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NUMERICAL ANALYSIS OF THE NATURAL CONVECTIVE HEAT TRANSFER BETWEEN TWO VERTICAL ISOTHERMAL WAVY SURFACES

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Abstract. *This article consists of a numerical analysis of the natural convective heat transfer from two wavy vertical surfaces separated horizontally. The horizontal distance that separates the surfaces is relatively small in such a way that the flow from both surfaces influences the heat transfer rates from each. The waves considered in this work have a rectangular shape with constant height, having the effect of increasing the heat transfer rate of the heated plates to an adjacent fluid, in this case, ambient air. The main objective of this article is to determine the natural convective heat transfer rates from the vertical wavy surfaces to the ambient air. Effects on the natural convective heat transfer from the individual surfaces of each plate as well as from the total surface of each plate were analyzed according to the following variables: dimensionless height of the waves, dimensionless width of the waves, dimensionless horizontal distance between the surfaces and the Rayleigh number. Results were obtained numerically using the standard* $k-\varepsilon$ *turbulence model including effects of buoyant forces with the aid of the commercial CFD solver ANSYS FLUENT©. Depending on the results obtained, geometric recommendations of the situation under analysis can be made that provide the greatest improvement in the natural convective heat transfer rate from the vertical wavy surfaces.*

Keywords: *natural convective heat transfer, wavy surfaces, heat transfer enhancement,* $k-\varepsilon$ *turbulence model*

1. INTRODUCTION

Natural convective heat transfer occurs in many practical situations and remains an important area of study due to its engineering applications, i.e., heat exchangers, heat treatment processes, drying processes, among others. Yao (1983) studied natural convection along a vertical sinusoidal surface through transformation method. Bhavnani and Bengles (1990) evaluated the effect of surface geometry on a laminar natural convection heat transfer from a vertical flat plate with roughness and one of their conclusions is that the enhancement of a heat transfer occurs using transverse roughness elements with proper sizing and shape selection. Yao (2006) concluded that the total heat transfer rate for the vertical wavy surfaces is indeed much greater than that of a flat plate. Oosthuizen (2010) studied about laminar and turbulent flow in a vertical isothermal wavy surface and noted that the condition under which transition from laminar to turbulent flows occurs is not strongly affected by the presence of surface waviness.

Increasing the heat transfer rate in a given situation involving natural convective flows is often difficult to accomplish and one method to enhance natural convective heat transfer rate is the use of wavy surfaces. The enhancement of the heat transfer rate by using these surfaces arises from the increase in the surface area exposed to the fluid to which the heat is being transferred. As mentioned by Bhavnani and Bengles, the enhancement of the heat transfer rate will depend on the proper sizing (shape and relative size) of the wavy surface. Many wavy shapes have been considered in past studies but the most common shapes considered remain rectangular, triangular (also known as saw-tooth) and sinusoidal waves.

The improvement of the heat transfer rate due to using a wavy surface will also depend on the flow situation being considered; for example, flow over a plane surface or flow over a cylinder and on the thermal boundary conditions at the surface. In this paper, attention will be restricted to external natural convective flows - that is, flow situations in which there are no constraining boundary surfaces close enough that surfaces considered to have any significant influence on the natural convective flow over these surfaces. The two surface boundary conditions most commonly considered are those in which there is a uniform temperature over the surface and those in which there is a uniform heat flux over the surface. Another factor that influences the natural convective heat transfer rate from a surface is its orientation - that is, is it horizontal or is it vertical, or is it inclined to the vertical and whether, when inclined, it is facing upward or downward (Oosthuizen, 2016).

The purpose of the present article is to undertake a numerical study of natural convective heat transfer from two thin, two-sided, two-dimensional vertical plates having a uniform surface temperature. The surface shape is wavy, and attention has been given to the case where the surface waves have a rectangular shape with constant height. The importance of the present work is related to the increase of the natural convective heat transfer rate in situations where the implementation of a forced convection would be difficult to apply or even impossible. This may be the case, for example, when cooling a circuit board assembly or in cases where the thermal management of electronic components must be performed in more detail.

2. PHYSICAL SITUATION

The physical situation analyzed in this study can be visualized in Fig. 1:

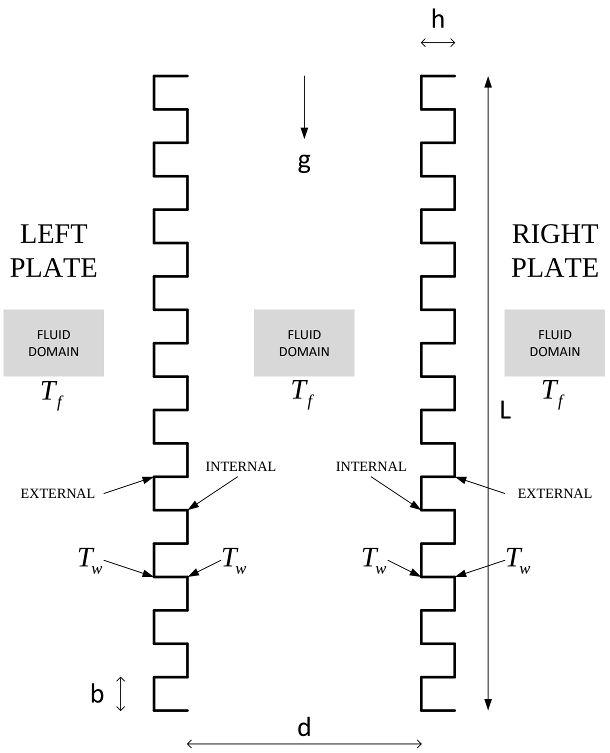


Figure 1. Two vertical wavy surfaces.

