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SYNTHESIS OF THE TRANSFER MATRIX OF THERMOACOUSTIC CORES TOWARD AN INVERSE PROBLEM

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Abstract. *The performance of thermoacoustic devices, either engines or refrigerators, depends essentially on the design of its main component: the thermoacoustic core (TAC), which comprises a porous material, heat exchangers and a thermal buffer tube. Usual porous materials of the most efficient devices are complex in its internal geometry, which makes convenient its experimental characterization as an implicit component of the entire TAC, to be treated as a black box in the form of a transfer matrix. From this measured data, a particular TAC can then be inserted into a simulated waveguide network so that to constitute a thermoacoustic engine (TAE), whose performance can be analytically predicted from usual transfer matrix relations. Nonetheless, selecting the best porous material for a certain TAE configuration by experimental means is highly time demanding due to the laborious iterative process involved. On the other hand, for complex porous materials, analytical or numerical approaches are so far impractical. The purpose of this work is to explore experimental data from previous works, where TAC acoustic transfer matrices had been measured, aiming to circumvent the need of such iterative experimental process in the quest for a better porous material. Those matrices are adequate and directly edited with close observation of the resulting new simulated - and artificial - performance. They are evaluated with respect to the actual physical system and corresponding boundary conditions by contrasting their values to the real transfer matrix data. The corresponding determinant, which measures the degree of nonreciprocity, is proposed as reference for selecting the synthesized transfer matrices as a preceding step toward the optimal design of porous materials from an inverse problem approach.*

Keywords: *thermoacoustics, thermoacoustic core, transfer matrix synthesis, inverse problem, porous material design*

1. INTRODUCTION

Thermoacoustics is a branch of acoustics that studies the interaction of heat over acoustic oscillations and vice-versa. Pressure oscillations of fluids caused by heat transfer mechanisms can be stimulated by combustion phenomena as well as by thermal coupling between oscillating fluids and differentially heated solid surfaces (Rott, 1969). Although these interactions may be undesired in some engineering applications involving combustion and noise, such as in internal combustion systems, gas turbines and rocket engines, this phenomenon has been used in well-known applications such as in glassblowing and cryogenics, and further exploited to the development of thermoacoustic devices, where this mechanism is not only desirable but essential (Swift, 2002). Since the mathematical foundations of thermoacoustics were established, several engines and refrigerators were successfully developed (Garrett *et al.*, 1993; Swift, 1988).

The conversion of thermal energy into acoustic energy, or vice-versa, constitutes the so-called thermoacoustic effect. This interaction takes place in the thermal and viscous boundary layers of an acoustically oscillating gaseous fluid in contact with a solid surface. The fluid must be confined in a resonating waveguide network so that to conform a stationary or a traveling wave. For engines, part of the heat transferred from a solid surface by convection to the neighbor gas parcels is converted in their acoustic oscillation along a temperature gradient, which is a spontaneous generation of an acoustic field (Bannwart, 2014). The other way around works for refrigerators or heat pumps, where the source of energy is no longer thermal, but mechanical, in the form of an imposed acoustic field. The magnitude of energy transport can be promoted by increasing the area of exposition between gas and solid surface by means of the use of the internal walls of a porous material adequately placed inside the device. Heat exchangers surrounding the porous material delimit the inhomogeneous temperature profile. The waveguide segment comprising the porous material and the surrounding heat exchangers is the main component. It is denominated thermoacoustic core (TAC), as can be seen within the green dashed box in Fig. 1, where a scheme of the device considered in this work is shown.

