across different measuring stations, a new concept is introduced: the discrepancy between signals. This quantity is defined in such a way that higher amplitude would not imply in higher correlation – which often happens using a simple cross-correlation and may lead to wrong results. Instead, the better the correspondence, the lower the discrepancy factor. The discrepancy factor is defined in Eq. 8.

$$D_x^y = \left| 1 - \frac{R(x,y)}{R(x,x)} \right|, \tag{8}$$

where D_x^y is the discrepancy factor between signals x and y and R is the cross-correlation peak value.

Discrepancy factors are compared for the current unit cell and the two adjacent ones: its leading and trailing unit cells. Additionally, one more unit cell from the second signal is used in the comparisons as a margin of safety. Figure 2 depicts the nomenclature used to define analyzed unit cells – represented by its bubbles – and Fig. 3 summarizes the algorithm to find the best match, where discrepancies are compared.

After properly identified across the pipeline, a correspondence matrix is used in order to compare the hydrodynamic parameters of the unit cell in every measuring station. Hence, evolution of hydrodynamic parameters may be retrieved for individual unit cells rather than analyzing average values. Furthermore, it is possible to analyze the behavior of unit cells after a coalescence takes place, which is a critical matter for numerical models.

3. RESULTS AND DISCUSSION

Results will be presented separately for each experimental approach, but the discussion will regard both – as they are not exclusive, but complementary. As there are many data points processed and results, a sample point with moderate aeration (J_L = 1 m/s, J_G = 1 m/s) is chosen to show the capabilities of the applied procedure. Moreover, since there are many different hydrodynamic parameters to analyze, slug length is chosen, as one of the main goals of the novel methodology proposed is to establish a more fair comparison between experimental data and numerical models.

Before presenting average results, it is important to check whether the analyzed data set is enough to describe the flow; in other words, data must be statistically independent to effectively represent the studied phenomena. Therefore, cumulative average values and standard deviations are investigated, as statistical independence is attained when average values converge. Cumulative analysis should be carried for every experimental point. A sample experimental point cumulative velocity and standard deviation is chosen in order to represent the procedure, as shown in Fig. 4.

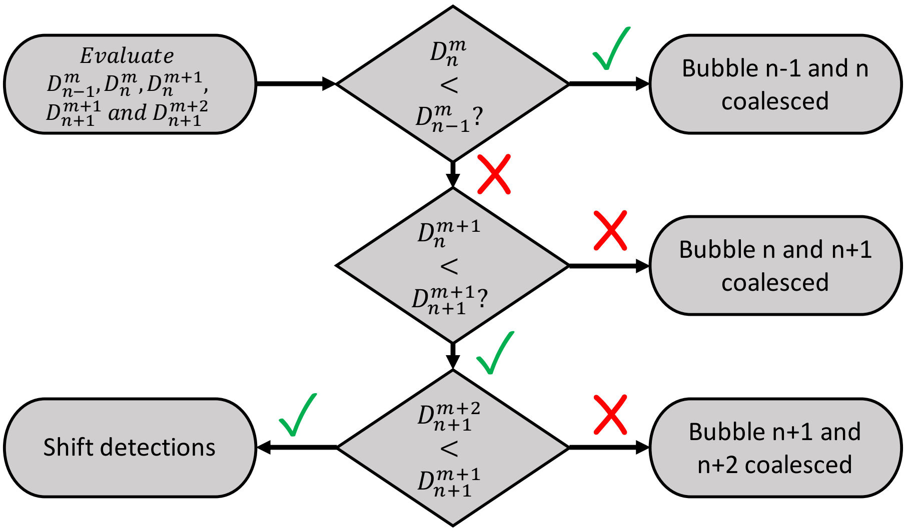

Figure 3. Flowchart of the matching algorithm.