The situation under analysis consists of two thin, two-sided, two-dimensional vertical plates having a uniform surface temperature T_w. The surface shape of both plates is wavy and attention has been given to the case where the surface waves have a rectangular shape with constant height *b* and width *h*. The nineteen rectangular waves of both plates are equally spaced. The plates are in contact with a surrounding fluid at constant temperature T_f. For a heated surface, $T_w > T_f$ and both the top and bottom surfaces of both plates will exchange energy with the surrounding fluid by natural convection. The vertical surfaces have unit length *L* and unit depth *w*. The horizontal distance between plates is denoted by d. The purpose of this study is to calculate the heat transfer rate by natural convection between the heated surfaces and the surrounding fluid. The mean Nusselt numbers related to the natural convective heat transfer from the external and internal surfaces of each plate can be calculated using the Newton's law of cooling and the definition of the mean Nusselt number based on the horizontal distance *d* between the surfaces, that is:

$$\overline{\mathrm{Nu}}_{d\,(left,external/internal)} = \frac{q_{(left,external/internal)}d}{A(T_w - T_f)k} \tag{1}$$

$$\overline{\mathrm{Nu}}_{d\,(right,external/internal)} = \frac{q_{(right,external/internal)}d}{A(T_w - T_f)k} \tag{2}$$

where $\overline{\mathrm{Nu}}_d$ is the mean Nusselt number based on *d* and on the mean heat transfer rate, q is the mean heat transfer rate, *k* is the thermal conductivity of the fluid and *A* is the total heat transfer area, calculated using *L*, *b*, *h* and the number of waves, according to Fig. 1.

3. SOLUTION PROCEDURE

In obtaining the numerical results discussed above, the mean flow has been assumed to be steady. The Boussinesq approximation has been used (i.e., fluid properties have been assumed to be constant except for the density that change with temperature in momentum equation). This gives rise to the buoyancy forces and the density change being assumed to be proportional to the temperature change. Radiation heat transfer effects have been neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. In order to deal with this, the $k-\varepsilon$ turbulence model with standard wall functions and with full account being taken of buoyancy force effects has been used.

The mathematical model consists of an equation for the turbulent kinetic energy κ, Eq. (3), and a transport equation for the dissipation of turbulent kinetic energy ε, Eq. (4):

$$\frac{\partial(\rho\kappa)}{\partial t}+\text{div}\,(\rho\kappa\text{U})=\text{div}\left[\frac{\mu_t}{\sigma_\kappa}\text{ grad }\kappa\right]+2\mu_t S_{ij}\cdot S_{ij}-\rho\varepsilon \tag{3}$$

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\text{div}\,(\rho\varepsilon\text{U})=\text{div}\left[\frac{\mu_t}{\sigma_\varepsilon}\text{ grad }\varepsilon\right]+C_{1\varepsilon}\frac{\varepsilon}{\kappa}2\mu_t S_{ij}\cdot S_{ij}-C_{2\varepsilon}\rho\frac{\varepsilon^2}{\kappa} \tag{4}$$

Equations (3) and (4) contains five adjustable constants, e.g., C_μ, σ_k, σ_ε, $C_{1\varepsilon}$ and $C_{2\varepsilon}$. The standard $k-\varepsilon$ turbulence model uses values for these constants obtained through comprehensive curve adjustments for a wide range of turbulent flows, i.e., $C_\mu=0.09$, $\sigma_k=1.00$, $\sigma_\varepsilon=1.30$, $C_{1\varepsilon}=1.44$ and $C_{2\varepsilon}=1.92$. U is the velocity vector, μ_t is the turbulent viscosity and S_{ij} is the deformation rate. The horizontal wavy surfaces has unit depth w and unit width L maintained at a uniform surface temperature $T_w=310$ K. The surrounding fluid is air at a temperature $T_f=290$ K at atmospheric pressure in all cases.

The governing equations subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT©. The numerical approach used here in order to determine when turbulence develops which involves solving the Reynolds averaged governing equations together with a turbulence model, in which the effects of buoyancy forces are taken into account for all conditions considered and then monitoring the results obtained with increasing Rayleigh numbers to determine when significant turbulence effects develop. This approach has been used quite extensively in the study of forced convective flows, e.g., see (Schmidt and Patankar, 1991) and (Zheng *et al*., 1998). The solutions presented in this work all basically have the following parameters:

1. The Rayleigh number, Ra_d, based on the reference length scale d of the heated surface and the difference between the temperature of the heated surface, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$Ra_d=\frac{g\beta(T_w-T_f)d^3}{\nu\alpha} \tag{5}$$

2. The dimensionless height of the waves, $B=b/L$;
4. The dimensionless width of the waves, $H=h/L$;
5. The dimensionless vertical distance between the plates, $D=d/L$, and
5. The Prandtl number, Pr.

