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THERMAL ANALYSIS OF CORE-ANNULAR FLOW WITH TWO IMMISCIBLE LIQUIDS

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Abstract. *Studies among the annular flow in mineral origin products have the outcome as an extremely interesting solution for reducing the global viscosity of the flow. In the present paper, a core-annular flow with two immiscible liquids is thermally analyzed using the Generalized Integral Transformation Technique (GITT). The dimensionless steady-state problem is defined in a semi-infinite circular duct with prescribed wall temperature and a single domain. Through mathematical operations, a generalized formulation is described and the results are analyzed. The integral balance is used to calculate the Nusselt number, whose distribution is studied during the duct flow for the simulated cases as well as its convergence rate by GITT. The results are compared with the literature and the thermal profile is presented in particular positions.*

Keywords: *Heat Transfer, Liquid-liquid Flow, Generalized Integral Transform Technique*

1. INTRODUCTION

Immiscible flows encompass a large range of applications for the liquid-liquid segment, the transportation of heavy oils, in special, is a very popular utilization (Preziosi et al., 1989; Shahidi and Özbelge, 1995; Bannwart, 2001; Su, 2006; Ghosh et al., 2009). From the many types of liquid-liquid immiscible flows, the core-annular flow (CAF) is an efficient method to reduce the flow's viscosity and ease its transport. In this type of flow, one fluid forms an annular film on the pipe wall while the other flows in the pipe center (Angeli and Hewitt, 2000), the fluids under this regime usually have significantly different viscosities and relatively small density range (Bannwart, 2001).

The classical Graetz problem was previously studied for the case of thermally-developing core-annular flow of immiscible fluids (Bentwicw and Sideman, 1964; Su, 2006; Lindemer et al., 2015). The work of Lindemer et al. (2015), for instance, showed a combined numerical-analytical solution for the laminar core-annular flow of two Newtonian fluids, where the solution may be highly affected by the inlet temperature distribution in some cases. Su (2006) developed an exact analytical solution for the hydrodynamically fully-developed laminar core-annular flow of two immiscible liquids, where, considering a circular pipe, the cases of prescribed constant wall temperature and the pipe cooled by an external flow were presented.

The Generalized Integral Transform Technique (GITT) is an established hybrid numerical-analytical methodology for diffusion-advection problems (Cotta, 1993). Specifically for the Graetz problem, several works are available in the literature (Chalhub and Sphaier, 2011a; Sphaier, 2012; Knupp et al., 2013; Braga et al., 2014; Knupp et al., 2015; Chalhub et al., 2016; Knupp et al., 2018; de Barros and Sphaier, 2019; Knupp et al., 2020). Previously, Chalhub and Sphaier (2011b) and Chalhub et al. (2016) evaluated the Graetz problem in parallel plates comparing the achieved GITT results with previous studies and implemented Finite Volume Method (FVM) and Finite Difference Method (FDM) numerical solution, respectively. From the previous studies, the GITT presents, on the overall, very good convergence rates for downstream positions.

In this present work, an extension of the thermal entry problem for the core-annular flow of two immiscible liquids in a circular pipe is proposed using a single domain, semi-infinite channels and prescribed wall temperature boundary condition. The Generalized Integral Transform Technique (GITT) using a single domain was the methodology chosen for a hybrid analytical-numerical approach and the integral balance for the local Nusselt number distribution. The achieved results are compared with existing data from the literature.

2. MATHEMATICAL FORMULATION

Consider the steady-state heat transfer of the laminar core-annular flow of two immiscible fluids in a circular tube. The fluids are assumed to be Newtonian with constant properties, dynamically developed and thermally developing. In the current work, the axial diffusion is neglected. The schematics of the proposed extended Graetz problem is represented in Figure 1 and the mathematical formulation is described by Equations (1):