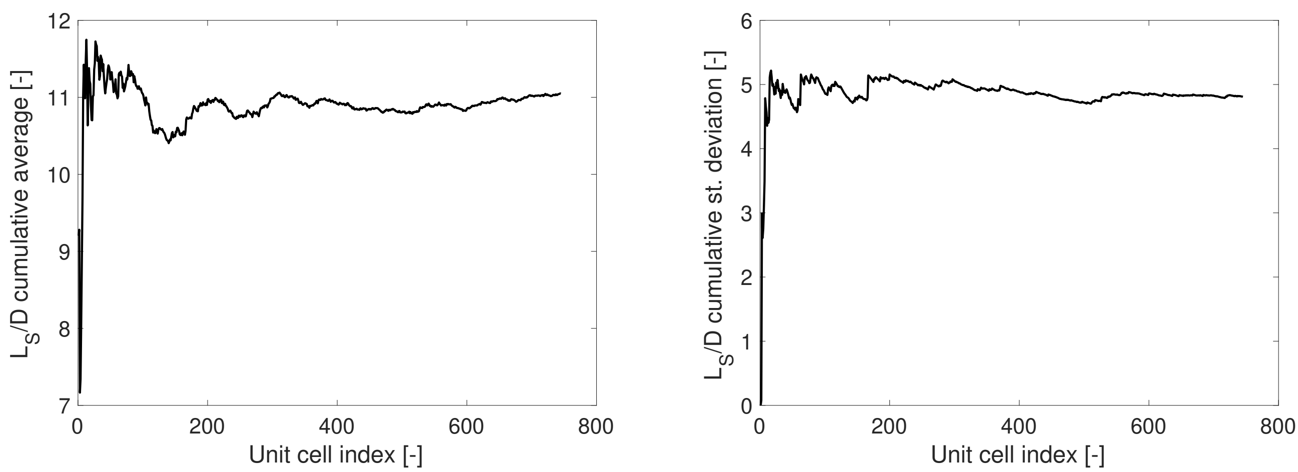

Figure 4. Slug length cumulative average and standard deviation.

