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## NUMERICAL MODELING OF HEAT AND MASS TRANSFER DURING THE WOOD DRYING PROCESS

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**Abstract.** Wood is widely used, due to its diversity of species and physical-chemical characteristics, for furniture manufacturing, packaging, and many other products. However, to improve the final quality of the product and to minimize the probability of the occurrence of fungi and bacteria, or defects in its structure in the final product, there is a need to carry out the wood drying process. This process occurs through the transfer of heat and mass between air and wood. Therefore, mathematical models assist in the control and analysis of this process parameters, enabling an assertive analysis about the quality of the wood product regarding its drying rate. For this reason, the present paper aims to study the behaviour of pine wood in its drying process through the development of computational code, using the TDMA solver, with time-stepping time scanning, and the Matlab software, solving the equations decoupled and applying the finite difference method. This study includes a literature review, code development, validation and refining of the proposed model, analysis of the pinewood drying process and verification of the results. This model was compared with the code proposed by Simo-Tagne *et al.* (2016), which evaluated Frake wood. With that, the results of the two species were compared. Furthermore, drying was studied considering the condition of Dirichlet and Neumann. The result obtained was a code capable of evaluating the drying process with satisfaction, a product with moisture content lower than 25%, with wood type Frake and Pine having similar behaviour.

**Keywords:** numerical methods, wood, mass transfer, heat transfer, finite difference, computational modelling.

### 1. INTRODUCTION

The applicability of wood is mainly due to the great diversity of species and the different drying processes, once each type of wood has anatomical characteristics and thermophysical properties that directly influence its industrial use (Barmpoutis *et al.*, 2018).

Moisture removal helps to eliminate fungi and pests, reduce stains and rot. Also, it facilitates handling, transportation, some processes, such as sanding and makes dimensional variation difficult. Other important factors improved with the wood drying are the improvement of the mechanical resistance, thermal, acoustic and electrical insulation properties (Bonin, 2017). Therefore, drying is an essential process for importation and exportation of wood, once it eliminates the risk of transporting and transferring harmful microorganisms that can threaten some native species to the destination countries.

That is, drying increases the biological durability of the wood by removing water from the product and provides greater dimensional stability (Kuzman *et al.*, 2015). Since, the chances of suffering mechanical defects, colouring problems and damage caused by fungi and other pests are mitigated when the present moisture in the wood is less than 20% (Woodproducts, 2020).

However, once it is a delicate and complex process, all operating conditions must be adequately controlled, especially drying speed and temperature, to make the process effective, and thus avoid any damage to the product (Perré *et al.*, 2014). Since the wood first loses surface moisture, from the edges towards the centre, it generates tension and traction gradients, and compression perpendicular to the fibres, due to the moisture gradient that causes different shrinkage (Santos, 2012). It means that the wood shrinks during drying and develops deformations and stresses that can lead to unusable products, once dimensional changes are directly impacted by the moisture content present in the wood (Perré *et al.*, 2014). Therefore, the main defects are cracks, warping, hardening and collapse (Santos, 2012).

Considering that the drying occurs by the surficial water removal, due to the heat transfer between the wood and the air and, by mass transfer, as it is the displacement of the wood internal water to its surface and later, to the air current (Santos, 2012). Computer simulation helps to trace the wood characteristics, establishing the process model that best suits its conditions, indicating which parameters are adopted, as well as the behaviour as the drying process occurs. The objective of this paper is to analyze the processes of heat and mass transfer during the drying of pine-type wood,

developing a computational code, to investigate, relate and control the influences of existing variables in this process on the final characteristics of the product. Therefore, the objective is to relate the drying time to the percentage of moisture in the wood, evaluating in 2 different boundary conditions. Therefore, the first is the condition of Neumann and the second the condition of Dirichlet, that is, prescribed temperature and humidity.

## 2. MODEL

The physical problem consists of a piece of wood, which can be represented in the Cartesian system with three-dimensional geometry, being: length (Z), width (Y) and thickness (W). For the development of the computational code, simplifications were applied to the computational model. These are:

- (i) Heat and mass transfer of the lateral faces are disregarded, that is, one-dimensional domain;
- (ii) Liquid water and air are considered incompressible;
- (iii) The samples are homogeneous; chemically inert and with constant sizes;
- (iv) The losses inherent to the equipment are negligible;
- (v) The gravity effect and hydraulic conductivity of the wood are neglected.
- (vi) At the beginning of the drying process, the free water (on the sample surface) evaporates. On the other hand, bound (chained) water is extracted at the moment it reaches the saturation point of the fibres.

