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**GENERALIZED INTEGRAL TRANSFORMS AND KIRCHHOFF
TRANSFORMATION IN NONLINEAR HEAT CONDUCTION**

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Abstract. *The Generalized Integral Transform Technique (GITT) is combined with the Kirchhoff transformation in solving transient nonlinear heat conduction problems, considering temperature dependence on both the thermal conductivity and the thermal capacitance. The hybrid methodology is employed with a convergence acceleration strategy based on a quasi-steady filtering scheme. The approach is demonstrated for a test case based on boundary conditions of the second and first kinds. The results were critically compared against numerical data obtained using the method of lines (MOL). The approach can be directly applied in the thermal analysis of nuclear fuel rods or plates, considering temperature dependent thermophysical properties.*

Keywords: *Generalized integral transform technique, Kirchhoff transformation, nonlinear problems, heat conduction, method of lines, nuclear fuel*

1. INTRODUCTION

The present work is motivated by the search of more precise and cost-effective simulation tools for nonlinear heat conduction problems, especially for computationally intensive tasks that require hundreds or thousands of evaluations of a specific formulation, such as in optimization, inverse problem analysis, real time simulation, machine learning, and simulation under uncertainty. Accounting for the temperature dependence of thermophysical properties becomes mandatory in various classes of materials under more extreme operational conditions (Ozisik, 1993). In such cases, the adoption of properties linearization approximations can lead to significant deviations of the simulations from the actual physical behavior (Sphaier et al., 2017). The thermal analysis of nuclear fuel rods provides a very relevant example of this class of problems (Pontedeiro et al., 2007; Pontedeiro et al., 2008; Dourado et al., 2017; Cotta et al., 2018), due to the very large temperature gradients achieved and the marked variation of physical properties within the typical wide temperature ranges.

In light of the intrinsic limitation of classical analytical approaches in handling such class of nonlinear problems, an interesting alternative to the traditional time-consuming purely numerical methods is the advancement of hybrid numerical-analytical methodologies, that combine the accuracy and mild computational costs of analytical methods with the flexibility and generality more typical of numerical tools. One such hybrid approach is the so-called Generalized Integral Transform Technique (GITT) (Cotta, 1990, 1993, 1994; Serfaty and Cotta, 1990, 1992; Cotta and Mikhailov, 1997; Cotta et al., 2016; Cotta et al., 2017), first employed in the solution of nonlinear diffusion problems in (Cotta, 1990). The formalism behind the GITT for nonlinear diffusion problems involves the choice of an eigenvalue problem with linear coefficients, though arbitrarily variable in the space coordinates, that are characteristic representations of the original nonlinear coefficients, for instance by evaluating them at the initial or steady-state known temperature distributions. Then, after the proposition of the integral transform pair and respective orthogonality property, the original partial differential equation is integral transformed to yield a coupled system of nonlinear ordinary differential equations

for the transformed potentials, which requires a numerical solution methodology for initial value problems, for instance, in the case of a transient formulation. After that, the analytical inversion formula is recalled to reconstruct the desired temperature field throughout the domain. An important aspect in the practical implementation of the GITT is the need to improve the convergence behavior of the resulting eigenfunction expansions, aimed at reducing computational costs through the adoption of transformed ODE systems with lower truncation orders. In the present class of problems, the nonlinear coefficients which are not fully represented in the eigenvalue problem, lead to a behavior typical of source terms in delaying convergence of the expansions. However, there are convergence acceleration schemes that originate from the use of filters based on simplified representations of the original problem, as summarized in Almeida and Cotta (1996) and Cotta and Mikhailov (1997), and afterwards extended in the form of local-instantaneous transient filtering schemes (Macedo et al., 1999).