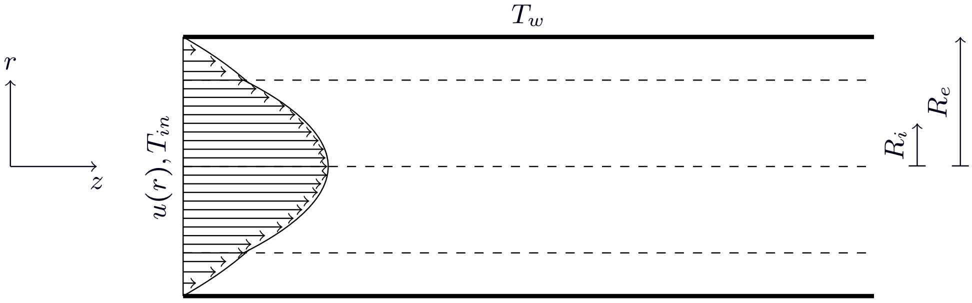

Figure 1. Schematic representation of the problem.

$$u(r)w(r)\frac{\partial T(r,z)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(k(r)\cdot r\,\frac{\partial T(r,z)}{\partial r}\right) \tag{1a}$$

for $\quad 0 \le r \le R_e \quad$ and $\quad z \ge 0$

with the following boundary conditions:

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0; \qquad T(R_e, z) = T_w; \tag{1b}$$

$$T(r,0) = T_{\text{in}}; \qquad |T(r, z \to +\infty)| < +\infty \tag{1c}$$

where T is the temperature, T_{in} is the prescribed uniform inlet fluid temperature which is considered the same for both fluids, u is the velocity profile, w is the heat capacity (ρc_p), k is the thermal conductivity, and R_e is the circular tube's radius. Even though w and k are considered constant within each fluid, the values are different between the inner and outer fluids, therefore, in a single domain approach, w and k depend on the radial direction r.

The dimensionless temperature is defined as

$$\theta(\eta,\xi) = \frac{T(r,z) - T_w}{T_{\text{in}} - T_w} \tag{2}$$

and the following dimensionless parameters are given as

$$k^*(\eta) = \frac{k(r)}{k_i}, \quad w^*(\eta) = \frac{w(r)}{w_i}, \quad \beta = \frac{R_i}{R_e}$$

$$\xi = \frac{z\,k_i}{4R_e\,u_{\text{avg}}\,w_i}, \quad \eta = \frac{r}{R_e}, \quad u^*(\eta) = \frac{u(r)}{u_{\text{avg}}}.$$

where ξ and η are dimensionless versions of x and z, respectively, k^* is the dimensionless thermal conductivity, w^* is the dimensionless heat capacity, β is the radius ratio, u^* and u_{avg} are the dimensionless and average velocity, respectively. k_i, w_i, and R_i refers to the thermal conductivity, heat capacity, and radius of the inner fluid.

The dimensionless form of the mathematical formulation is given by:

$$u^*(\eta)w^*(\eta)\frac{\partial\theta(\eta,\xi)}{\partial\xi} = \frac{4}{\eta}\frac{\partial}{\partial\eta}\left(k^*(\eta)\cdot\eta\,\frac{\partial\theta(\eta,\xi)}{\partial\eta}\right) \tag{3a}$$

for $\quad 0 \le \eta \le 1 \quad$ and $\quad \xi \ge 0$

with the following boundary conditions:

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = 0; \qquad \theta(1,\xi) = 0; \tag{3b}$$

$$\theta(\eta,0) = 1; \qquad |\theta(\eta, \xi \to +\infty)| < +\infty. \tag{3c}$$

Based on Su (2006), the following dimensionless function groups are defined as:

$$u^*(\eta) = \frac{u}{u_{\text{avg}}} = \begin{cases} u_i^*(\eta) = \dfrac{2\left[1-\beta^2+\tilde{\mu}(\beta^2-\eta^2)\right]}{\beta^4(\tilde{\mu}-1)+1}, & \text{if} \quad 0 \le \eta \le \beta \\ u_e^*(\eta) = \dfrac{2(1-\eta^2)}{\beta^4(\tilde{\mu}-1)+1}, & \text{if} \quad \beta < \eta \le 1 \end{cases} \tag{4}$$

$$w^*(\eta) = \begin{cases} 1, & \text{if} \quad 0 \le \eta \le \beta \\ \tilde{w} = \dfrac{w_e}{w_i}, & \text{if} \quad \beta < \eta \le 1 \end{cases} \tag{5}$$