In Eq. (5), Ra_d is the Rayleigh number based on *d*, *g* is the gravitational acceleration, β is the bulk coefficient of thermal expansion, *d* is the horizontal distance between the plates, ν is the kinematic viscosity of the fluid and α is the thermal diffusivity of the fluid. Results have only been obtained for a Prandtl number of 0.71, i.e., effectively the value for heated surface at 310 K and air at 290 K.

Before obtaining numerical results, a mesh independence study was carried out using the highest Rayleigh number value, i.e., 10^{12}, for a case where $B=H=0.0526232$ and $D=0.1$. All meshes were created with the aid of the GAMBIT© software. Results of the mesh independence test can be seen in Tab. 1:

Table 1. Mesh independence test results.

Number of elements	$\overline{\mathrm{Nu}}_{d\,(left,total)}$	$\overline{\mathrm{Nu}}_{d\,(right,total)}$
102700	256.150776	244.147581
147888	254.981838	245.070740
201292	256.371869	247.287302
262912	257.898331	249.543655
332748	259.960960	252.021460
410800	261.404575	253.452438
497068	262.485099	254.984226

According to Tab. 1, for 410.800 elements, the mean Nusselt numbers remained approximately constant. This number of elements was then used in all numerical simulations. In all simulations, the mean Nusselt number integrated over the surface was monitored to ensure convergence and to verify that the simulation reached the steady state. The complete computational domain is 1.5m width and 5.0m high. The configuration of the ANSYS FLUENT© solver was based on the work of Oliveira and Oosthuizen (2018), Oliveira and Oosthuizen (2019), Oosthuizen (2016a) and Oosthuizen (2016b), having already been extensively tested and validated with results of numerical and experimental works by these authors. The convergence criteria used for all variables in numerical simulations was 10^{-5}.

4. RESULTS AND DISCUSSION

The different mean total Nusselt numbers, related to the right/left plates and internal/external sides should show differences because of the orientation of the surface waves, both in the lower and upper locations. As differences in the mean total Nusselt numbers for the external right/left and internal right/left are expected to be small because of the significant number of waves (19), a symmetric case was run to assess uncertainty in the numerical procedure.

Initially, have been determinate five distances to evaluate whether these distances should be enough to generate some interaction between the plates during the heat transfer process from heated surfaces to surrounding fluid. It is important to note that the plates are positioned with the same orientation. The initial distances chosen was *d* equal to 0.1, 0.2, 0.3, 0.4 and 0.5m with both, dimensionless height and width, of rectangular wave equals to 0.052632 and using a range for Ra from 10^6 to 10^{12}. Based on these measures, an equal number of computational grids was created to simulate the situation purpose for authors and after obtaining the results, it was noted that for these distances there is no interaction between the plates, as shown in Fig. 2:

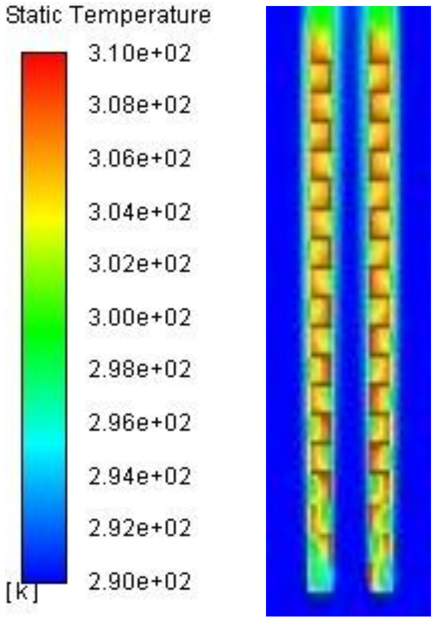


Figure 2. Temperature distribution along the plates for d = 0.1 m and Ra = 10^6.

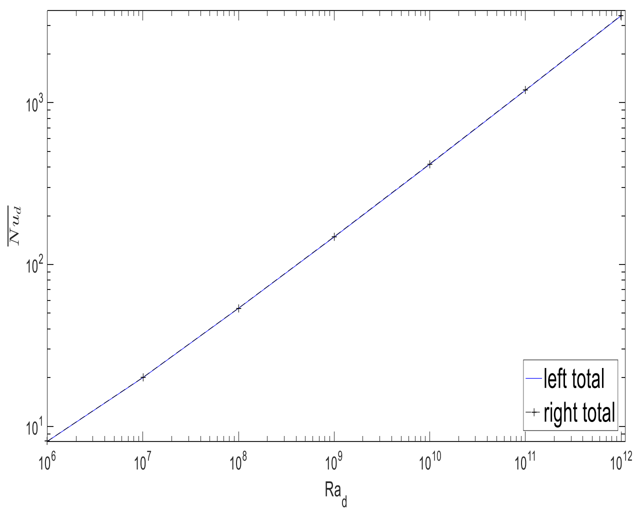
New distances were defined as being 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, 0.03, 0.02 and 0.01m with the same measures of height and width. Creating new computational grids and then solving in ANSYS FLUENT© resulted in the heat transfer rate increasing considerably at over 80W between 0.05m and 0.04m until beyond 650W between 0.02m and 0.01m for both sides (for this reason only the data between 0,,05 ≤ d ≤ 0,01m will use in this study, as shown in tables 2-3). This indicates that the distance between the plates is an important parameter for heat transfer rate, due to a greater interaction of plates as well as the Rayleigh number (the heat transfer increases with higher Ra). It can also be noted from Tables 2-3 that the heat transfer rates both left and right sides are practically the same. In Fig.3 can be noted the behavior of Nusselt in function of Rayleigh.