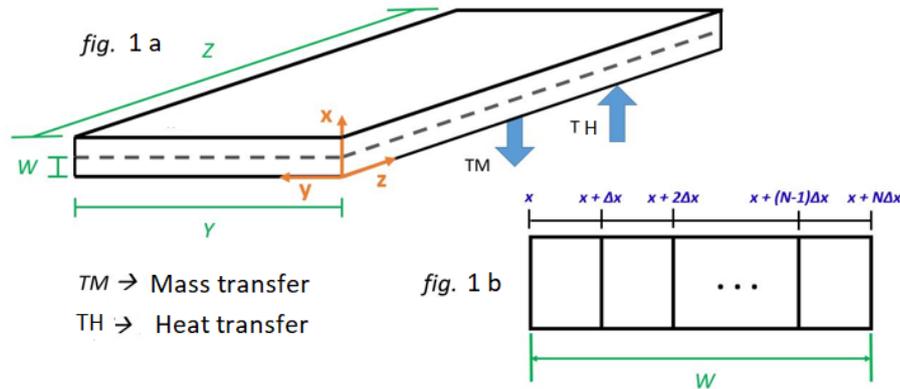


Figure 1. Three-dimensional and one-dimensional geometry representing a piece of wood.

The heat transfer equation (Simo-Tagne *et al.*, 2016) is expressed by,

$$\frac{\partial T}{\partial t} \rho_s C_p = (\lambda + \alpha_t \rho(E + L)D_H) \frac{\partial^2 T}{\partial x^2} + \rho(E + L)D_H \frac{\partial^2 H}{\partial x^2} \quad (1)$$

where T is the temperature (K), t is the time (s),  $\rho_s$  is the specific mass of dry wood ( $\text{kg}/\text{m}^3$ ),  $C_p$  is the specific heat of the wood ( $\text{kJ}/(\text{kg}\cdot\text{K})$ ),  $\lambda$  is the thermal conductivity of wood ( $\text{W}/(\text{m}\cdot\text{K})$ ),  $\alpha_t$  is the solet effect coefficient (K-1),  $\rho$  is the specific mass of wet wood ( $\text{kg}/\text{m}^3$ ),  $E$  is the heat desorption of the adsorbed water ( $\text{J}/\text{kg}$ ),  $L$  is the heat latent vaporization ( $\text{J}/\text{kg}$ ),  $D_H$  is the binary mass diffusion coefficient of wood and water ( $\text{m}^2/\text{s}$ ), x is the position (m), H is the wood pores average humidity (%),  $h_c$  is the convection heat transfer coefficient ( $\text{W}/(\text{m}^2\cdot\text{K})$ ) and  $T_{ar}$  is the air temperature (K).

The initial condition will be  $T(x, 0) = 10^\circ\text{C}$ .

Regarding the boundary conditions we have:

### Case 1:

$$\frac{\partial T}{\partial x} \lambda = h_c(T_{ar} - T) + \rho_s L D_H \frac{\partial H}{\partial x} \text{ in } x=0 \quad (2)$$

**Case 2:**

$$T(0, t) = 305 \text{ K} \quad (3)$$

The mass transfer equation (Simo-Tagne *et al.*, 2016) is calculated by

$$\frac{\partial H}{\partial t} = D_H \frac{\partial^2 H}{\partial x^2} + \alpha_t D_H \frac{\partial^2 T}{\partial x^2} \quad (4)$$

Which the initial condition will be  $H(x, 0) = 75\%$ .  
Regarding the boundary conditions, we have:

**Case 1:**

$$D_H \frac{\partial H}{\partial x} = h_m (X_{eq} - H) \text{ in } x = 0 \quad (5)$$

where  $X_{eq}$  is the water content in the balance (kg of water/kg of dry wood) and  $h_m$  is the mass transfer coefficient by convection (m/s).

**Case 2:**

$$H(0,t) = 12\% \quad (6)$$

In the middle plane, we have the symmetry condition that is, in  $x = W$ , the heat flow and the moisture diffusion flow are null, as follow,

$$\frac{\partial T}{\partial x} = 0 \quad (7)$$

$$\frac{\partial H}{\partial x} = 0 \quad (8)$$

**2.1 Simulation Parameters**

In this study, a unidimensional medium with symmetry and the transient regime will be used. The simulation parameters were based on the study by Simo-Tagne *et al.* (2016). These are: for an air velocity equal to 0.25 m / s there is a laminar flow, which has the number of Reynolds ranging from 237 to 2300; air temperature equal to 25°C; relative humidity equal to 50%, the sample size is 36.5×11.7×3.0 cm, but will be transformed into one-dimensional, measuring 3.0 cm in length; initial humidity equal to 75%, the initial wood temperature of 10°C and drying process duration of 300h. Besides was used the TDMA (Tri Diagonal Matrix Algorithm) to the solution of the linear system resulting in the discretization of both equations, heat and mass transfer. Time-stepping and MATLAB software as a solver, solving the equations decoupled and applying the finite difference method. Beyond that, other parameters are shown in Tab. 1 that are important to the solution of the process.