$$k^*(\eta) = \begin{cases} 1, & \text{if} \quad 0 \le \eta \le \beta \\ \tilde{k} = \dfrac{k_e}{k_i}, & \text{if} \quad \beta < \eta \le 1 \end{cases} \tag{6}$$

where $u^*(\eta)$, $u_i^*(\eta)$, $u_e^*(\eta)$ and u_{avg} are the dimensioless, internal, external and average velocities, respectively. For this current problem, the $u^*(\eta)$ dimensionless velocity profile is given according to Su (2006). μ is the dynamic viscosity and $\tilde{\mu}$ is the viscosity ratio (μ_e/μ_i). $w^*(\eta)$ is the dimensionless heat capacity and $\tilde{w}$ is the heat capacity ratio (w_e/w_i). k^* is the dimensionless thermal conductivity and $\tilde{k}$ is the heat capacity ratio (k_e/k_i).

The local Nusselt number for this problem, in terms of the dimensionless variables, can be calculated from:

$$\text{Nu}(\xi) = 2\tilde{k}\frac{(\partial\theta/\partial\eta)_{\eta=1}}{\theta(\xi,1)-\theta_m(\xi)} \tag{7}$$

where the dimensionless bulk temperature is given by:

$$\theta_m(\xi) = (T_{\text{in}} - T_w)\frac{\displaystyle\int_0^1 u^*(\eta)w^*(\eta)\theta(\eta,\xi)\eta\,\mathrm{d}\eta}{\displaystyle\int_0^1 u^*(\eta)w^*(\eta)\eta\,\mathrm{d}\eta} + T_w \tag{8}$$

3. METHODOLOGY

The hybrid numerical-analytical solution of the core-annular problem is accomplished by employing the Generalized Integral Transform Technique (GITT) (Cotta, 1990, 1993). The solution procedure is initiated by defining the transformation pair:

$$\bar{\theta}_n(\xi) = \int_0^1 \theta(\eta,\xi)\Psi_n(\eta)\eta\,\mathrm{d}\eta \tag{9a}$$

$$\theta(\eta,\xi) = \sum_{n=1}^{\infty}\frac{\bar{\theta}_n(\xi)\Psi_n(\eta)}{N_n} \tag{9b}$$

The eigenfunctions Ψ_n are solutions of a Sturm-Liouville type problem. For this problem, the following Helmholtz formulation is chosen:

$$\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}\Psi_n(\eta)}{\mathrm{d}\eta}\right) + \lambda_n^2\Psi_n(\eta) = 0 \tag{10a}$$

$$\Psi_n'(0) = 0 \tag{10b}$$

$$\Psi_n(1) = 0 \tag{10c}$$

which leads to infinite nontrivial solutions of the form:

$$\Psi_n(\eta) = J_0(\lambda_n\eta) \quad \text{for} \quad n = 1, 2, 3, \ldots \tag{11}$$

where J is the Bessel function of first kind.

The eigenvalues λ_n and the normalization integrals N_n related to the above eigenfunctions are obtained from the following relations:

$$J_0(\lambda_n\eta) = 0 \tag{12}$$

$$N_n = \int_0^1 \Psi_n^2(\xi)\eta\,\mathrm{d}\eta = \frac{1}{2}\left(J_0(\lambda_n)^2 + J_1(\lambda_n)^2\right) \tag{13}$$

The transformation of the given problem is accomplished by multiplying Eq. (3) by the eigenfunction $\Psi_n(\eta)$, integrating the equation within the domain $(0 \leq \eta \leq 1)$ and applying the inversion formula (9b) to the non-transformable terms. This process yields the following coupled ODEs system:

$$\sum_{m=1}^{\infty} A_{n,m} \cdot \bar{\theta}'_m(\xi) + \sum_{m=1}^{\infty} C_{n,m} \cdot \bar{\theta}_m(\xi) = 0 \tag{14a}$$

$$\bar{\theta}_n(0) = b_n = \int_0^1 \Psi_n(\eta)\eta \,\mathrm{d}\eta \tag{14b}$$