Table 2: Total heat transfer rate (W) for left side varying Ra from 10^6 to 10^{12} for $0,05 \leq d \leq 0,01$m

	0,05	0,04	0,03	0,02	0,01
10^6	355,771	438,470	577,395	860,360	1711,747
10^7	789,481	1019,255	1430,342	2276,156	4747,821
10^8	2013,993	2671,670	3835,036	6240,634	13374,063
10^9	5427,400	7284,625	10575,204	17454,091	38254,023
10^{10}	15071,354	20361,673	29773,902	49647,509	110585,734
10^{11}	42728,635	57948,006	85230,136	142861,320	320867,220
10^{12}	122896,914	167032,375	246012,840	412114,720	840634,910

Table 3: Total heat transfer rate (W) for right side varying Ra from 10^6 to 10^{12} for $0,05 \leq d \leq 0,01$m

	0,05	0,04	0,03	0,02	0,01
10^6	354,637	439,478	578,819	865,032	1742,783
10^7	791,231	1023,261	1434,842	2293,363	4831,307
10^8	2019,794	2681,358	3846,495	6284,819	13640,717
10^9	5451,381	7320,788	10626,109	17615,174	39026,497
10^{10}	15140,824	20467,634	29943,245	50137,443	112811,594
10^{11}	42934,996	58277,385	85643,004	144312,031	324508,240
10^{12}	123536,766	167861,258	247348,310	413884,690	841267,310



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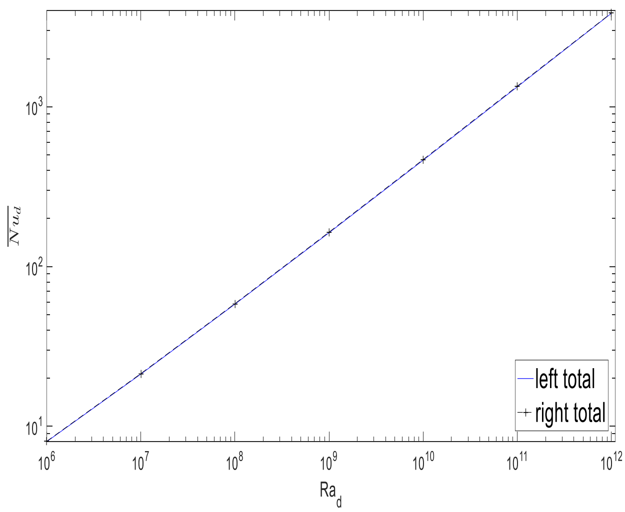

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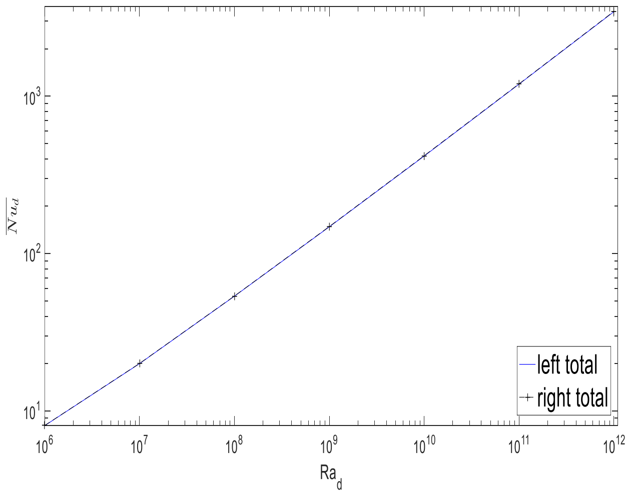
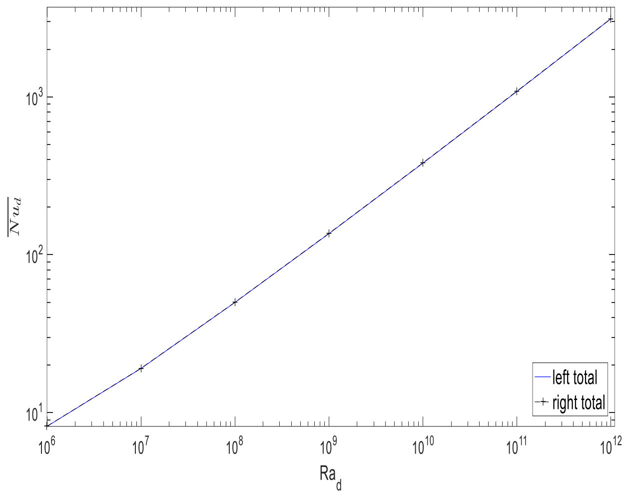


(c)