In addition, nonlinear heat conduction problems can be reformulated through the well-known Kirchoff transformation (Ozisik, 1993), when the nonlinearity associated with the temperature dependent thermal conductivity can be removed from the governing equation, and collapsed to the nonlinear thermal capacitance, explicitly introducing the nonlinear thermal diffusivity in the new formulation (Özisik, 1993; Soares, 1995; Pontedeiro et al. 2007; Pelegrini et al., 2015). For a steady state problem with first or second kind boundary conditions, the Kirchoff transformation leads to a linear heat conduction problem since the thermal diffusivity is not required. In terms of the GITT methodology, it has been demonstrated (Pontedeiro et al., 2007) that the combination of the integral transforms approach with the Kirchoff transformation can offer improved convergence behavior, since the new diffusion term is exactly integral transformed and the temperature dependence of the thermal diffusivity is much less pronounced than for the conductivity and capacitance independently. This work further explores the solution of nonlinear transient heat conduction by combining the GITT approach with the Kirchoff transformation, by avoiding an implicit transformed transient term, and in addition adopting an explicit quasi-steady filter for convergence acceleration. The proposed approach is demonstrated for a test case based on boundary conditions of the second and first kinds, considering temperature dependent thermophysical properties.

2. ANALYSIS

A one-dimensional nonlinear transient heat conduction problem is here selected to illustrate the proposed approach, with prescribed heat flux and prescribed temperature at each boundary, given by:

$$\rho(T)Cp(T)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(T)\frac{\partial T(x,t)}{\partial x}\right) + g(x,t) \quad 0 < x < L, t > 0 \quad (1)$$

$$T = f(x) \quad 0 \leq x \leq L, t = 0 \quad (2)$$

$$k(T)\frac{\partial T}{\partial x} = -q''(t) \quad x = 0, t > 0 \quad (3)$$

$$T = T_L \quad x = L, t > 0 \quad (4)$$

where, $\rho(T)$, $Cp(T)$ and $k(T)$ are temperature dependent thermophysical properties, namely specific mass, specific heat and thermal conductivity, $g(x,t)$ is the internal heat generation, $q''(t)$ the heat flux. The thermophysical properties are defined at the present test case as:

$$k(T) = k_0(1 + \beta T), \quad \rho(T) = \rho_0(1 + \delta T), \quad Cp(T) = Cp_0(1 + \gamma T)$$

The following dimensionless groups are introduced:

$$\eta = \frac{x}{L}; \quad \tau = \frac{\alpha_0 t}{L^2}; \quad \theta = \frac{T - T_\infty}{q_0'' L / k_0}; \quad \alpha_0 = \frac{k_0}{\rho_0 Cp_0}; \quad G(\eta, \tau) = g(x, t)L / q_0''; \quad (5-9)$$

Therefore, the dimensionless problem formulation is written as follows:

$$\bar{\rho}(\theta)\bar{Cp}(\theta)\frac{\partial \theta(\tau, \eta)}{\partial \tau} = \frac{\partial}{\partial \eta}\left(\bar{k}(\theta)\frac{\partial \theta(\tau, \eta)}{\partial \eta}\right) + G(\tau, \eta) \quad 0 < \eta < 1, t > 0 \quad (10)$$

$$\theta(0, \eta) = F(\eta) \quad 0 \leq \eta \leq 1, t = 0 \quad (11)$$

$$\bar{k}(\theta)\frac{\partial \theta}{\partial \eta} = -\omega(\tau) \quad \eta = 0, t > 0 \quad (12)$$

$$\theta = \theta_L \quad \eta = 1, t > 0 \quad (13)$$

where,

$$\bar{k}(\theta) = \frac{k(\theta)}{k_0} = a_0 + a_1\theta; \quad \bar{\rho}(\theta) = \frac{\rho(\theta)}{\rho_0} = b_0 + b_1\theta; \quad \bar{Cp}(\theta) = \frac{Cp(\theta)}{Cp_0} = c_0 + c_1\theta; \quad (14-16)$$

$$a_0 = 1 + \beta T_\infty; \quad a_1 = \beta q_0'' L / k_0; \quad b_0 = 1 + \gamma T_\infty; \quad b_1 = \gamma q_0'' L / k_0; \quad c_0 = 1 + \delta T_\infty; \quad c_1 = \delta q_0'' L / k_0; \quad (17-22)$$

2.1 Kirchhoff transformation

Applying the Kirchhoff transformation on the dimensionless temperature, a new dependent variable is constructed as:

$$U = \int_{\theta_0}^{\theta} \bar{k}(\theta') d\theta' = \int_0^{\theta} (a_0 + a_1\theta') d\theta' = a_0\theta + \frac{a_1}{2}\theta^2 \quad (23)$$