$$|\bar{\theta}_n(\xi \to \infty)| < +\infty \tag{14c}$$

where the coefficients $A_{n,m}$ and $C_{n,m}$ are given by:

$$A_{n,m} = \frac{1}{N_m} \int_0^1 u^*(\eta) w^*(\eta) \Psi_n(\eta) \Psi_m(\eta)\, \eta \,\mathrm{d}\eta \tag{14d}$$

$$C_{n,m} = \frac{4}{N_m} \left[\lambda_n^2 \int_0^1 k^*(\eta) \Psi_n(\eta) \Psi_m(\eta)\, \eta \,\mathrm{d}\eta - \int_0^1 \frac{\mathrm{d}k^*}{\mathrm{d}\eta} \Psi'_n(\eta) \Psi_m(\eta) \eta \,\mathrm{d}\eta \right] \tag{14e}$$

Coefficient $A_{n,m}$ integration has no closed-form analytical solution, thus a numerical integration must be performed. Conversely, coefficient $C_{n,m}$ is solved analytically. In order to solve system (14) the infinite series must be truncated to a finite number of terms $n_{\max}$, also known as truncation order. After the truncation, the system can be represented as a matrix form equation:

$$\boldsymbol{A}\boldsymbol{\theta}'(\xi) + \boldsymbol{C}\boldsymbol{\theta}(\xi) = 0 \tag{15a}$$

$$\boldsymbol{\theta}(0) = \boldsymbol{b}, \qquad |\boldsymbol{\theta}(\xi \to \infty)| < +\infty \tag{15b}$$

where $\boldsymbol{A}$ and $\boldsymbol{C}$ are matrices generated by coefficients $A_{n,m}$ and $C_{n,m}$ respectively, and $\boldsymbol{\theta}$ is a vector containing the unknown transformed potentials.

$$\boldsymbol{\theta}(\xi) = \left(\bar{\theta}_1(\xi), \bar{\theta}_2(\xi), \bar{\theta}_3(\xi), \ldots, \bar{\theta}_{n_{\max}}(\xi) \right). \tag{16}$$

The matrix equation can be reduced to the following form:

$$\frac{\mathrm{d}\boldsymbol{\theta}(\xi)}{\mathrm{d}\xi} = \boldsymbol{M}\boldsymbol{\theta} \tag{17}$$

where

$$\boldsymbol{M} = -\boldsymbol{A}^{-1}\boldsymbol{C} \tag{18}$$

The solution of the matrix equation can be obtained analytically if the eigenvalues and eigenvectors of $\boldsymbol{M}$ are calculated, yielding:

$$\theta_n = \sum_{m=1}^{n_{\max}} G_{n,m} c_m \exp(\omega_m \xi) \quad \text{for} \quad n = 1, 2, 3, \ldots, n_{\max} \tag{19}$$

in which $G_{n,m}$ are the coefficients of a matrix containing the eigenvectors of $\boldsymbol{M}$ as columns, ω_m are the eigenvalues of $\boldsymbol{M}$, and c_m are arbitrary constants and are calculated directly from the inlet boundary condition:

$$\theta_n(0) = \sum_{m=1}^{n_{\max}} G_{n,m} c_m = b_n \quad \text{for} \quad n = 1, 2, 3, \ldots, n_{\max} \tag{20}$$

After obtaining the transformed temperatures $\bar{\theta}_n$, the original temperature can also be computed by using the inversion formula on Equation (9b) and truncating it in $n_{\max}$ terms in the sum.

In order to obtain the Nusselt number, shown on Equation (7), the derivative $(\partial\theta/\partial\eta)_{\eta=1}$ must be computed. Therefore an integral balance approach (Baohua and Cotta, 1993; Sphaier, 2012; Cotta et al., 2018), is used in this work to avoid the direct derivation of the series and achieve better convergence rates. This approach is performed by integrating the original equation (3a) over the domain. By doing so, an alternative to the derivative mentioned is obtained:

$$\left(\frac{\partial\theta}{\partial\eta} \right)_{\eta=1} = \frac{1}{4k^*(1)} \left[\int_0^1 u^*(\eta) w^*(\eta) \frac{\partial\theta}{\partial\xi} \eta \,\mathrm{d}\eta \right] \tag{21}$$