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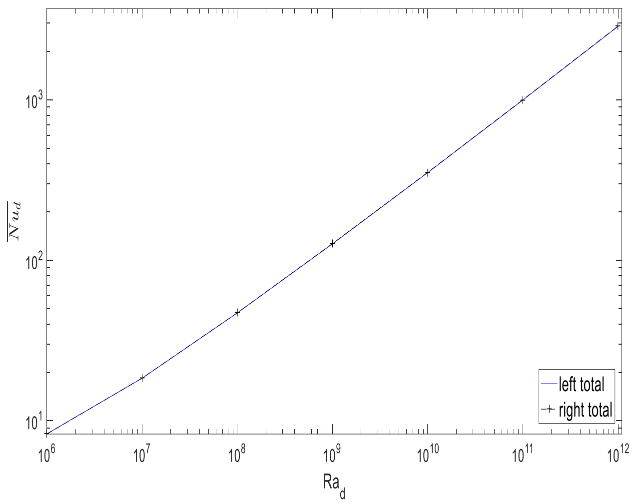
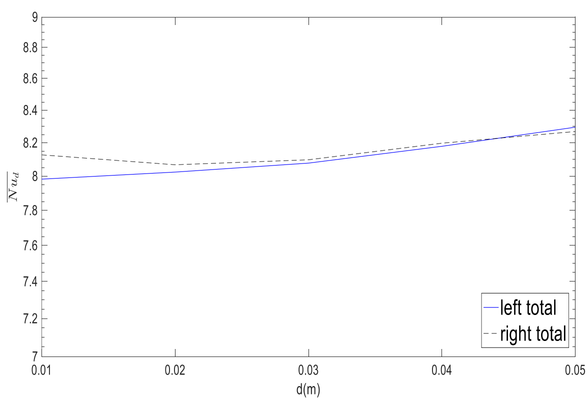
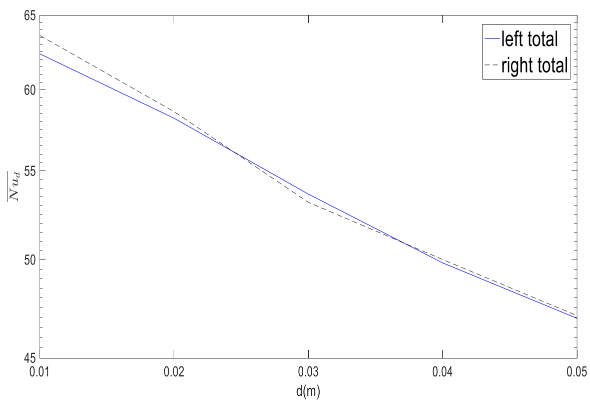


(e)

Figure 3. Variation of the mean total Nusselt number for the left/right plates with the Rayleigh number from 10^6 to 10^{12} for $B = H = 0.052632$ and (a) $d = 0.01m$, (b) $d = 0.02m$, (c) $d = 0.03m$, (d) $d = 0.04m$ and (e) $d = 0.05m$.



(a)



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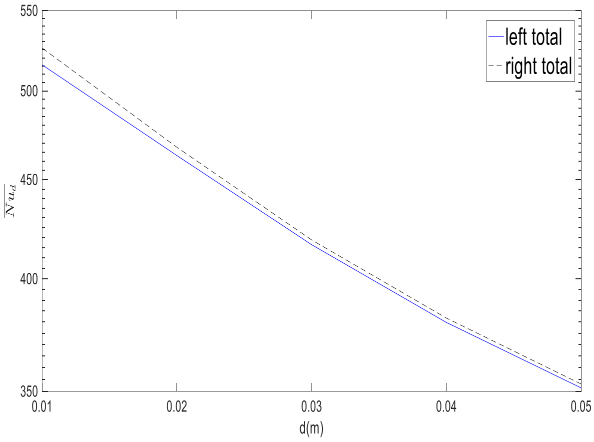
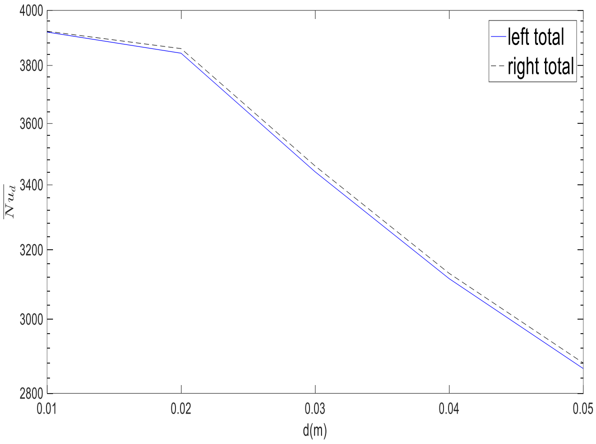




(c) **(d)**

Figure 4**.** Variation of the mean total Nusselt number for the left/right plates with the Rayleigh number equal to (a) 10^6, (b) 10^8, (c) 10^{10} and (d) 10^{12} for $B = H = 0.052632$ and $0.01m \leq d \leq 0.05m$.

Evaluating the Fig.4, note the influence of distance as well as Rayleigh number over heat transfer rate. As shown, increasing the distance between plates decreases the heat transfer rate and, subsequently, the Nusselt number also decreases. However, Fig. 4-a shows otherwise, that is, Nusselt number slightly increases when the plates are removed.