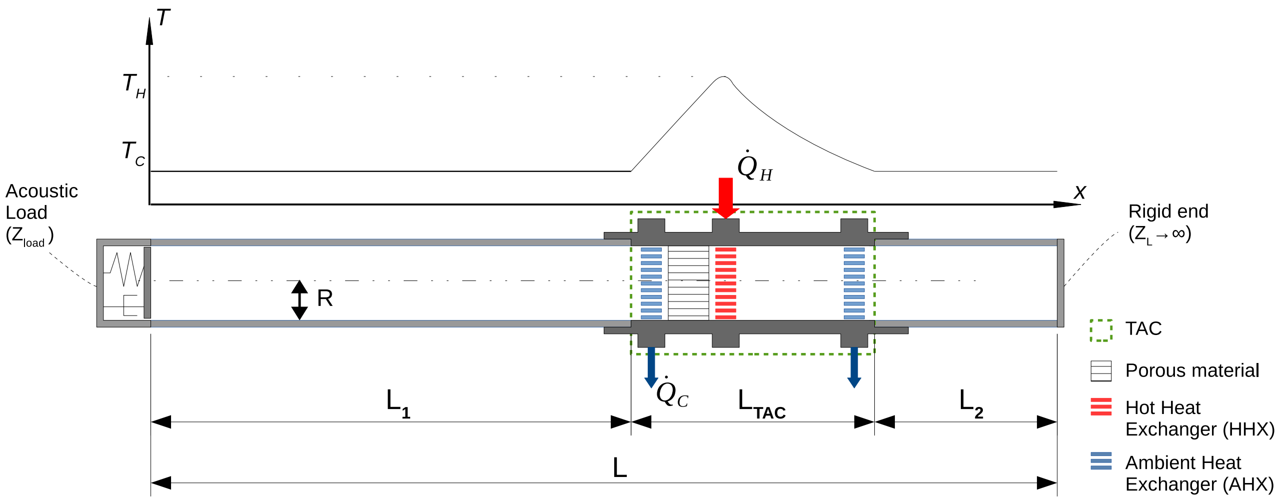

Figure 1. Sketch of the thermoacoustic engine (TAE) considered. Adapted from Bannwart (2014).

The TAC of this investigation was previously characterized in works of Bannwart *et al.* (2013). It was sided by two ambient heat exchangers (AHXs), and a hot heat exchanger (HHX) was placed inside it. By inserting power into the system with a heating power supply ($\dot{Q}_H$) and rejecting part of the heat to a cold reservoir ($\dot{Q}_C$), several temperature gradients were established in the porous material along the axial direction of the waveguide. Regardless of the wave type, self-sustained oscillations can be set when the solid is subject to a temperature gradient high enough for the vibrational energy stemming from thermal expansion to surpass the viscous and thermal losses along the pore paths and the resonating cavity. Conceptually speaking, this net acoustic work could eventually be employed to produce electricity -for example, with the use of a linear alternator- or to directly drive a thermoacoustic refrigeration by means of a membrane.

The waveguide network configuration determines whether the resonant acoustic field established in the gaseous medium is stationary or progressive. Correspondingly, the porous material is commonly referred to as a *stack* or a *regenerator*, respectively, if the device operates with a standing wave (SW) or a traveling wave (TW) (Swift, 1988).

Many efforts have been done to make thermoacoustic refrigerators more efficient in order to compete with conventional vapor-compression refrigeration technologies. Still, there is plenty of room for their improvement until they become commercially attractive (Tassou *et al.*, 2010). Nonetheless, thermoacoustics suits well for regeneration applications because it can operate with a low thermal potential at low-temperature heat sources, as the working fluid remains in one phase during the complete cycle. The present work is concerned with the study of standing wave thermoacoustic engines (SW TAEs). However, the focus is not the whole device but the TAC, where the acoustic power is actually produced.

Acoustic characteristics of a thermoacoustic core can be described by its TAC transfer matrix ($\mathbf{T}_{TAC}$), accounting for dissipation and propagation of the acoustic wave. Bannwart *et al.* (2013) developed a method for characterizing acoustically both stack-based and regenerator-based TACs in an accurate way, by measurements of their impedance matrices. From the measured $\mathbf{T}_{TAC}$-matrices, it is possible to predict the performance of the TAE by calculating a parameter known as thermoacoustic amplification gain (G). Therefore, the best performing porous material for a given engine configuration can then be found from a set of characterized TACs.

Still, it is desirable that only a few porous materials are tested experimentally and their performance compared. For that to be possible, a cheaper and simpler a priori design path should be developed. By arbitrarily perturbing the coefficients of $\mathbf{T}_{TAC}$ looking for a higher gain, Gomes and Bannwart (2018) have investigated the possibility of synthesizing $\mathbf{T}_{TAC}$-matrices. Ideally, one would like to know what geometrical (or even thermophysical characteristics) a porous material should have in order to give an expected performance. Though this constitutes an inverse problem where parameters of a model ($\mathbf{T}_{TAC}$) are determined from their derived output data (G). Thus the focus of our work is to further enable such an a priori design of porous materials (particularly here, *stacks*). Applying an inverse problem solution as employed in Bannwart (2014) and Guédra *et al.* (2015), *stack* geometrical parameters could be roughly estimated from synthesized $\mathbf{T}_{TAC}$-matrices, previously selected according to predefined limits of tolerance properly established from experimental data.