Finally, the local Nusselt number can be obtained by replacing Eq. (21) in Eq. (7) and applying the inverse formula (9b):

$$\mathrm{Nu}(\xi) = 2\tilde{k}\,\frac{\displaystyle\sum_{m=1}^{n_{\max}} \frac{\bar{\theta}'_m}{4N_m k^*(1)} \int_0^1 u^*(\eta) w^*(\eta) \Psi_m(\eta)\,\eta\,\mathrm{d}\eta}{\displaystyle\sum_{m=1}^{n_{\max}} \frac{\bar{\theta}_m}{N_m}\left(\Psi(1) - \frac{\displaystyle\int_0^1 u^*(\eta) w^*(\eta) \Psi_m(\eta)\,\eta\,\mathrm{d}\eta}{\displaystyle\int_0^1 u^*(\eta) w^*(\eta)\,\eta\,\mathrm{d}\eta}\right)} \tag{22}$$

where the integrals are solved analytically.

4. RESULTS AND DISCUSSION

After illustrating the problem and explaining the used methodology, in this section, the obtained results are shown. The convergence analysis for the local Nusselt number is initially presented for the thermal entry region in Table 1 for different values of β. Then, in Figure 2, the Nusselt number behavior is shown for $\tilde{k} = 5$, $\tilde{w} = 2.5$, $\tilde{\mu} = 0.02$ and $n_{\max} = 1000$.

Table 1. GITT Convergence for local Nusselt using the integral balance with $\tilde{k} = 5$, $\tilde{w} = 2.5$ and $\tilde{\mu} = 0.02$.

$n_{\max}$	$\xi = 0.01$	$\xi = 0.1$	$\xi = 0.5$	$\xi = 1$	$\xi = 10$	$\xi = 100$	$\xi = 1000$
	$\beta = 0.7$						
50	20.6021	11.3440	11.3438	11.3438	11.3438	11.3438	11.3438
100	20.5242	11.3043	11.3040	11.3040	11.3040	11.3040	11.3040
200	20.4858	11.2844	11.2842	11.2842	11.2842	11.2842	11.2842
500	20.4626	11.2725	11.2723	11.2723	11.2723	11.2723	11.2723
1000	20.4549	11.2686	11.2683	11.2683	11.2683	11.2683	11.2683
	$\beta = 0.8$						
50	13.3512	8.89187	8.89173	8.89173	8.89173	8.89173	8.89173
100	13.2975	8.86366	8.86352	8.86352	8.86352	8.86352	8.86352
200	13.2708	8.84963	8.84949	8.84949	8.84949	8.84949	8.84949
500	13.2548	8.84121	8.84107	8.84107	8.84107	8.84107	8.84107
1000	13.2494	8.83840	8.83826	8.83826	8.83826	8.83826	8.83826
	$\beta = 0.9$						
50	9.42711	6.98656	6.98638	6.98638	6.98638	6.98638	6.98638
100	9.40363	6.96643	6.96625	6.96625	6.96625	6.96625	6.96625
200	9.39214	6.95654	6.95636	6.95636	6.95636	6.95636	6.95636
500	9.38527	6.95065	6.95046	6.95046	6.95046	6.95046	6.95046
1000	9.38298	6.94868	6.94850	6.94850	6.94850	6.94850	6.94850
	$\beta = 1$						
50	4.91608	3.65807	3.65679	3.65679	3.65679	3.65679	3.65679
100	4.91607	3.65807	3.65679	3.65679	3.65679	3.65679	3.65679
200	4.91607	3.65807	3.65679	3.65679	3.65679	3.65679	3.65679
500	4.91607	3.65807	3.65679	3.65679	3.65679	3.65679	3.65679
1000	4.91607	3.65807	3.65679	3.65679	3.65679	3.65679	3.65679

As can be seen, good convergence rates are seen for a single fluid, $\beta = 1$. For $\beta = 1$ and $n_{\max} = 100$, the solution achieved the six-digit precision in all the selected positions of ξ. For other values of β, however, worse rates are seen for inlet near positions due to the boundary condition discontinuity and does not achieve the six-digit precision convergence. Downstream positions present a satisfying convergence rate, with 3 to 4 digit-precision. For $\beta = 0.7$, however, at the upstream positions, the solution converged with a three-digit precision at the downstream and upstream positions.