To evaluate the effects of width over the heat transfer rate, the distance *d* between the plates was fixed at 0.05m and the dimensionless height of the waves B was kept in 0.052632 and varying the dimensionless width with follow values: 0.013158, 0.026313, 0.078948, and 0.105264. New computational grids are created and solving then in ANSYS FLUENT©. The total transfer rates are shown in Tables 4-5.

Table 4: Total heat transfer rate (W) for left side varying Ra from 10^6 to 10^{12} for $0,013158 \leq H \leq 0,105264$

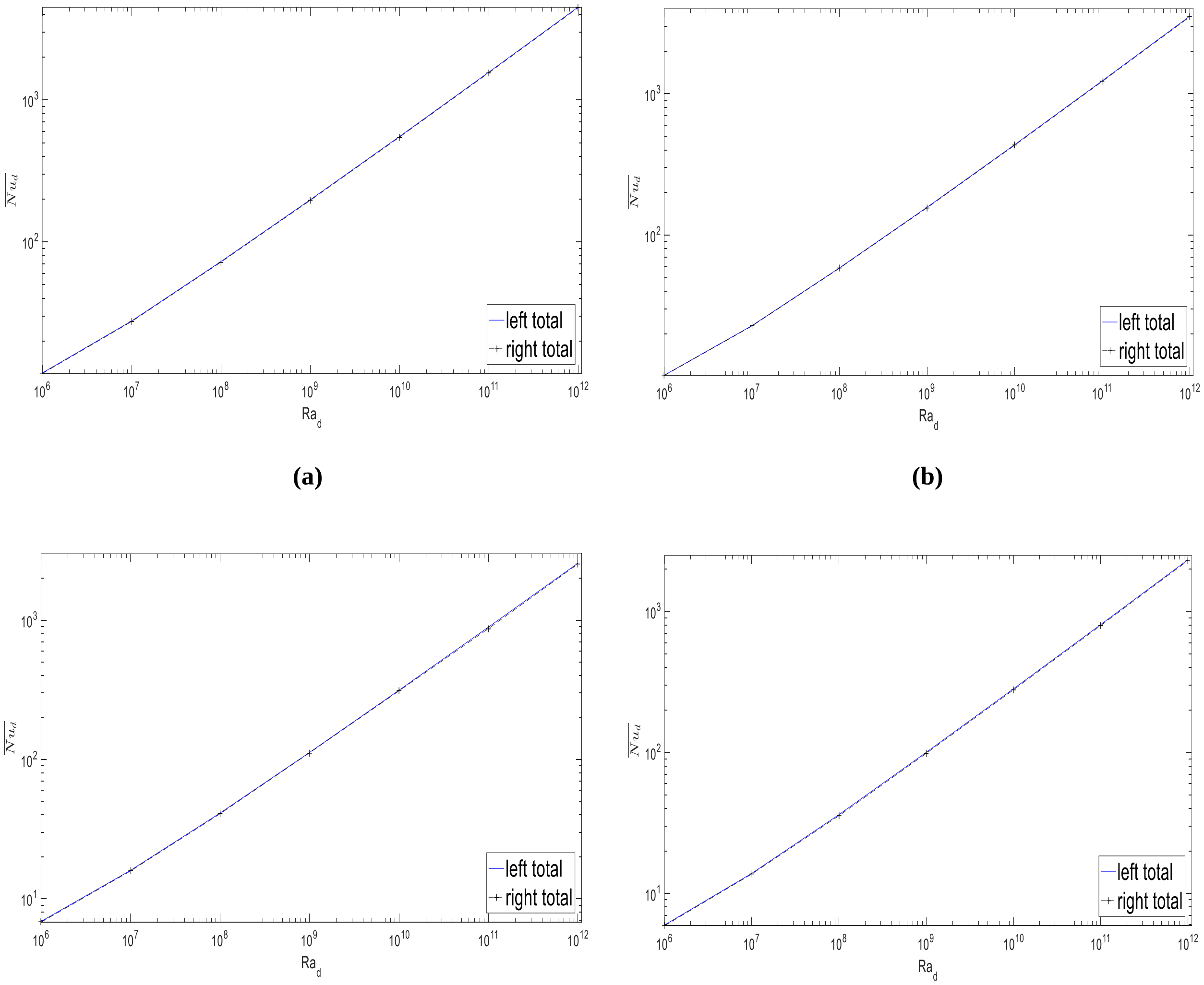
	0.013158	0.026313	0.052632	0.078948	0.105264
10^6	314,991	324,936	355,771	370,792	386,836
10^7	732,427	727,989	789,481	862,252	899,614
10^8	1920,357	1867,999	2013,993	2222,183	2348,353
10^9	5230,664	5013,991	5427,400	6049,406	6492,531
10^{10}	14604,041	13895,678	15071,354	16931,210	18316,885
10^{11}	41478,375	39342,363	42728,635	48066,295	52311,051
10^{12}	119218,840	112825,597	122896,914	138272,175	150801,257

Table 4: Total heat transfer rate (W) for right side varying Ra from 10^6 to 10^{12} for $0,013158 \leq H \leq 0,105264$