Table 1. Process parameters to Pine wood.

Parameters	Expressions
Thermal conductivity of wood (W/(mK))	$\lambda = \frac{\rho}{\rho_t} (0.2003 + 0.00548H) + 0.02378 \text{ H in } \%$
Specific heat capacity of wood (kJ/(kgK))	$C_p = \frac{C_{po} + 0.01 H C_{pw}}{1 + 0.01 H} + H(-0.06191 + 2.36 * 10^{-4}T - 1.33 * 10^{-4}H)$ $C_{po} = 0.1031 + 0.003867 T \text{ and } C_{pw} = 4.19 \text{ kJ}/(\text{kg K}) \text{ where } T(\text{K})$
Water content at the fibres saturation point (kg/kg)	$H_s = 0.3161 - 1.327 * 10^{-3}(T - 273.15) \text{ where } T(\text{K})$
Latent heat of vaporization (J/kg)	$L = (3335 - 2.91 T) * 10^3 T \text{ where } T(\text{K})$
Heat desorption of the adsorbed water (J/kg)	$E = 1170.4 * 10^3 \exp(-0.14 H) \text{ H in } \%$
Mass diffusion coefficient of the wood (m <sup>2</sup> /s)	$D_H = 5.22 * 10^{-10}$
Density of the water (kg/m <sup>3</sup> )	$\rho_t = -0.0038 T^2 - 0.0505 T + 1002.6$

Activation energy of the wood (J/mol)	$E_b = 4.18 (9200 - 7000 X_{eq})$
Equilibrium water content (kg/kg)	$X_{eq} = \frac{1}{0.09764 T - 11.533384} \ln \left[ - \frac{\ln (HR)}{-0.03738 T + 16.35834} \right]$ where T(K) and HR in decimal
Effect solet coefficients (K <sup>-1</sup> )	$\alpha_t = \frac{-E_b}{(0.09764 T - 11.533384) R T^2 \ln (HR)}$
Universal gases constante (J/(mol K))	R = 8.314
Mass transfer coefficient (m/s)	$h_m = 1.312 \times 10^{-7}$
Air velocity in the drying-cupboard (dryer) (m/s)	$V_a = 0.25$
Heat transfer coefficient (W/(m <sup>2</sup> K))	$h_c = 6.72$
Density of wood (kg/m <sup>3</sup> )	$\rho_s = 500$
Density of wet wood (kg/m <sup>3</sup> )	$\rho = 2.357 H + 450.6 H \text{ in } \%$

### 3 NUMERICAL METHOD

To solve the problem equations, the Finite Difference Method (MDF) was applied, which is a numerical method used for numerical calculations for the partial derivatives present in differential equations, which has as a basic principle the approximation through algebraic expressions, based on the expansion of the Taylor series. Each one of the terms obtained from the mathematical model is according to the distribution of the computational mesh (Mariani et al., 2008; Perussello et al., 2011; Perussello et al., 2013; Fontana, 2019).

For this, an implicit method was adopted because it has greater numerical stability. Considering that the meshes in time and space are evenly spaced and calculated as, respectively,  $\theta = j\Delta t$ , where  $\Delta t$  is a non-zero constant,  $j = 0, 1, 2, \dots, N_t$  where  $N_t$  is the number of steps in time, while  $x_i = i\Delta x$ , that is,  $x_{i+1} - x_i \equiv \Delta x$ ,  $\forall i \in \{0, 1, 2, \dots, N-1\}$ , where  $\Delta x$  is a non-zero constant that represents the spacing between the nodes of the numerical grid of the x-axis and  $N$  is the total space intervals,  $N = W/\Delta x$ , where  $W$  is the total length of the wood studied piece. Thus, the sample is divided into  $2N+2$ . Thus, the values of the function  $T(x, t)$ , see Eq. (1), and  $H(x, t)$ , see Eq. (4) in nodes discrete are defined by  $T_i^j \stackrel{\text{def}}{=} T(x_i, \theta)$  e  $H_i^j \stackrel{\text{def}}{=} H(x_i, \theta)$  and, still applying TDMA have the equations in the discrete form for heat and mass transfer, respectively, presented by

$$(1+2B) T(i,j+1) = B [T(i-1,j+1) - 2T(i,j+1) + T(i+1,j+1)] + C [H(i-1,j) - 2H(i,j) + H(i+1,j)] + T(i,j) \quad (9)$$

$$(1+2A) H(i,j+1) = AH(i-1, j+1) + AH(i+1, j+1) + D[T(i-1,j) - 2T(i,j) + T(i+1,j)] + H(i,j) \quad (10)$$

Where:

$$A = \frac{D_H \Delta t}{\Delta x^2} \quad (11)$$

$$B = \frac{\Delta t (\lambda + \alpha_t \rho (E + L) D_H)}{\rho_s C_p \Delta x^2} \quad (12)$$

$$C = \frac{\Delta t \rho (E + L) D_H}{\rho_s C_p \Delta x^2} \quad (13)$$

$$D = \frac{\alpha_t D_H \Delta t}{\Delta x^2} \quad (14)$$

The code validation was performed through the study by Simo-Tagne *et al.* (2016), which studies the one-dimensional mass and heat transfer for the wood species *Frake (Terminalia Superba)* and *Ayous (Triplochiton Scleroxylon)*, whose results are evaluated in drying time, humidity percentage and temperature of the samples.