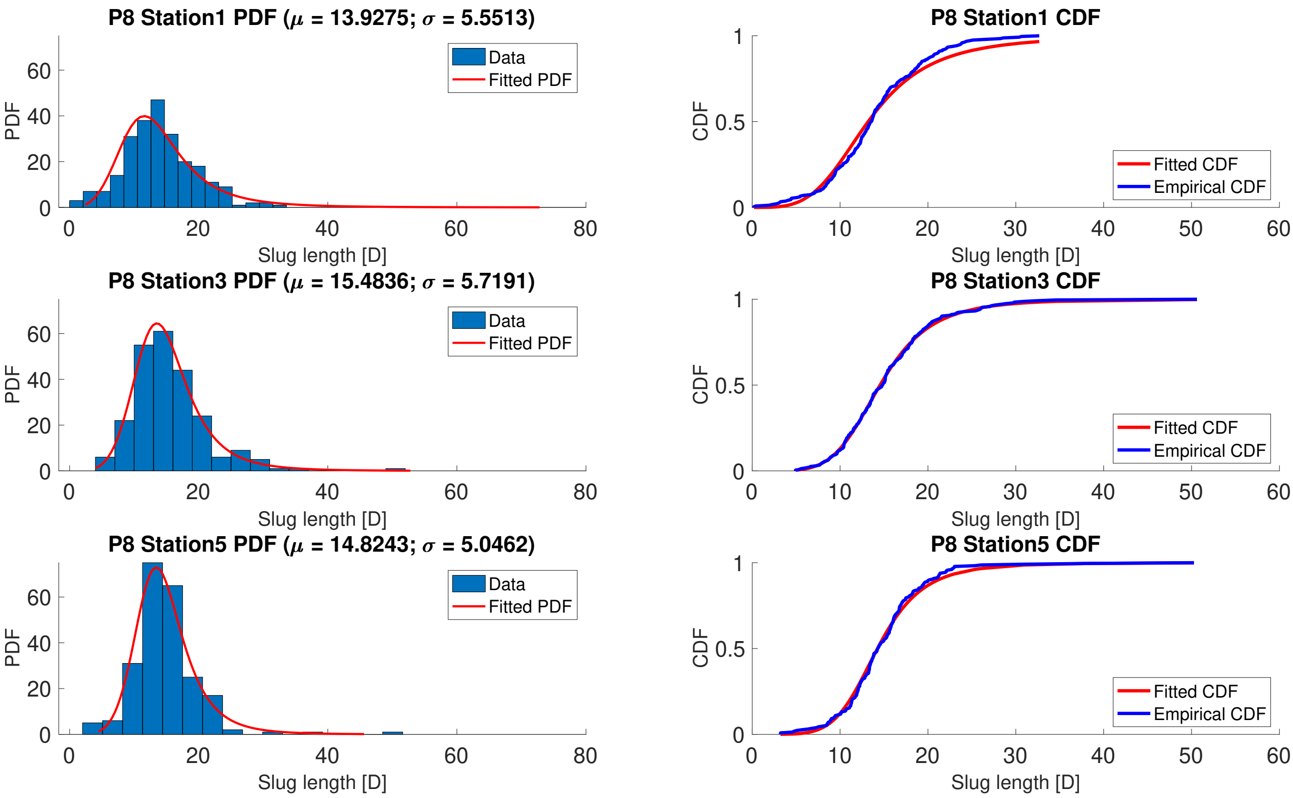

Figure 5. Histograms, PDFs, CDFs and log-logistic fits for slug length.

Noticeable spikes are found when the number of samples is low. With increasing samples, though, it can be seen that the slug length average, as well as its standard deviation, tends to converge and does not vary significantly for more than 400 samples. Aside from ensuring data validation, this result may be used by researchers going for image processing and analysis, which may reduce the total processing time by picking only enough samples to process and analyze.

Slug length histograms along with fitted probability density functions (PDF) and cumulative density functions (CDF) are shown in Fig. 5. Even though literature suggests a log-normal distribution for this parameter, log-logistic distribution was chosen to represent slug length distribution, as it was found to better represent the peak and the heavier tails.

Figure 6 exposes Lagrangian referential results for slug length.

Although Eulerian referential results report little change in slug length average (μ) and standard deviation (σ) – which would suggest a steady flow –, Lagrangian referential results demonstrate the ongoing changes: roughly half of the slugs have their length decreased whilst the other half experiments opposite behavior. A simple mass balance is enough to explain this phenomenon: since bubble velocities are not equal, as bubbles get closer to each other, liquid slugs tend to decrease; the liquid, though, does not disappear instantly, but is rather placed in the next liquid slug – hence, when a liquid slug length decreases, another liquid slug should increase. The process keeps going back and forth across the pipeline. Gas expansion, responsible for bubble expansion, can also play a minor role in the process.

The alternative behavior on slug length difference exposes another phenomenon ongoing during the flow: the oscillatory behavior of bubbles, as reported by Mao and Dukler (1989). Slug length can be understood as the distance between two consecutive bubbles. Hence, velocities may be correlated according to Eq. 9.

$$V_{TB}^{trailing} = V_{TB}^{leading} - \frac{dL_S}{dt}. \tag{9}$$

As liquid slug lengths oscillate back and forth across the pipeline due to liquid mass conservation, bubbles are expected to oscillate in a similar fashion.

4. CONCLUSION

The present study has proposed a novel methodology for data analysis, which can be used simultaneously to the widely used Eulerian analysis proposed in literature. Moreover, the proposed methodology was used in order to suit numerical simulations needs, extracting experimental parameters which would otherwise be impossible to measure – such as the behavior and phenomena behind the slug length oscillation.

The current work inserts itself in an interesting gap: although the phenomena investigated with the Lagrangian ref-