2. TRANSFER MATRIX SYNTHESIS

In the current transfer matrix modeling, the TAC is seen as a *black box* and treated as a two-port system, whose coefficients were obtained from acoustic measurements of previous works (Bannwart *et al.*, 2013). The acoustic field (acoustic pressure and volume velocity) propagation through the TAC is described by four complex coefficients, which contribute to relating each input of this two-port system to each of its outputs (Gentemann *et al.*, 2003). Thus, all internal features of the TAC are contained in these coefficients, not mattering how complex the porous material geometrical and thermmophysical properties are.

2.1 Synthesis by perturbations

Gomes and Bannwart (2018) proposed a new approach aiming to obtain better performing thermoacoustic engines by means of interfering the $\mathbf{T}_{TAC}$-matrices. By arbitrarily perturbing transfer matrices of measured TACs in both magnitude and phase arbitrarily, multiple $\mathbf{T}_{TAC}$-matrices may be constituted. Each of the $\mathbf{T}_{TAC}$-matrices synthesized in this manner could correspond to a virtual TAC characterized by its new and artificial T-coefficients, namely: T_{pp}, T_{pu}, T_{up}, and T_{uu}.

However, estimating the performance of a TAC from transfer matrices must be done by mounting it into a waveguide network, composing a TAE. Each segment is then characterized by its own transfer matrix, which relates the acoustic field from one end to the other. Gomes and Bannwart (2018) used a TAE configuration of Bannwart (2014), chosen according to specific criteria.

In the works of Bannwart (2014), the TAEs are constituted by a TAC surrounded by two straight waveguides, each one varying in many different lengths. $\mathbf{T}_{TAE}$ is then obtained by the sequential product of the transfer matrices of each one of the three elements. The thermoacoustic gain is a ratio between the acoustic power produced within the TAC and the power dissipated along the surrounding waveguides. Further details on the measurement methods, analytical modeling and performance calculations from $\mathbf{T}$-matrices can be found in both Bannwart (2014) and Bannwart *et al.* (2013).

Each TAC transfer matrix coefficient (T_{ij}) is written as a phasor in its original form as $|T_{ij}|\exp^{\mathrm{i}\{\arg(T_{ij})\}}$, where T_{ij} is the original $\mathbf{T}_{TAC}$-matrix coefficient of line i and column j, and $|T_{ij}|$ and $\arg(T_{ij})$ correspond, respectively, to the magnitude and phase of each T-coefficient. Correspondingly, each coefficient is associated with one mode of perturbation. Thus, magnitude perturbations are done in a way that the magnitude of each original individual coefficient is modified by means of a magnitude coefficient (mc), which multiplies $|T_{ij}|$, such that

$$T'_{ij}(\omega,\dot{Q}_H) = mc(\omega,\dot{Q}_H)T_{ij}(\omega,\dot{Q}_H) = mc(\omega,\dot{Q}_H)\cdot|T_{ij}|\exp^{\mathrm{i}\{\arg(T_{ij})\}}, \tag{1}$$

where $T'_{ij}(\omega,\dot{Q}_H)$ is the coefficient obtained by perturbing the original magnitude of T_{ij} at the angular frequency ω and heating regime $\dot{Q}_H$. Similarly, the phase of each coefficient is modified by a phase coefficient (pc), which is summed to the original phase by $\arg(T_{ij})$ in the exponent, such that

$$T'_{ij}(\omega,\dot{Q}_H) = \exp^{\mathrm{i}\left\{pc(\omega,\dot{Q}_H)\right\}}T_{ij}(\omega,\dot{Q}_H) = |T_{ij}|\exp^{\mathrm{i}\left\{\arg(T_{ij})+pc(\omega,\dot{Q}_H)\right\}}, \tag{2}$$

where $T'_{ij}(\omega,\dot{Q}_H)$ is the coefficient obtained by perturbing the original phase of T_{ij}.

The approach proposed by Gomes and Bannwart (2018) yields a new transfer matrix by perturbing a coefficient, in either magnitude or phase, while the others are kept unchanged. Thus, eight different synthesized $\mathbf{T}_{TAC}$-matrices can be obtained from a single original matrix when matrices are synthesized based on single perturbations (2 perturbation modes $\times$ 4 coefficients).

In principle, any value of mc and pc could be chosen for perturbation. Nonetheless, discrepant values could be obtained due to the large differences of the coefficients, leading to physical inconsistencies. To avoid this, a simple comparison with real matrix data was done. Coefficients mc and pc were calculated by comparing transfer matrix coefficients from two TACs characterized, each with a different porous material: a ceramic catalyst (CCat) and a Nichrome foam (NiCr). Therefore, for example, $mc_{up}(\omega,\dot{Q}_H)$ is obtained by dividing $|T_{up}|$ of the CCat TAC by $|T_{up}|$ of the NiCr TAC in the same operating conditions of ω and $\dot{Q}_H$.