After validating the model, a study of the computational mesh was carried out to refine it and improve the accuracy of temperature and humidity values. This verification was done by modifying the number of nodes that constitute it. Therefore, the mesh was tested with 12 (coarse mesh), 48 (intermediate mesh) and 192 (refined mesh) knots. Such quantities were defined following a refining standard of 4.

Also, to verify the convergence for quantitative analysis, the performance indicators were calculated, which are R<sup>2</sup> (coefficient of determination) and RMSE (Root-mean-square-error) according to

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \widehat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (15)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \widehat{y}_i)^2} \quad (16)$$

where n is the number of observations,  $\bar{y}$  is the mean,  $y_i$  and  $\widehat{y}_i$  are the i-th observed and predicted values, respectively

#### 4 RESULTS AND DISCUSSION

The developed code was validated with the code proposed by Simo-Tagne et al. (2016). Because of that, it was tested with the parameters of the Frake wood, which were shown in the article by Simo-Tagne et al. (2016). The result obtained was similar, as can be seen in Figs. 2 and 3, to results for temperature and moisture, respectively.

Furthermore, in Tab. 2 you can check the values of the performance indicators:  $R^2$  and RMSE related to each of the computational meshes. Based on this, it was identified that the values become more accurate with the use of the refined mesh, composed of 192 knots.

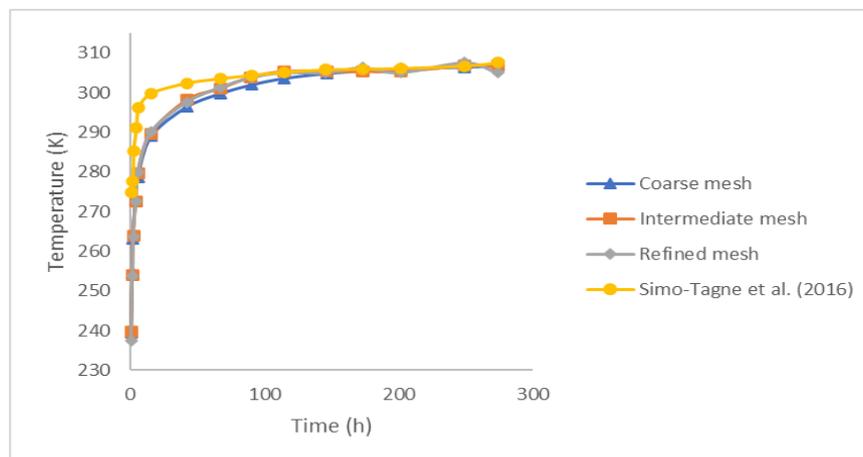


Figure 2. Temperature profiles over mesh study to Frake wood.

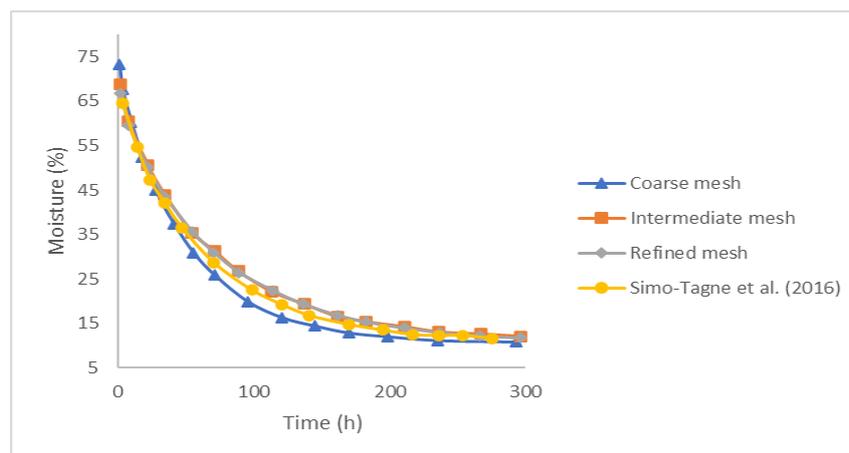


Figure 3. Moisture profiles over mesh study to Frake wood.

Table 2. Performance indicators for Frake wood.