From the data of the Table 1, the flow achieves the thermally developed regime before $\xi = 0.5$. This information can be concluded by the local Nusselt which does not vary after $\xi = 0.5$ for the presented cases. The figure 2 presents graphically the local Nusselt behavior in logarithmic scale for the β tested values and $n_{\max} = 1000$. As can be seen, higher values of β display lower local Nusselt values.

The used methodology is also verified with the previous study of Su (2006). Table 2 indicates the comparison between the GITT summing 1000 terms and the available data for the local Nusselt indicated in the literature after achieving the

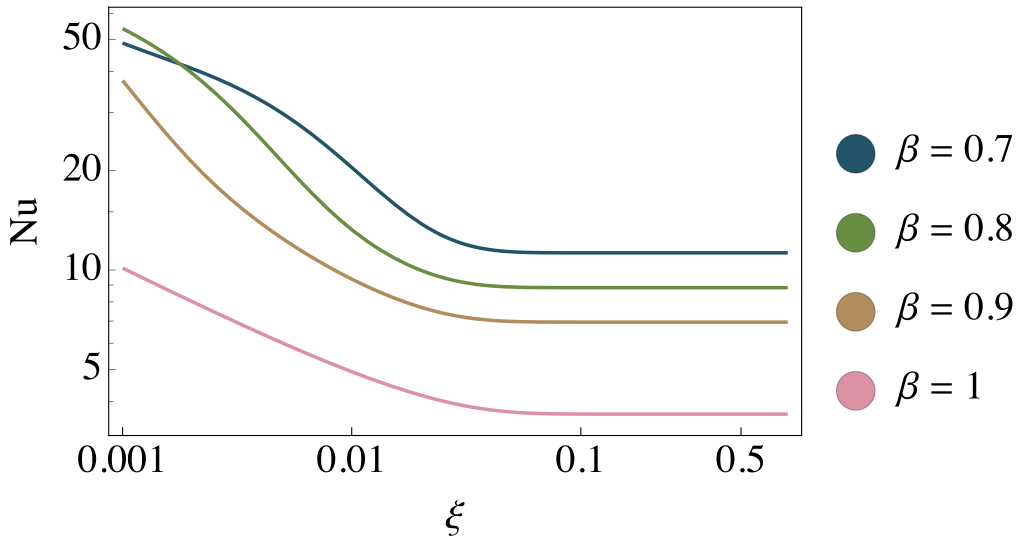

Figure 2. Local Nusselt calculated using the integral balance with $\tilde{k} = 5$, $\tilde{w} = 2.5$, $\tilde{\mu} = 0.02$ and $n_{\max} = 1000$.

thermally developed regime. For the single fluid, the solution matches the with a six-digit precision. Even though the $n_{\max} = 1000$ did not achieve the six-digit precision for other cases, the overall solution approaches to the literature data, endorsing its verification purposes. In those cases, the solutions using different methodologies achieved a three to four-digit precision.

Table 2. Comparison between the GITT for $\xi = 1000$ and Su (2006) data for local Nusselt with $\tilde{k} = 5$, $\tilde{w} = 2.5$, $\tilde{\mu} = 0.02$, $n_{\max} = 1000$ and different β values.

Comparison	$\beta = 1$	$\beta = 0.9$	$\beta = 0.8$	$\beta = 0.7$
$n_{\max} = 1000$	3.65679	6.94850	8.83826	11.2683
Su (2006)*	3.65679	6.94653	8.83545	11.2643

Finally, Figure 3 shows the thermal profile in η-direction for $\xi = 0.01, 0.05$ and 0.1. Flows with different values of β are plotted and indicate the thermal behavior for each different flow. When $\eta = 0$ and $\xi = 0.01$, the flow's temperature is the inlet temperature, as η increases, the fluid is cooled until achieving the wall temperature. As the fluid moves away from the inlet, the temperature is progressively reduced as expected. The annular thermal profiles are shown for $\beta = 0.7$, 0.8, and 0.9. As observed on Figure 3, single fluid flow is defined when $\beta = 1$.