2.2 Intrinsic concern in synthesized transfer matrices

An immediate concern of this approach by perturbation is the fact that the resulting $\mathbf{T}_{TAC}$-matrices do not necessarily correspond to the operating conditions of their original matrices. As the operating conditions are associated not only to internal acoustic phenomena (i.e. wave establishment with frequency ω), but also to the boundary conditions (such as the heating power supply $\dot{Q}_H$); altering a coefficient of the transfer matrix may carry unrealistic changes in either ω or $\dot{Q}_H$. This poses a problem when one wants to synthesize $\mathbf{T}_{TAC}$-matrices with higher gains while maintaining the original heating power and temperature profile along the TAC.

One of the first synthesized $\mathbf{T}_{TAC}$-matrices of Gomes and Bannwart (2018), for instance, would have violated the principle of energy conservation. This was verified by evaluating G and observing that a positive value was obtained when no heating power was supplied to the TAC. They observed an supposed increase in the gain from a negative value to a positive value at homogeneous ambient temperature, which should correspond to the establishment of a non-null $\dot{Q}_H$. Therefore, such initial perturbation, if applied without criteria, would result in unrealistic boundary conditions, as no corresponding TAC would produce acoustic power without heat supply. Consequently, such particular synthesized matrix, exempted of any proper adaptation, would not accomplish the purpose of obtaining a better TAC without changing its surroundings.

3. RECIPROCITY AND NONRECIPROCITY

Ten Wolde (2010) stated that the general reciprocity principle, earlier formulated by Helmholtz (1860), "is a general property of all stable, lumped, linear, passive, dynamical systems which only contain bilateral elements". From this statement, it follows directly that a system attending those conditions should be reciprocal ($\det(\mathbf{T}) = 1$), in other words, its determinant should be unitary. As a consequence, any deviation from unity corresponds to a nonreciprocal system. The latter might have one or more characteristics that violate any of those properties for a reciprocal system.

The thermoacoustic systems can be reciprocal only if at homogeneous temperature profile, i.e., they are not operating, as kept in thermal equilibrium. This property was notably employed by Bannwart *et al.* (2013) to experimentally validate the accuracy of their Impedance Method, which ensured its employment in nonreciprocal conditions involving three heat exchangers. When energy is supplied into a specific region of the system, the latter becomes an active element, which results in reciprocity. Even though the thermoacoustic effect onsets due to acoustic instability, it becomes a stable process during steady state operation. Phenomena such as streaming, locally turbulent structures and nonlinearities resulted from high pressures may also lead to affect the reciprocity. However, these are indeed undesirable effects and, thus, have to be avoided as opposed to the heating power supply.

The TAEs considered here are under the linear limit of thermoacoustics, as the acoustic pressures involved are small (Swift, 2002; Bannwart, 2014). Therefore, the TAC is a reciprocal system at ambient temperature, and it is nonreciprocal if some boundary condition is changed, i.e. power is supplied or environmental temperature is in transient contrast. For this reason, the heat input can be assumed the essential cause for nonreciprocity in TACs at low acoustic pressures, which is our case (Bannwart *et al.*, 2013). As the focus of this work is on modifications of the TAC for bettering the system performance and not in modifying its surroundings, it is important to make sure that such changes of the $\mathbf{T}_{TAC}$-matrix are not rooted in boundary condition changes.

3.1 Nonreciprocity and heating power supply relationship

In order to guarantee a constant heat input while synthesizing $\mathbf{T}_{TAC}$-matrices, it would be convenient that some matrix operation could be used to check the heating condition associated with it. Due to the relationship of heating power supply $\dot{Q}_H$ with the determinant of the $\mathbf{T}_{TAC}$-matrix (from here on called nonreciprocity, or $\bar{S}$) was chosen for associating the heat input to the matrix values which characterize the system. Nonreciprocity is then calculated as:

$$\bar{S} = \det(\mathbf{T}_{TAC}) = T_{pp}T_{uu} - T_{pu}T_{up}. \tag{3}$$

Though useful, this criterion has no practical application if employed in this manner exclusively for TACs at ambient condition. Operating TAEs demand TACs to operate with significant temperature gradients and, therefore, their $\mathbf{T}$-matrices will be far from unitary reciprocity ($\bar{S} = 1 = S$). One must then find a way of making it usable for heated TACs and, to the extent that it is possible, for high heat input levels.