The dimensionless temperature can be recovered from the new variable by taking the positive root of θ in terms of U :

$$\theta = f(U) = \frac{-a_0 + \sqrt{a_0^2 + 2a_1U}}{a_1} \quad (24)$$

The dimensionless diffusion equation (10) and the boundary and initial conditions (11-13) are then rewritten in this new variable U as:

$$\frac{\partial U}{\partial \tau} = \alpha(f(U)) \left\{ \frac{\partial^2 U}{\partial \eta^2} + G(\tau, \eta) \right\} \quad 0 < \eta < 1, t > 0 \quad (25)$$

$$U = a_0 F(\eta) + \frac{a_1}{2} (F(\eta))^2 = \bar{F}(\eta) \quad 0 \leq \eta \leq 1, \quad t = 0 \quad (26)$$

$$\frac{\partial U}{\partial \eta} = -\omega(\tau) \quad \eta = 0, \quad t > 0 \quad U = a_0 \theta_L + \frac{a_1}{2} (\theta_L)^2 = U_L \quad \eta = 1, \quad t > 0 \quad (27,28)$$

where in eq. (25) $\alpha(f(U)) = \frac{\bar{k}(f(U))}{Cp(f(U))\bar{\rho}(f(U))}$ is the dimensionless thermal diffusivity.

To obtain the solution of the problem defined by the Eqs. (25-28) through the Generalized Integral Transform Technique (GITT), an explicit quasi-steady filter is adopted that allows for the homogenization of the boundary conditions, in the following form:

$$U(\tau, \eta) = U_F(\tau, \eta) + U_H(\tau, \eta) \quad (29)$$

The formulation of the filter problem $U_F(\tau, \eta)$ is proposed as:

$$\frac{\partial^2 U_F}{\partial \eta^2} = 0 \quad 0 < \eta < 1, \quad t > 0 \quad (30)$$

$$\frac{\partial U_F}{\partial \eta} = -\omega(\tau) \quad \eta = 0, \quad t > 0 \quad U_F = a_0 \theta_L + \frac{a_1}{2} \theta_L^2 = U_L \quad \eta = 1, \quad t > 0 \quad (31,32)$$

Analytically solving Eqs. (30-32), the expression for the filter becomes:

$$U_F(\tau; \eta) = \omega(\tau)(1 - \eta) + U_L \quad (33)$$

The formulation for the filtered potential $U_H(\tau, \eta)$ is such that:

$$\frac{\partial U_H}{\partial \tau} = \alpha(f(U)) \frac{\partial^2 U_H}{\partial \eta^2} + \bar{G}(\tau, \eta) \quad 0 < \eta < 1, t > 0 \quad (34)$$

$$U = \bar{F}(\eta) - U_F(\eta; 0) \quad 0 \leq \eta \leq 1, \quad t = 0 \quad (35)$$

$$\frac{\partial U_H}{\partial \eta} = 0 \quad \eta = 0, \quad t > 0 \quad U_H = 0 \quad \eta = 1, \quad t > 0 \quad (36,37)$$

where,

$$\bar{G}(\tau, \eta) = \alpha(f(U))G(\tau, \eta) - \omega'(\tau)[1 - \eta] \quad (38)$$

2.2 The generalized integral transform technique (GITT)

To develop the solution of system (34-38) with homogeneous boundary conditions, the GITT methodology is applied, starting with the proposition of the eigenfunction expansion basis.

2.2.1 Eigenvalue problem

$$\frac{\partial^2 \psi_i}{\partial \eta^2} + \mu_i^2 \psi_i = 0, \quad 0 < \eta < 1 \quad (39)$$

$$\frac{\partial \psi_i}{\partial \eta} = 0, \quad \eta = 0 \quad (40)$$

$$\psi_i = 0, \quad \eta = 1 \quad (41)$$

Solving the auxiliary problem of Eqs. (39-41), the eigenfunctions are given by:

$$\psi_i(\eta) = \text{Cos}(\mu_i \eta); \quad i = 1, 2, 3, \dots \quad (42)$$

The eigenvalues of the auxiliary problem are given by:

$$\mu_i = (2i-1) \frac{\pi}{2}; \quad i = 1, 2, 3, \dots \quad (43)$$

The eigenfunctions of Eq. (42) satisfy the orthogonality property:

$$\int_0^1 \bar{\psi}_i(\eta) \bar{\psi}_j(\eta) d\eta = \begin{cases} 0, & i \neq j \\ N_i, & i = j \end{cases} \quad (44)$$