	0.013158	0.026313	0.052632	0.078948	0.105264
10^6	312,341	323,643	354,637	366,066	382,592
10^7	725,576	723,825	791,231	853,322	889,575
10^8	1901,429	1855,157	2019,794	2194,680	2306,927
10^9	5176,523	4971,091	5451,381	6013,422	6384,556
10^{10}	14450,084	13754,360	15140,824	16740,655	18022,167
10^{11}	41042,992	38913,406	42934,996	46919,629	51502,031
10^{12}	117983,566	111544,601	123536,766	137610,856	148759,309

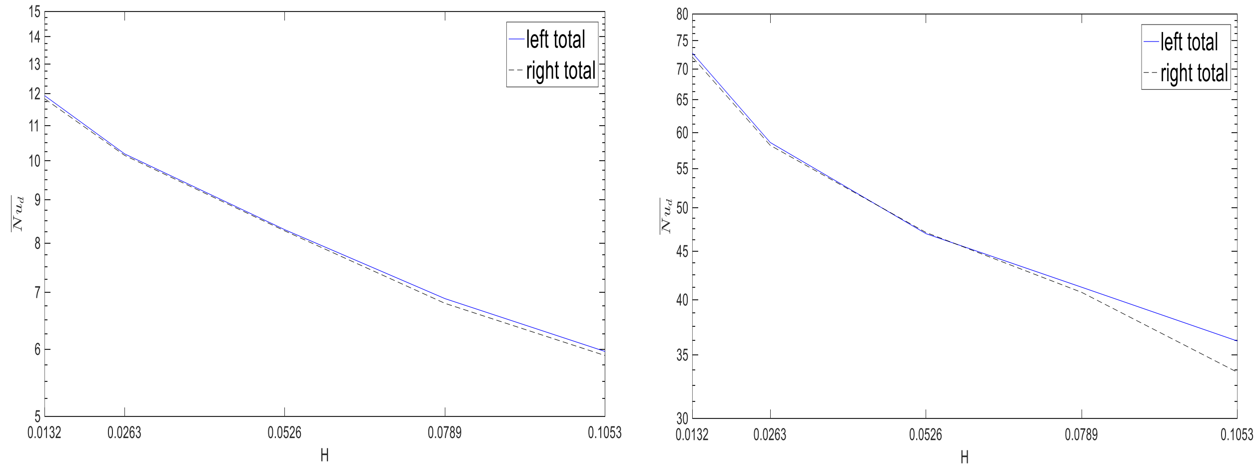
After obtaining results (Tables 4-5 and Fig. 5) it can be noted that the mean total Nusselt number increases since increasing the heat exchange area leads to an improvement in the heat transfer rate. Increasing the areas from

1,263168m^2 to 3,105288m^2 the mean gain for Ra = 10^6 is 70W on both sides and for Ra = 10^{12} the mean gain is over than 30kW.

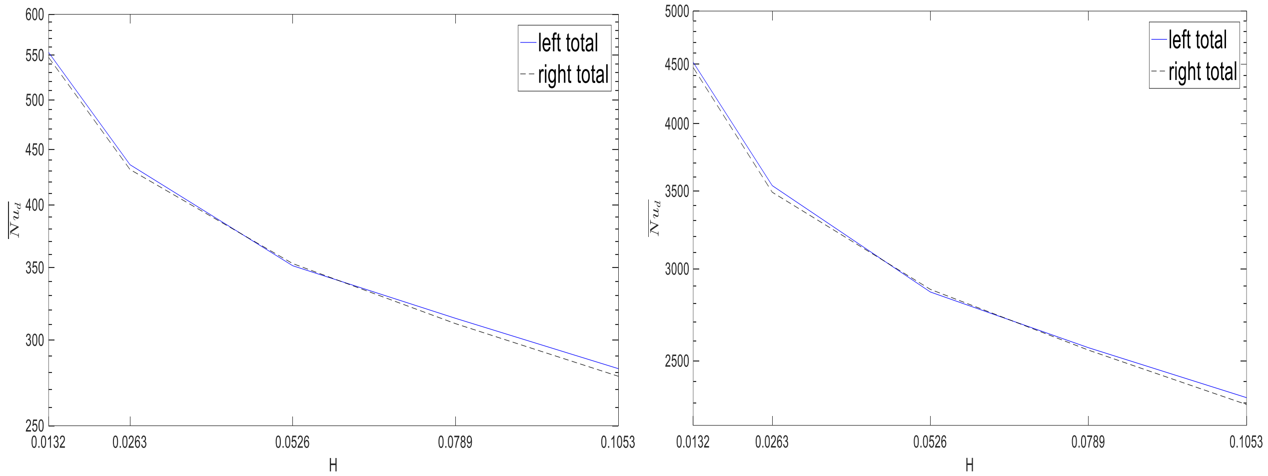


(a) **(b)** **(c)** **(d)**

Figure 5. Variation of the mean total Nusselt number for the left/right plates with the Rayleigh number for *B = 0.052632, D = 0.05* and (a) *H = 0.013158*, (b) *H = 0.026313*, (c) *H = 0.078948*, (d) *H = 0.105264*.



(a) **(b)**



(c) **(d)**

Figure 6. Variation of the mean total Nusselt number for the left/right plates with the Rayleigh number equal to (a) 10^6, (b) 10^8, (c) 10^{10} and (d) 10^{12} for $B = 0.052632$, $D = 0.05$ and $0.013158 \leq H \leq 0.105264$.