Bannwart (2014) identified a proportionality between an increasing heating supply and deviation from reciprocity when employing the latter for evaluating his characterization method. With the purpose of better understanding this relationship between both parameters, we utilized $\mathbf{T}_{TAC}$-matrices from both characterized materials at the frequency of $70Hz$. Calculated nonreciprocity values are shown in Fig. 2 for both CCat and NiCr TACs as the heating power supply is increased from 0 to $81W$ (in steps of $9W$). Note that both materials obey $|\bar{S}| = 1$ and $\arg(\bar{S}) = 0$ at ambient condition.

As one can see, there seems to be a direct proportionality between the magnitude of nonreciprocity and the heat input, at least in the range in which the TACs were characterized. On the other hand, the phase of nonreciprocity appears to have a somewhat asymptotic growth, despite some fluctuations from the measurements of the NiCr TAC. It is interesting to note that the CCat TAC, which gives the highest performance in SW TAEs, has a lower deviation from S (i.e. $|\bar{S}| = 1$). Regarding $\arg(\bar{S})$, exactly the contrary seems to happen in heating regimes when the TACs can produce useful work (from $27W$ on, in TAEs with total waveguide length around $2m$, as the ones analyzed by Gomes and Bannwart (2018)).

3.2 Nonreciprocity of synthesized transfer matrices

In order to evaluate the validity of the synthesized $\mathbf{T}_{TAC}$-matrices from Gomes and Bannwart (2018), their determinants were calculated for magnitude as well as in phase, as shown in Fig. 3. Each line corresponds to $\mathbf{T}_{TAC}$-matrices synthesized by single perturbations using mc and pc, as explained in Subsection 2.1.

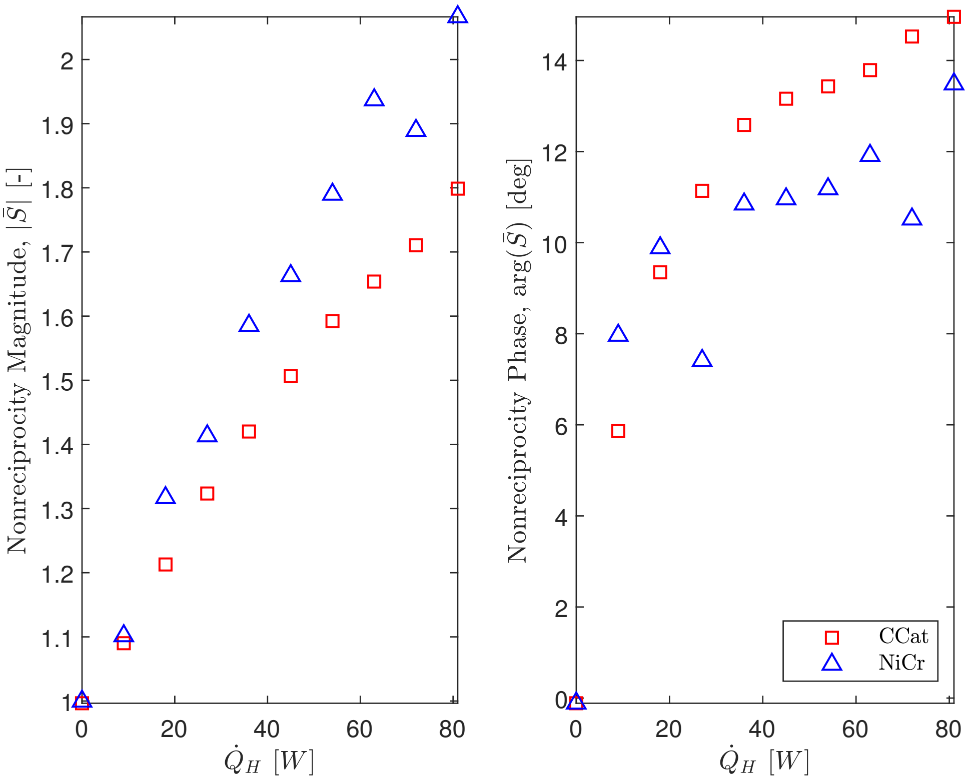

Figure 2. Magnitude and phase of the determinant of $\mathbf{T}_{TAC}$-matrices for a CCat TAC (red square) and a NiCr TAC (blue triangle) as a function of the heating power supply $\dot{Q}_H$.