Meshes	Temperature (K)		Moisture (%)		$\Delta t(s)$	$\Delta x(m)$	no. nodes
	$R^2$	RMSE	$R^2$	RMSE			
Coarse	0.998050	14.409178	0.876288	7.332675	3400	0.001250000	12
Intermediate	0.998081	1.016672	0.929328	5.197081	850	0.003125000	48
Refined	0.998928	0.850093	0.980963	2.456262	4900	0.000078125	192

As can be seen, the values for  $R^2$  increase with the mesh refinement, just as the RMSE decreases. Being close to 1 and tending to zero, respectively. This fact proves in a quantitative way that the model developed is close to that proposed by the reference model, and thus, it can be considered that it presents values for temperature and humidity consistent with those verified in the real process. That is, in these analyzes it is possible to identify advances in congruence gradually.

In turn, considering case 1 of the study, Figs. 4 and 5 present the results of refining the computational mesh for Pine type wood. Tab. (3) shows the results of Pine' performance indicators.

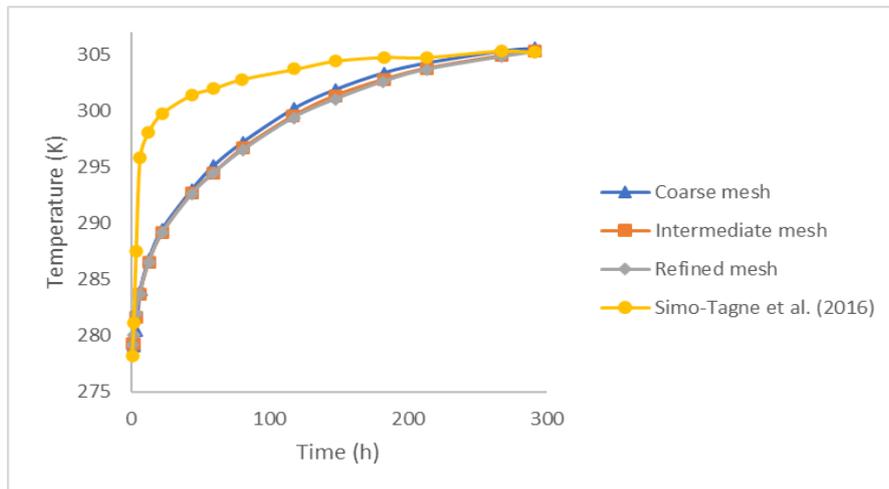


Figure 4. Temperature profiles over mesh study to Pine wood.

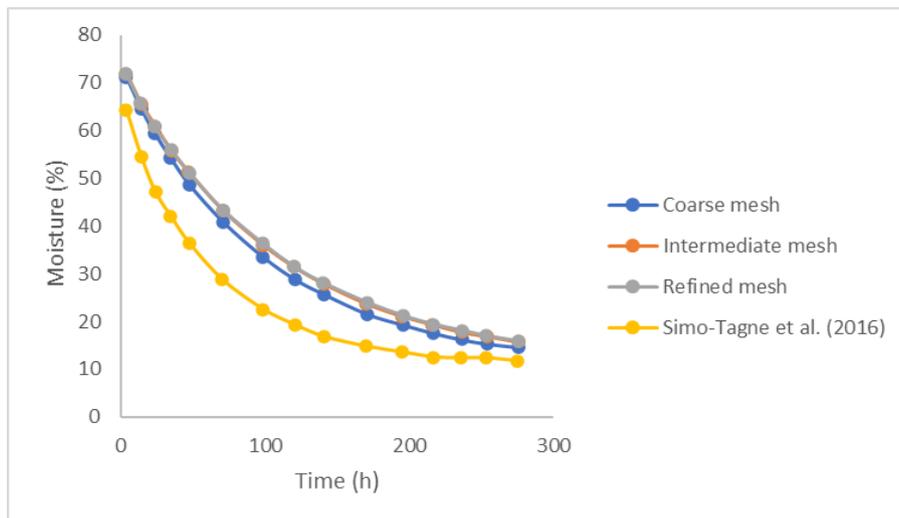


Figure 5. Moisture profiles over mesh study to Pine wood.

Table 3. Performance indicators for Pine wood.

Meshes	Temperature (K)		Moisture (%)		$\Delta t(s)$	$\Delta x(m)$	no. nodes
	$R^2$	RMSE	$R^2$	RMSE			
Coarse	0.725683	6.306113	0.772721	8.851684	3400	0.001250000	12
Intermediate	0.996797	0.534143	0.988906	1.852483	850	0.003125000	48
Refined	0.999584	0.252896	0.999880	0.217024	4900	0.000078125	192

In the case of Pine wood, the values for performance indicators are better for the refined mesh, once it has the highest  $R^2$  value and the lowest for RMSE both for humidity and temperature. As a result, was chosen the 192-knit mesh. In this case, Figs. 6 and 7 show the evolution of temperature and moisture content during the drying of Pine wood referring to case 1 of the study.