In Fig. 6 it can be noted that increasing the width results in Nusselt number declining, even with an increase in the heat exchange area and in the heat transfer rate. This can be explained seeing Eq. 1-2 and concluded that the improvement of heat transfer rate is less significant when compared in heat exchange area.

In general, in natural convective heat transfer situations, correlations between the mean Nusselt number and other parameters of interest can be written with the aid of a power-law expression, i.e.:

$$\overline{\mathrm{Nu}}_d = C\mathrm{Ra}_d^m D^n H^p \tag{6}$$

where the constants C, m, n and p can be obtained using the least-square method. Using the numerical results, the following correlations can be written:

$$\overline{\mathrm{Nu}}_{d\ \mathrm{(left,external)}} = 0.0312\mathrm{Ra}_d^{0.654} D^{0.278} H^{0.231} \tag{7}$$

$$\overline{\mathrm{Nu}}_{d\ \mathrm{(left,internal)}} = 0.0767\mathrm{Ra}_d^{0.945} D^{0.199} H^{0.593} \tag{8}$$

$$\overline{\mathrm{Nu}}_{d\ \mathrm{(right,external)}} = 0.0548\mathrm{Ra}_d^{0.998} D^{0.756} H^{0.176} \tag{9}$$

$$\overline{\mathrm{Nu}}_{d\ \mathrm{(right,internal)}} = 0.0654\mathrm{Ra}_d^{0.175} D^{0.537} H^{0.768} \tag{10}$$

5. CONCLUSIONS

- The distance between the plates is an important parameter, since its reduction leads to a significant increase in the heat transfer rate.
- As well as distance, Rayleigh number is also an important parameter to heat transfer rate. For some cases, the difference between heat transfer rate to Rayleigh from 10^6 to 10^{12} is above 400 times.
- The values of heat transfer rate for both left and right sides presented results very close.
- The variation of width brought an increase to heat transfer rate.
- Nusselt increases when compared with Rayleigh but when compares with distance between plates and with dimensionless width it shows a decreasing behavior.
- Mean Nusselt can be written exponentially in function of Rayleigh, dimensionless distance D and dimensionless width H.

- The ratio between Nusselt and Rayleigh numbers allows to set the flow to desired parameters or limited by project and also allows to estimate the heat transfer rate for the studied and/or applied flow.

6. REFERENCES

Bhavnani, S.H. and Bergles, A.E., 1990. “Effect of surface geometry and orientation on laminar natural convection heat transfer from a vertical flat plate with transverse roughness elements”. *Int. J. Heat Mass Transfer*, Vol. 33, p. 965-981.

Oosthuizen, P.H., 2010. “A numerical study of laminar and turbulent natural convective heat transfer from an isothermal vertical plate with a wavy surface”. In *Proceedings of the ASME 2010 International Mechanical Engineering Congress & Exposition* - IMECE2010, Vancouver, British Columbia, Canada.

Oosthuizen, P.H., 2016. “External natural convective heat transfer from bodies having a wavy surface for conditions under which laminar, transitional and turbulent flow can exist”. *Advances in Heat Transfer*, Vol. 48, p. 261-317.

Oosthuizen, P.H., 2016a. “A numerical study of natural convective heat transfer from a horizontal isothermal surface with rectangular surface roughness elements”. In *Proceedings of the First Pacific Rim Thermal Engineering Conference* – PRTEC 2016, Hawaii’s Big Island, USA.

Oosthuizen, P.H., 2016b. “A numerical study of the effect of triangular waves on natural convective heat transfer from an upward facing heated horizontal isothermal surface”. In *ASME 2016 International Mechanical Engineering Congress and Exposition*, Phoenix, USA.

Oliveira, S.D.R. and Oosthuizen, P.H., 2018. “A numerical study of the simultaneous natural convective heat transfer from the top and bottom surfaces of a thin horizontal plane having a wavy surface”. In *26th Annual Conference of the Computational Fluid Dynamics Society of Canada* – CFD 2018, Winnipeg, Canada.

Oliveira, S.D.R. and Oosthuizen, P.H., 2019. “A numerical study of the simultaneous natural convective heat transfer from the top and bottom surfaces of a thin inclined plate having a wavy surface with constant height”. In *4th Thermal and Fluids Engineering Conference* – TFEC 2019, Las Vegas, USA.

Schmidt, R.C. and Patankar, S.V., 1991. “Simulating boundary layer transition with low-Reynolds-Number k-ε turbulence models: Part 1 – an evaluation of prediction characteristics”. *ASME Journal of Turbomachinery*, Vol. 113, p. 10-17.

Yao, L.S., 1983. “Natural convection along a vertical wavy surface”. *Journal of Heat Transfer*, Vol. 105, p. 465-468.

Yao, L.S., 2006. “Natural convection along a vertical complex wavy surface”. *International Journal of Heat and Mass Transfer*, Vol. 49, p. 281-286.

Zheng, X., Liu, C., Liu, F. and Yang, C.I., 1998. “Turbulent transition simulating using the k-ω model.” *International Journal for Numerical Methods in Engineering*, Vol. 42, p. 907-926.

7. RESPONSIBILITY NOTICE

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