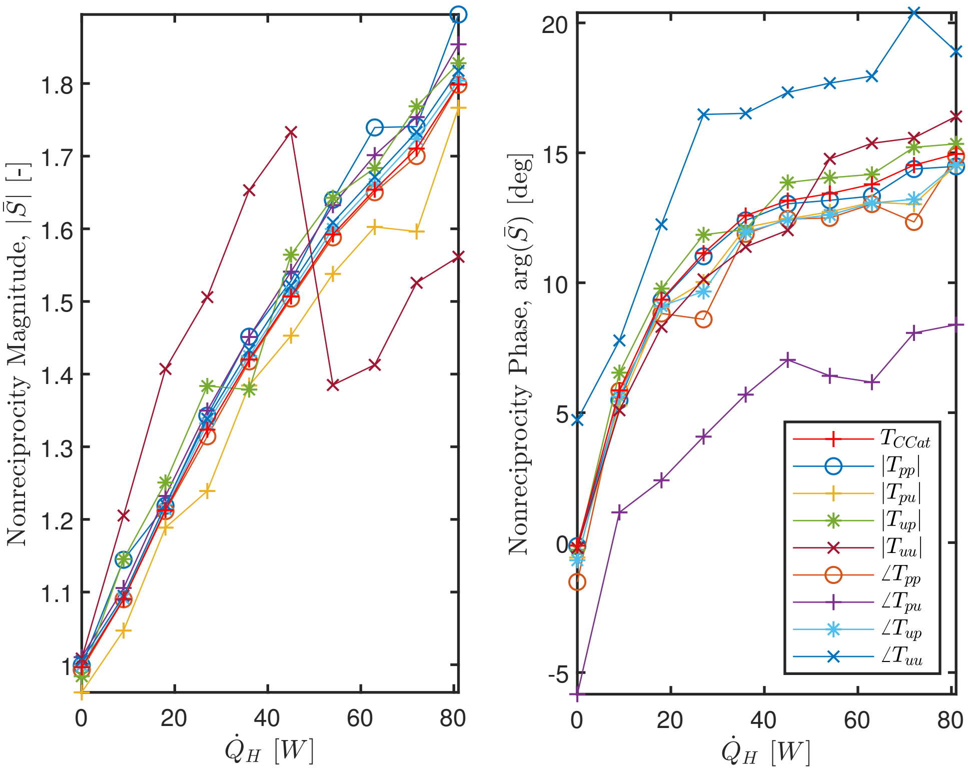

Figure 3. Nonreciprocity magnitude ($|\bar{S}|$, at left) and nonreciprocity phase ($\arg(\bar{S})$, at right) of the synthesized transfer matrices of Gomes and Bannwart (2018) compared to their respective original values at 70 Hz as a function of $\dot{Q}_H$.

It becomes clear that some $\bar{S}$ values at $\dot{Q}_H = 0W$ are out of the expected reciprocal condition ($\bar{S} = 1$, or S) when looking at the phase plot. At least four of the synthesized matrices at $0W$ have $\arg(\bar{S})$ values which significantly depart from 0. For example, the $\mathbf{T}_{TAC}$-matrices synthesized from single perturbations of the phase of T_{pp}, T_{pu} and T_{uu}

differ from zero by values higher than ± 0.01 (which corresponds to an error of 10%). The latter one (perturbation on $\angle T_{uu}$) corresponds precisely to the invalid $\mathbf{T}_{TAC}$-matrix synthesized by Gomes and Bannwart (2018), even if it does not correspond to the greatest deviation from $\arg(\bar{S})$.

Figure 4 shows $\bar{S}$ values calculated for synthesized $\mathbf{T}_{TAC}$-matrices based on the single perturbation of $\arg(T_{uu})$. These are represented in blue lines with crosses, in contrast to their original $\mathbf{T}_{TAC}$-matrices of a CCat TAC (red lines with plus marks). One can also see that the nonreciprocity curves from $\mathbf{T}_{TAC}$-matrices synthesized as described in Subsection 2.1 are subject to more oscillations than the experimental ones. Such discrepant difference might signal that the approach chosen for this synthesis not necessarily may synthesize matrices which correspond to the same synthetic (hypothetical) TAC under several heating power supply values but actually to several TACs, each operating at a specific $\dot{Q}_H$.