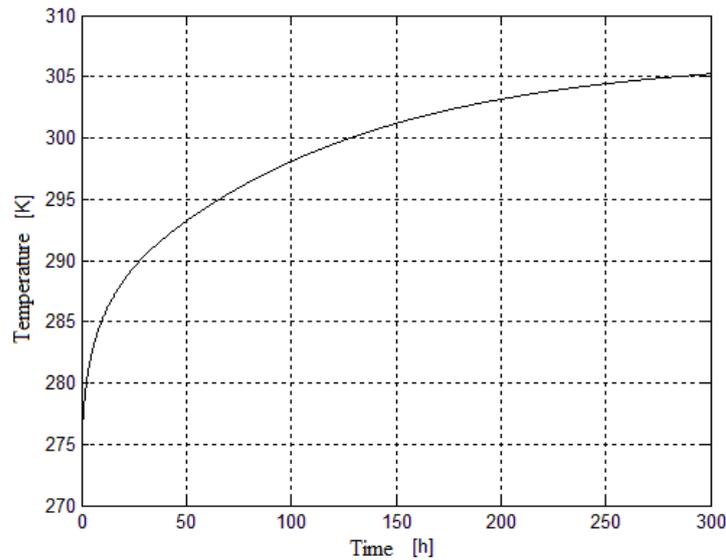


Figure 6. Temperature profile for case 1 to Pine wood.

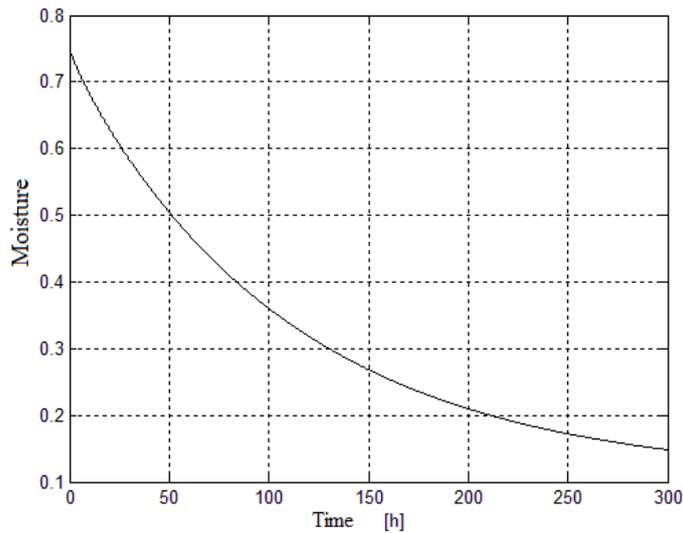


Figure 7. Moisture content profile for case 1 to Pine wood.

Thus, it can be verified that the temperature (K) increases as time (h) elapses, and consequently, the moisture percentage (%) present in the wood decreases. Indicating the expected behaviour for the drying process, since the moisture present at the end was less than 25%. As approached in the literature review, this value represents a significant increase in dry wood quality.

So, it can be noticed a small difference between the results for the temperature evolution and the moisture content between Pine and Frake wood, this is due to the change in specific masses, in the mass diffusion coefficient and the heat transfer coefficients and of mass. However, this variation is small, because both species have the same botanical origin, that is, they are softwood, which are light and softwoods made up of a simple structure (Desch, H.E *et al.*, 2016). The Figs. 8 and 9 show the evolution of temperature and moisture content for case 2. These results refer to the refined mesh.

The curves in case 2 show coherent behaviours and results that differ from case 1, especially regarding drying time. To reach 12% humidity was required a drying time approximately 3 times longer. This fact is linked to the assumption of the absence of convection and heat conduction, which accelerate the removal of moisture from the wood. On the other hand, this time increase leads to increased operating costs for drying in general, among which can be mentioned: storage space, energy and production quantity. These measurements can be seen in Figs 10 and 11 to Pine wood.

In addition, when analyzing the behavior of Frake wood in the same way as pine wood. That is, through the methods of cases 1 and 2, obtains a result similar to that of Pine wood. Since through the conditions of case 2, the time required for the process to reach the same moisture content is longer, as showed in Figs 12 and 13.