$N_i(\eta)$ is the normalization integral, given by:

$$N_i = \int_0^1 \psi_i^2(\eta) d\eta = \frac{1}{2}; \quad i = 1, 2, 3, \dots \quad (45)$$

where the normalized eigenfunctions are given by:

$$\bar{\psi}_i(\eta) = \frac{\psi_i(\eta)}{\sqrt{N_i}} = \sqrt{2} \text{Cos}(\mu_i \eta) \quad i = 1, 2, 3, \dots \quad (46)$$

2.2.2 Integral transform pairs

$$\bar{U}_{Hi}(\tau) = \int_0^1 \bar{\psi}_i(\eta) U_H(\tau, \eta) d\eta, \quad \text{Transform} \quad (47)$$

$$U_H(\tau, \eta) = \sum_{i=1}^{\infty} \bar{\psi}_i(\eta) \bar{U}_{Hi}(\tau), \quad \text{Inverse} \quad (48)$$

2.2.3 Integral transformation of the original problem

Integral transforming equation (34):

$$\int_0^1 \bar{\psi}_i(\eta) \frac{\partial U_H}{\partial \tau} d\eta = \int_0^1 \bar{\psi}_i(\eta) \left\{ \alpha(f(U)) \frac{\partial^2 U_H}{\partial \eta^2} + \bar{G}(\tau, \eta) \right\} d\eta \quad (49)$$

It then results the following coupled system of ordinary differential equations for the transformed potentials $\bar{U}_{Hi}(\tau)$:

$$\frac{d\bar{U}_{Hi}}{d\tau} = \bar{G}_i(\tau) \quad \tau > 0 \quad (50)$$

with initial conditions:

$$\bar{U}_{Hi} = \int_0^1 \bar{\psi}_i(\eta) [\bar{F}(\eta) - U_F(0; \eta)] d\eta = B_i \quad (51)$$

$$\bar{G}_i(\tau) = \int_0^1 \bar{\psi}_i(\eta) \left\{ \alpha(f(U)) \frac{\partial^2 U_H}{\partial \eta^2} + \bar{G}(\tau, \eta) \right\} d\eta \quad (52)$$

3. TEST CASE

The ordinary differential equations system (50,51) was solved by a numerical algorithm for initial value problems with the aid of the subroutine DIVPAG with Gear's method, while the subroutine DFQRUL was used in the numerical integration of the coefficient $\overline{G}_i(\tau)$, both from the IMSL Library (2018). The transformed potential is then obtained and the dimensionless temperature is analytically recovered by the inversion formula, eq. (48).

The convergence analysis of the GITT solution was performed in terms of the dimensionless temperature at different values of η and τ , considering the following parameters:

$$\overline{k} = 1; \overline{Cp} = 1; \overline{\rho} = 1; \omega(\tau) = \text{Sin}(0.1\tau); \theta(0, \eta) = 1; \theta(\tau, 1) = 0; G(\tau, \eta) = [\exp(-0.1(1-\eta))][1-\tau/3]$$

Table 1 illustrates the convergence behavior of the GITT solution, where it is noted that the convergence rates for the dimensionless temperatures are indeed satisfactory, requiring truncation orders at most around 10 to 15 terms for attaining four significant digits in the analyzed cases. Table 2 shows the convergence analysis of the dimensionless temperature using the method of lines (MOL), for dimensionless times τ and position values η . Note that with around 15 to 20 mesh points, the results are already converged to four significant digits in the analyzed cases, agreeing with the GITT solution.

Table 1. Convergence analysis of the GITT solution for the test case.

$\theta(\tau, \eta); \tau = 0.3$						
η/NT	5	10	15	20	25	30
0.2	0.8019	0.8021	0.8021	0.8021	0.8021	0.8021
0.4	0.6920	0.6917	0.6917	0.6917	0.6917	0.6917
0.6	0.5156	0.5158	0.5159	0.5159	0.5159	0.5159
0.8	0.2824	0.2819	0.2820	0.2819	0.2819	0.2819
$\theta(\tau, \eta); \tau = 3$						
η/NT	5	10	15	20	25	30
0.2	0.2685	0.2685	0.2685	0.2685	0.2685	0.2685
0.4	0.2052	0.2052	0.2052	0.2052	0.2052	0.2052
0.6	0.1389	0.1389	0.1389	0.1389	0.1389	0.1389
0.8	0.0702	0.0702	0.0702	0.0702	0.0702	0.0702

Table 2. Convergence analysis of the MOL solution for the test case.