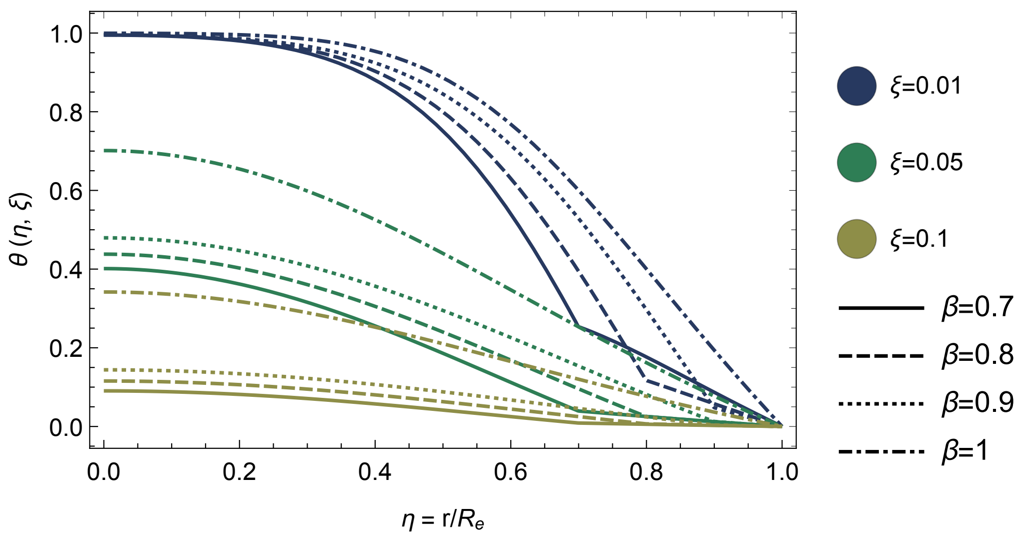

Figure 3. Temperature field of the core-annular flow at $\xi = 0.01$ for different values of β with $\tilde{k} = 5$, $\tilde{w} = 2.5$, $\tilde{\mu} = 0.02$ and $n_{\max} = 1000$.

5. CONCLUSION

This work presented a different approach for the extended Graetz problem in annular flows of two immiscible fluids in circular pipes with a semi-infinite domain. The methodology to obtain the hybrid analytical-numerical solution was based on the Generalized Integral Transform Technique (GITT) and single domain. The local Nusselt was obtained using the Integral Balance. The convergence analysis confirmed that good numerical convergence rates are seen for positions downstream, whereas worse rates are seen for upstream positions. Following the convergence analysis, illustrative results were presented, showing the variation of the local Nusselt number and the temperature profile for different β values. Finally, the thermally developed results were validated through comparisons with data from previous studies. The results matched closely with the literature confirming GITT is a well-suited methodology for core-annular flow problems. Moreover, this methodology could be easily applied in more complicated Graetz formulations such as including the axial diffusion.

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7. REFERENCES

Angeli, P. and Hewitt, G., 2000. “Flow structure in horizontal oil–water flow”. International journal of multiphase flow, Vol. 26, No. 7, pp. 1117–1140.

Bannwart, A.C., 2001. “Modeling aspects of oil–water core–annular flows”. Journal of Petroleum Science and Engineering, Vol. 32, No. 2-4, pp. 127–143.

Baohua, C. and Cotta, R., 1993. “Integral transform analysis of natural convection in porous enclosures”. International journal for numerical methods in fluids, Vol. 17, No. 9, pp. 787–801.

Bentwicw, M. and Sideman, S., 1964. “Temperature distribution and heat transfer in annular two-phase (liquid-liquid) flow”. The Canadian Journal of Chemical Engineering, Vol. 42, No. 1, pp. 9–13.

Braga, N., de Barros, L. and Sphaier, L., 2014. “Generalized integral transform solution of extended graetz problems with axial diffusion”. ICCM2014, pp. 1–14.