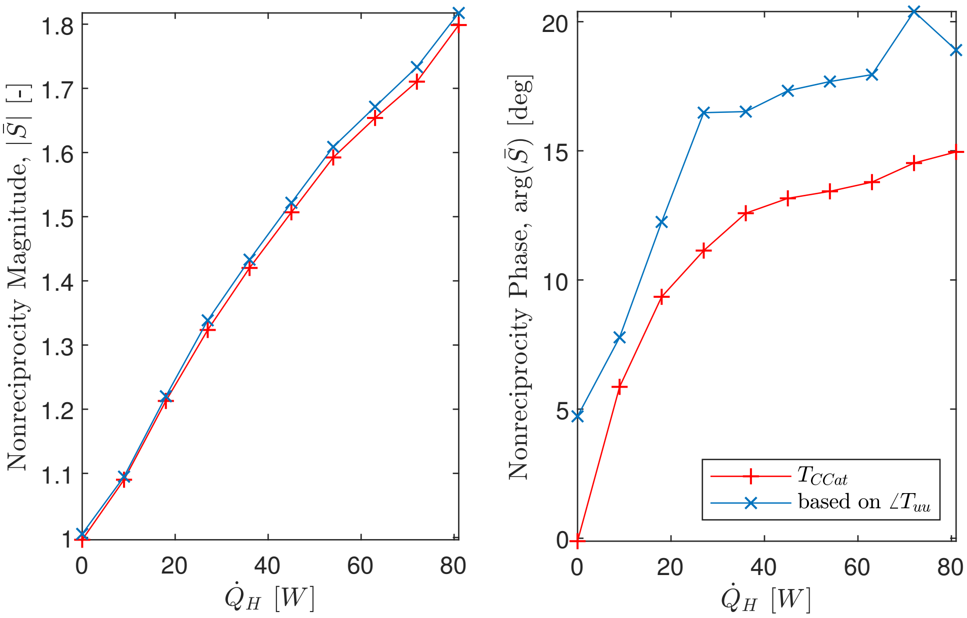

Figure 4. Nonreciprocity magnitude ($|\bar{S}|$) and nonreciprocity phase ($\arg(\bar{S})$) of the synthesized transfer matrices of Gomes and Bannwart (2018) compared to their respective original values of a CCat TAC at 70 Hz as a function of $\dot{Q}_H$.

By comparing Figs. 2 and 3, one can realize that most of the synthesized $\mathbf{T}_{TAC}$-matrices at regimes of useful work ($\dot{Q}_H > 27$ W) seem to lie within the values of nonreciprocity from experimental data. From this, it can be assumed that most of them could correspond to conceivable TACs. Hence, we propose the use of $\bar{S}$ values for selecting synthesized $\mathbf{T}_{TAC}$-matrices which could physically express the behavior of conceivable TACs.

4. THE USE OF NONRECIPROCITY TOWARD THE INVERSE PROBLEM

Inverse problems, generally speaking, consist in estimating model parameters of a system employing known outputs by means of a minimization function (Doutres *et al.*, 2005). They are opposed to forward problems, where a forward model is defined and has a unique solution that predicts results of measurements. For this reason, inverse problems are tied not only to input data (i.e. results of experiments), but also to a priori information on model parameters which are needed to select a single solution model from an infinite number of solution sets (Tarantola, 2005).

The inverse problem we are aiming toward in this work is of estimating two geometrical parameters (porosity and equivalent cylindrical pore radius) from transfer matrices, just like in the works of (Bannwart, 2014), where this sort of problem has been solved and explored with good agreement for measured $\mathbf{T}_{TAC}$-matrices of four porous material samples for TACs at homogeneous ambient temperature. We hereby propose to extend this solution by employing the same method, nevertheless for inhomogeneous temperature profile and from synthesized $\mathbf{T}_{TAC}$-matrices.

4.1 Approach toward the inverse problem

Bannwart *et al.* (2013) implemented a forward model for the TAC as a series of acoustics elements in steady-state regime, each one described by its own transfer matrix. They were later on explored in an inverse problem in Bannwart (2014) so that to calculate two geometrical properties of the porous material -the porosity and equivalent cylindrical pore radius- by means of a minimization function, developed in the works of Guédra (2012), involving the measured $\mathbf{T}_{TAC}$ matrices for $Q_H = 0W$; the tortuosity was assumed unitary. Guédra *et al.* (2015) extended this exploration including the calculation of the porous tortuosity for non-cylindrical pores by means of a the new minimization function, represented as $F(\mathbf{\Upsilon})$, nomenclature adopted in this work. The latter is a quadratic norm between each of the four T-coefficients of either experimental or iterative theoretical data:

$$F(\mathbf{\Upsilon}) = \sum_{i=1}^{4n} |\bar{Y}_i - Y_i(\mathbf{\Upsilon})|^2. \tag{4}$$

In Eq. (4) the superscript $^-$ denotes experimental data, $\bar{Y}_i$ and Y_i are vectors of elements obtained by sequentially concatenating each of the T-coefficients in each of the characterized frequencies, and $\mathbf{\Upsilon} = [\phi, r]$ is the estimated parameter vector, being ϕ the porosity and r the equivalent pore radius of the porous material of the theoretical model.