$\theta(\tau, \eta); \tau = 0.3$						
η/NM	5	10	15	20	25	30
0.2	0.8040	0.8022	0.8021	0.8021	0.8021	0.8021
0.4	0.6936	0.6918	0.6917	0.6917	0.6917	0.6917
0.6	0.5174	0.5160	0.5159	0.5159	0.5159	0.5159
0.8	0.2828	0.2820	0.2819	0.2819	0.2819	0.2819
$\theta(\tau, \eta); \tau = 3.0$						
η/NM	5	10	15	20	25	30
0.2	0.2697	0.2689	0.2686	0.2686	0.2686	0.2686
0.4	0.2061	0.2055	0.2052	0.2052	0.2052	0.2052
0.6	0.1396	0.1391	0.1390	0.1390	0.1390	0.1390
0.8	0.0706	0.0703	0.0702	0.0702	0.0702	0.0702

Next, the dimensionless temperature profiles are analyzed considering a flat wall with initial temperature condition $\theta(0, \eta) = 0$, isolated on $\eta = 0$, with a time and space dependent internal heat source $G(\tau, \eta) = \exp[-0.1(1-\eta)](1-\tau/3)$. Fig. 1(a) and Fig. 1(b) provide, respectively, the spatial distribution and time evolution of the dimensionless temperature, which are directly compared to the method of lines results, with excellent agreement throughout the solution domain.

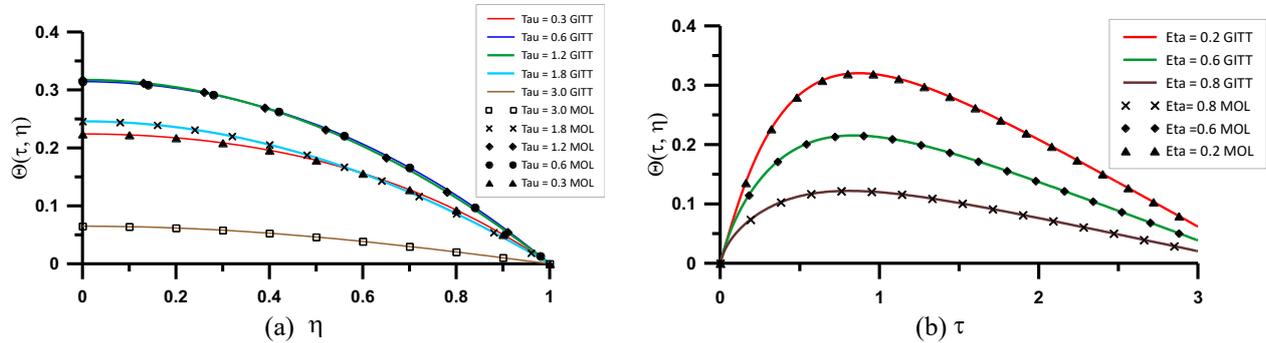


Figure 1. Dimensionless temperature profiles for isolated plane wall with heat generation. (a) dimensionless temperature profile across space coordinate and (b) evolution of dimensionless temperature along dimensionless time.

4. CONCLUSIONS

The generalized integral transformation technique (GITT) is applied in combination with the Kirchhoff transformation to solve transient non-linear heat conduction problems, avoiding an implicit transformed transient term and adopting a quasi-steady filter for convergence enhancement. A test case is considered and a convergence analysis of the proposed solution is presented, illustrating the enhancement promoted by the employed filter. Also, the hybrid solution is critically compared to a numerical solution by the method of lines (MOL), with excellent agreement. This approach can be readily applied to the thermal analysis of nuclear fuel element with temperature dependent thermophysical properties, aiming at the best possible convergence rates when considering time-consuming coupled neutronics and thermal hydraulics (Ribeiro et al., 2020).

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