Chalhub, D.J., Sphaier, L.A. and Alves, L., 2016. “Integral transform solution for thermally developing slip-flow within isothermal parallel plates”. Computational Thermal Sciences: An International Journal, Vol. 8, No. 2. doi: 10.1615/ComputThermalScien.2016015459.

Chalhub, D. and Sphaier, L., 2011a. “Analysis of integral transform solutions for thermally developing flow in microchannels with isothermal wall”. In ASME 2011 9th International Conference on Nanochannels, Microchannels, and Minichannels. American Society of Mechanical Engineers, pp. 523–530.

Chalhub, D. and Sphaier, L., 2011b. “Comparisons between integral transform, finite volumes and combined solution strategies for convective heat transfer problems”. In ASME/JSME 2011 8th Thermal Engineering Joint Conference. American Society of Mechanical Engineers, pp. T10058–T10058.

Cotta, R.M., 1993. Integral transforms in computational heat and fluid flow. CRC Press.

Cotta, R., 1990. “Hybrid numerical/analytical approach to nonlinear diffusion problems”. Numerical Heat Transfer, Part B: Fundamentals, Vol. 17, No. 2, pp. 217–226.

Cotta, R., Naveira-Cotta, C., Knupp, D., Zotin, J., Pontes, P. and Almeida, A., 2018. “Recent advances in computational-analytical integral transforms for convection-diffusion problems”. Heat and Mass Transfer, Vol. 54, No. 8, pp. 2475–2496.

de Barros, L. and Sphaier, L., 2019. “Improved lumped analysis of graetz problems with axial diffusion”. Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 41, No. 12, p. 564.

Ghosh, S., Mandal, T., Das, G. and Das, P., 2009. “Review of oil water core annular flow”. Renewable and Sustainable Energy Reviews, Vol. 13, No. 8, pp. 1957–1965.

Knupp, D., Cotta, R. and Naveira-Cotta, C., 2020. “Conjugate heat transfer: Analysis via integral transforms and eigenvalue problems”. Journal of Engineering Physics and Thermophysics, pp. 1–14.

Knupp, D.C., Cotta, R.M., Naveira-Cotta, C.P. and Kakaç, S., 2015. “Transient conjugated heat transfer in microchannels: Integral transforms with single domain formulation”. International Journal of Thermal Sciences, Vol. 88, pp. 248–257. doi:10.1016/j.ijthermalsci.2014.04.017.

Knupp, D.C., Mascouto, F.S., Abreu, L.A., Naveira-Cotta, C.P. and Cotta, R.M., 2018. “Conjugated heat transfer in circular microchannels with slip flow and axial diffusion effects”. International Communications in Heat and Mass Transfer, Vol. 91, pp. 225–233. doi:10.1016/j.icheatmasstransfer.2017.12.003.

Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M., 2013. “Conjugated convection-conduction analysis in microchannels with axial diffusion effects and a single domain formulation”. Journal of Heat Transfer, Vol. 135, No. 9, p. 091401. doi:10.1115/1.4024425.

Lindemer, M.D., Advani, S.G. and Prasad, A.K., 2015. "Graetz–brinkman problem in laminar core-annular flow of two immiscible liquids". International Journal of Thermal Sciences, Vol. 89, pp. 362–371.

Preziosi, L., Chen, K. and Joseph, D.D., 1989. "Lubricated pipelining: stability of core-annular flow". Journal of Fluid Mechanics, Vol. 201, pp. 323–356.

Shahidi, M.K. and Özbelge, T., 1995. "Direct contact heat transfer between two immiscible liquids flowing in a horizontal concentric annulus". International journal of multiphase flow, Vol. 21, No. 6, pp. 1025–1036.

Sphaier, L., 2012. "Integral transform solution for heat transfer in parallel-plates micro-channels: Combined electroosmotic and pressure driven flows with isothermal walls". International Communications in Heat and Mass Transfer, Vol. 39, No. 6, pp. 769–775. doi:10.1016/j.icheatmasstransfer.2012.05.010.

Su, J., 2006. "Exact solution of thermal entry problem in laminar core-annular flow of two immiscible liquids". Chemical Engineering Research and Design, Vol. 84, No. 11, pp. 1051–1058.

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