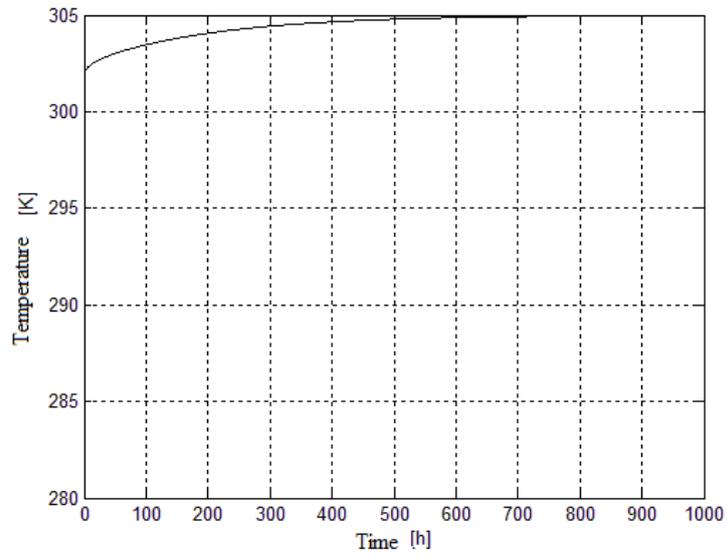


Figure 8. Temperature profile for case 2 to Pine wood.

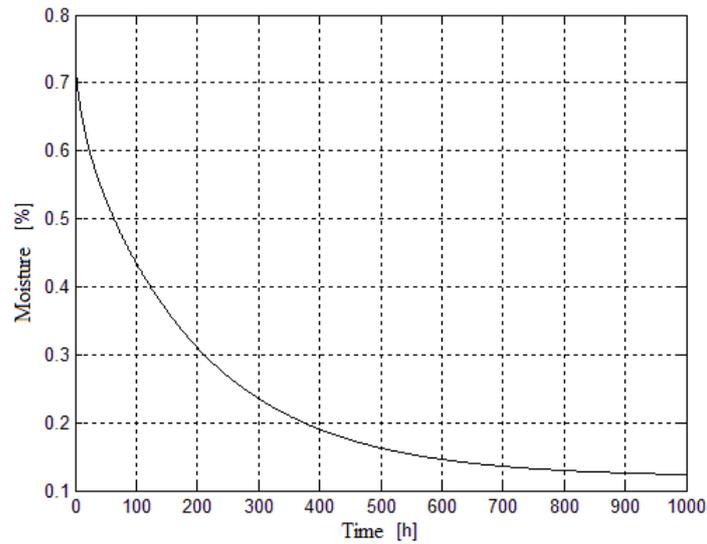


Figure 9. Moisture content profile for case 2 to Pine wood.

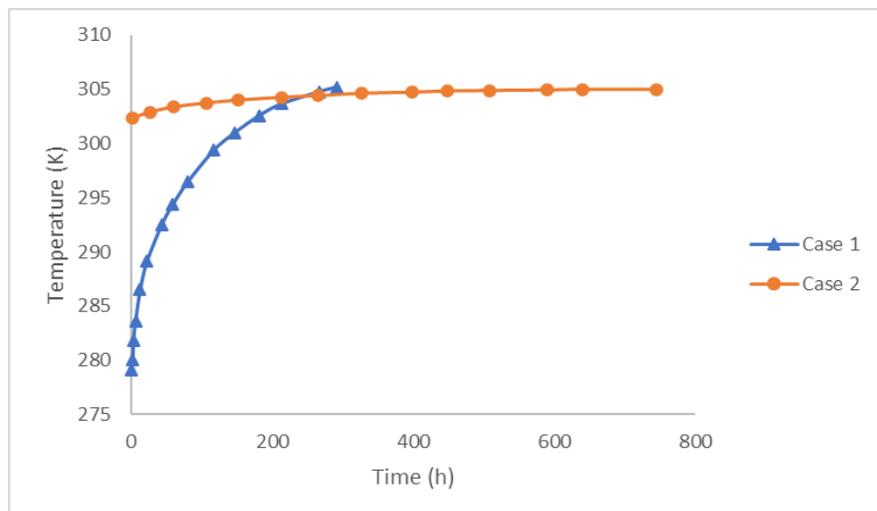


Figure 10. Temperature profile for case 1 to Pine wood.

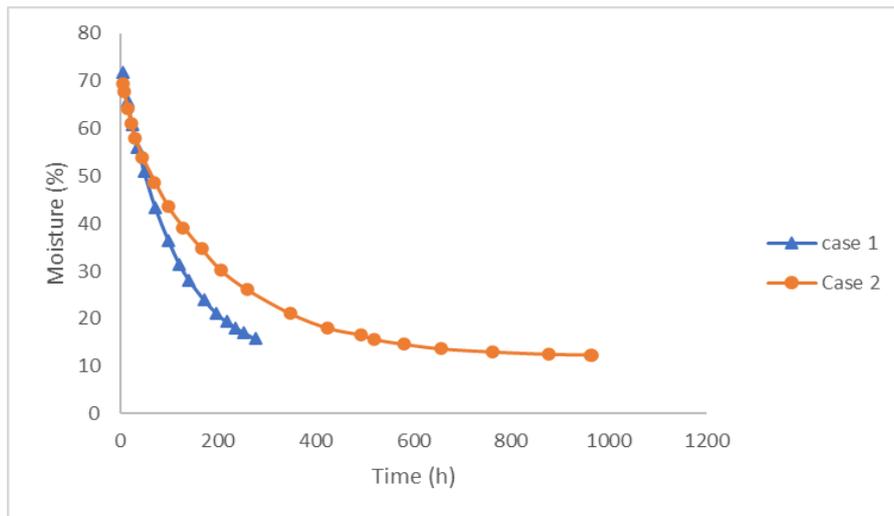


Figure 11. Moisture profile for case 1 to Pine wood.