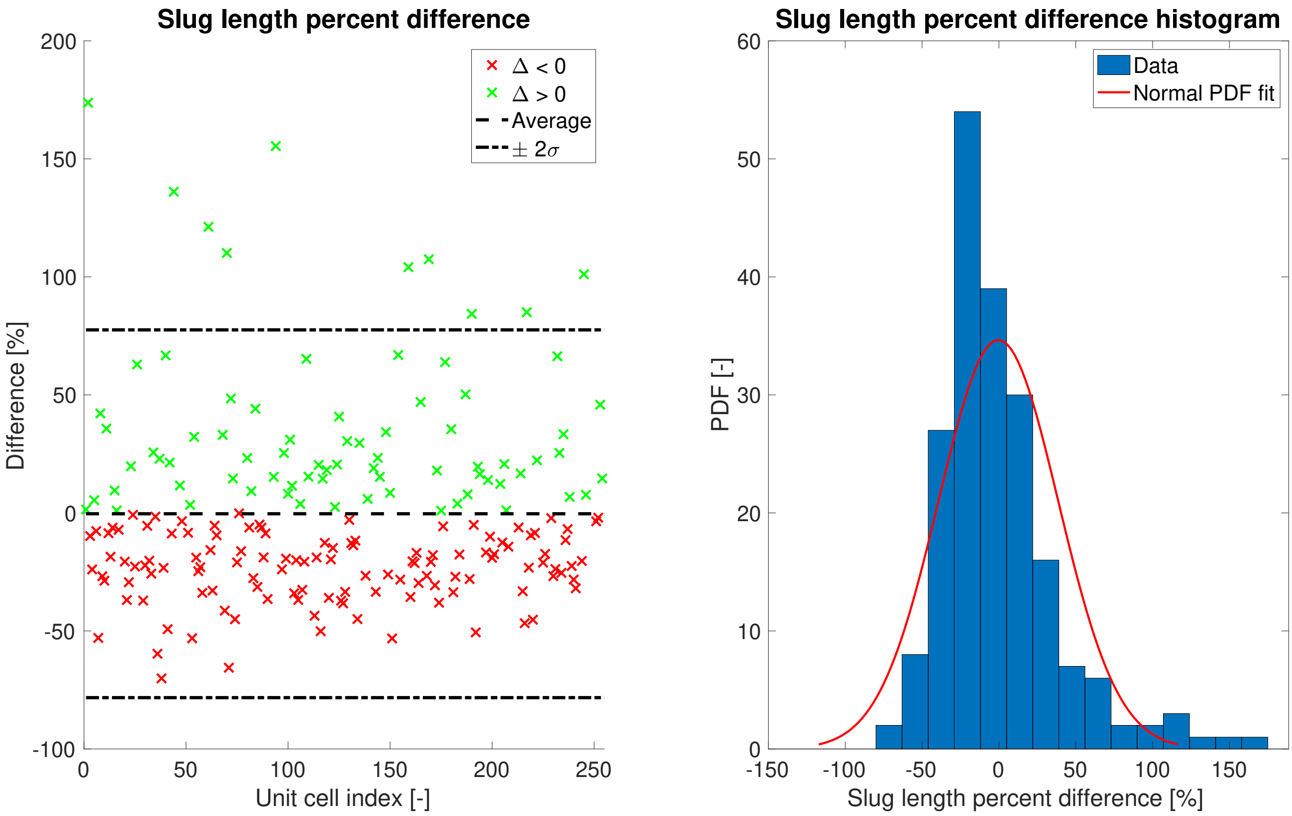

Figure 6. Slug length percent difference, histogram and normal fit between stations 1 and 5

erence frame are widely explored by numerical simulations, there are no experimental studies capable of provide these details in order to validate the findings. Numerical procedures often evaluate unit-cell hydrodynamic parameters and evaluate the histogram afterwards in order to compare to Eulerian experimental results, which could be hiding discrepancies that may end up compensating themselves – culminating to accurate average values, but failing to test the accuracy of the model itself.

Applying the proposed methodology, it was found that averaging results may mask an alternative behavior – intrinsic to the unsteadiness of slug flow – which could be limiting physical interpretation of phenomena. Slug lengths were found to oscillate at all times – even though their average was shown to differ slightly between measuring stations –, indicating that bubble movement would oscillate as well, which, indeed, is reported in literature.

5. ACKNOWLEDGEMENTS

The authors acknowledge the financial support of Petrobras to the development of this project.

6. REFERENCES

Al-Safran, E.M., Taitel, Y. and Brill, J.P., 2004. “Prediction of slug length distribution along a hilly terrain pipeline using slug tracking model”. *Journal of Energy Resources Technology, Transactions of the ASME*, Vol. 126, No. 1. ISSN 01950738. doi:10.1115/1.1649971.

Barnea, D. and Taitel, Y., 1993. “A model for slug length distribution in gas-liquid slug flow”. *International Journal of Multiphase Flow*, Vol. 19, No. 5. ISSN 03019322. doi:10.1016/0301-9322(93)90046-W.

Barros, H., Andrade, C.V., Czelusniak, F., Alves, R.F., Cozin, C., Marcelino Neto, M. and Morales, R., 2019. “Experimental analysis of two-phase slug flow evolution with a slight upward direction change”. In *Cong. B. Eng. Mec.* doi:10.26678/abcm.cobem2019.cob2019-1166.