Tortuosity above unity is an undesired feature for all thermoacoustic devices, as it affects thermal performance for the worse. The tortuous structure of regenerators makes their modeling and simulation difficult when dealing with TW TAEs (Bannwart *et al.*, 2013; Saat and Jaworski, 2017). According to Flores-Bonano *et al.* (2019), tortuosity is associated with the acoustic (and thermal) absorption capacity of porous media and therefore acts against the thermoacoustic effect. Despite augmenting the thermal contact, tortuosity leads to unwanted viscous and thermal relaxation losses (Doutres *et al.*, 2005); besides, it locally alters the phasing between pressure and velocity waves inside the TAC. Relevant tortuosity is intrinsically and unavoidably present in porous materials of thin pores in general, like fibrous materials, for example. Nevertheless, the overall benefits result prevalent for the case of a regenerative cycle, and such materials are of main interest for TW TAEs. In this work, the tortuosity is assumed unitary, as in Bannwart (2014), so that to rely in the same experimental data and referential parameters for the same inverse problem.

By applying synthesized $\mathbf{T}_{TAC}$-matrices representing TACs at desired heated conditions ($\dot{Q}_H > 0$) to this inverse problem solution, we will be able to possibly estimate porosity and equivalent porous radius of an equivalent porous material at those heating regimes. By estimating these parameters under ambient conditions (where no thermal effects are present) with fitted curves from experimental data, such geometrical properties of an equivalent porous material could be finally estimated, leading to a possibly better performing porous material design.

4.2 Selection by nonreciprocity and heat input estimation of synthesized TAC transfer matrices

The synthesized $\mathbf{T}_{TAC}$-matrices which would violate energy conservation, such as those mentioned in section 2.2 and also shown in Fig. 4, should naturally not even be considered as inputs for the inverse problem. Such matrices should be then discarded and smaller T-coefficient perturbations tried. This will lead to an iterative process where synthesized matrices are filtered by nonreciprocity curves, whose values indirectly function as physical restrictions. Thus, the selection by $\bar{S}$ values may avoid impossible systems violating conservation laws, and in particular the first law of thermodynamics (at 0 W), as well as prevent unrealistic acoustic field amplification values.

Moreover, nonreciprocity could be used as an estimation of the heat input associated to a synthesized matrix. The temperature profile is a consequence of the steady state heat transfer through the TAC and, therefore, it is also a function of the mechanical structure and thermophysical properties of the compounding materials. If the temperature profiles were known, Rott's linear equation of thermoacoustics could be solved to estimate the acoustic field throughout the whole device. Yet, the temperature distribution is very complex to be determined a priori, reason why it is easier to deal with heat input $\dot{Q}_H$.

5. CONCLUSIONS

The thermoacoustic core transfer matrix ($\mathbf{T}_{TAC}$) synthesis by coefficient perturbations produces matrices corresponding to predicted better performing TACs. However, the synthesized $\mathbf{T}_{TAC}$-matrices may have coefficient values that differ considerably from existing systems under similar conditions. In fact, sometimes these matrices may not correspond to realistic TACs, especially under low heating conditions. Such artificially obtained TACs may not obey restrictions imposed by the conservation laws.

To work around this issue, the determinant of the $\mathbf{T}_{TAC}$-matrix ($\bar{S}$) was proposed herein as a parameter for measuring the divergence of hypothetical TACs to existing real TACs. If applied to active nonreciprocal systems, $\bar{S}$ may be considered as a measurement of how distant a system under analysis is from a realistic TAC. Regarding the ambient temperature systems (i.e. passive), the deviation of S signals their invalidity. Thus $\bar{S}$ values may be used for selecting conceivable TACs from synthesized transfer matrices.

From transfer matrices of characterized TACs of previous works, nonreciprocity values were calculated for several thermal conditions, revealing a relationship of proportionality between heating power supply and deviation from reciprocity. We aim to explore this relationship by setting predefined limits which may serve as tolerances for selecting synthesized $\mathbf{T}_{TAC}$-matrices in future works. This may serve then as a preparatory step toward their application on inverse problems for the design of synthetic TACs.

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