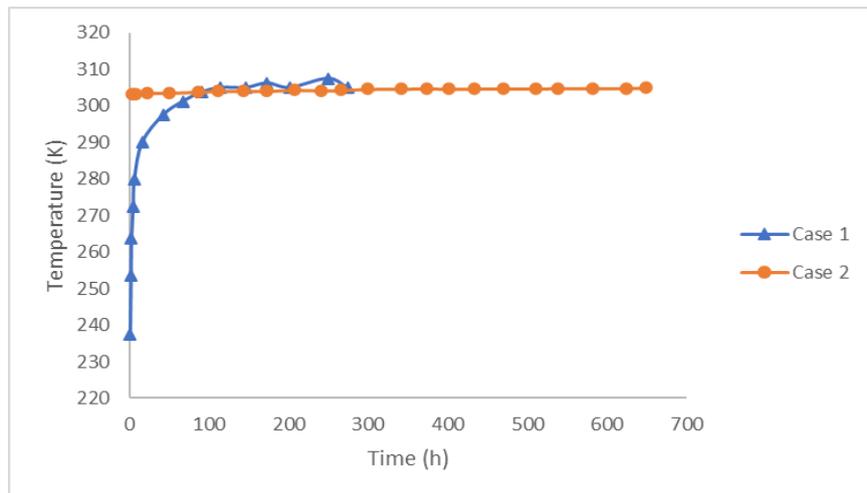


Figure 12. Temperature profile for case 1 to Frake wood.

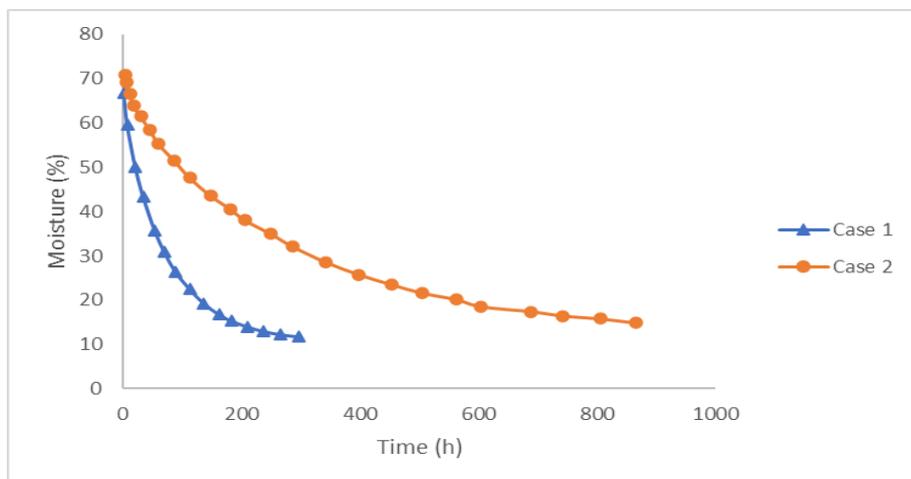


Figure 13. Moisture profile for case 1 to Frake wood.

## 5 CONCLUSIONS

Firstly, the validation of the computational code developed by comparing the initial results provided greater security in the results obtained for the pinewood, once the code development method had previously been tested and validated by the literature. Even more, the parameters used for the pine type wood were properly surveyed through a literature review, which, consequently, helps that the results found are within the expected. As well as it provides theoretical conformity for the study.

Besides, the results indicate that Pine and Frake wood have similar physical and chemical characteristics, presenting good final quality, if applied these drying conditions. The developed code describes the pinewood drying process with satisfaction, so it can be used as a tool to ascertain it.

The boundary conditions influence the drying process parameters since with the change of the Dirichlet condition to the Neumann condition there was an increase in operating costs. With this, it is extremely important to evaluate the initial and boundary conditions, so a more coherent and faithful analysis of the drying reality is obtained, and then allow the code to effectively assist business decisions.

In future studies, different numerical schemes may be applied, being implicit in time, such as Crank-Nicolson, Dufort-Frankel, Hyperbolization and Saul'yev or, other explicit models such as Euler. Then, compare the results by checking which one has better convergence, greater numerical stability and security of temperature and humidity values for pine wood.

## 6. ACKNOWLEDGEMENTS

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