Bendiksen, K.H., 1984. “An experimental investigation of the motion of long bubbles in inclined tubes”. *International Journal of Multiphase Flow*, Vol. 10, No. 4, pp. 467–483. ISSN 03019322. doi:10.1016/0301-9322(84)90057-0.

Benjamin, T.B., 1968. “Gravity currents and related phenomena”. *Journal of Fluid Mechanics*, Vol. 31, No. 2. ISSN 14697645. doi:10.1017/S0022112068000133.

Brill, J.P., Schmidt, Z., Coberly, W.A., Herring, J.D. and Moore, D.W., 1981. “Analysis of two-phase tests in large-diameter flow lines in Prudhoe Bay field”. *Society of Petroleum Engineers journal*, Vol. 21, No. 3. ISSN 01977520. doi:10.2118/8305-PA.

Collins, R., De Moraes, F.F., Davidson, J.F. and Harrison, D., 1978. "The motion of a large gas bubble rising through liquid flowing in a tube". *Journal of Fluid Mechanics*, Vol. 89, No. 3. ISSN 14697645. doi:10.1017/S0022112078002700.

Conte, M.G., Hegde, G.A., da Silva, M.J., Sum, A.K. and Morales, R.E., 2017. "Characterization of slug initiation for horizontal air-water two-phase flow". *Experimental Thermal and Fluid Science*, Vol. 87. ISSN 08941777. doi: 10.1016/j.expthermflusci.2017.04.023.

Dumitrescu, D.T., 1943. "Strömung an einer Luftblase im senkrechten Rohr". *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 23, No. 3. ISSN 15214001. doi:10.1002/zamm.19430230303.

Fabre, J., 2003. "Gas-Liquid Slug Flow". pp. 117–156. doi:10.1007/978-3-7091-2538-0_3.

Fanchi, J.R. and Christiansen, R.L., 2016. *Introduction to petroleum engineering*. ISBN 9781119193463. doi: 10.1002/9781119193463.

Fossa, M., Guglielmini, G. and Marchitto, A., 2003. "Intermittent flow parameters from void fraction analysis". *Flow Measurement and Instrumentation*, Vol. 14, No. 4-5. ISSN 09555986. doi:10.1016/S0955-5986(03)00021-9.

Mao, Z.S. and Dukler, A.E., 1989. "Experiments in Fluids An experimental study of gas-liquid slug flow". Technical report.

Mayor, T.S., Ferreira, V., Pinto, A.M. and Campos, J.B., 2008. "Hydrodynamics of gas-liquid slug flow along vertical pipes in turbulent regime-An experimental study". *International Journal of Heat and Fluid Flow*, Vol. 29, No. 4, pp. 1039–1053. ISSN 0142727X. doi:10.1016/j.ijheatfluidflow.2008.02.013.

Moissis, R. and Griffith, P., 1962. "Entrance effects in a two-phase slug flow". *Journal of Heat Transfer*, Vol. 84, No. 1. ISSN 15288943. doi:10.1115/1.3684284.

Nicklin, D., Wilkes, J. and Davidson, J., 1961. "Two-phase slug flow in vertical tubes". *Trans. Inst. of Chemical Engrs*, Vol. 40, p. 61.

Rodrigues, H., Morales, R., Rosa, E. and Mazza, R., 2009. "Modeling and simulation of two-phase horizontal and vertical slug flow patterns with a slug tracking numerical model". *Boletim Tecnico da Producao de Petroleo*, Vol. 4, No. 1, pp. 105–124. ISSN 18096751.

Shoham, O., 2006. "Mechanistic Modeling of Gas-Liquid Two-Phase Flow in Pipes". *Society of Petroleum Engineers*, p. 2006.

Taitel, Y. and Barnea, D., 2000. "Slug-tracking model for hilly terrain pipelines". *SPE Journal*, Vol. 5, No. 1. ISSN 1086055X. doi:10.2118/61445-PA.

Taitel, Y. and Barnea, D., 1990. "Two-Phase Slug Flow". *Advances in Heat Transfer*, Vol. 20, No. C. ISSN 00652717. doi:10.1016/S0065-2717(08)70026-1.

Taitel, Y. and Dukler, A.E., 1976. "A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow". *AIChE Journal*, Vol. 22, No. 1. ISSN 15475905. doi:10.1002/aic.690220105.

7. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.