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STATISTICAL ANALYSIS OF A BISTABLE FLOW VELOCITY SIGNAL

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Abstract. This paper presents a statistical analysis of velocity signals for two cylinders side-by-side in a bistable flow. Time series of transversal velocity were obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel. This study aims to characterize the bistable phenomena behavior by means of statistical quantities and probability density functions. Wavelet transforms are applied as a support tool for the statistical analysis. The results show a correlation between the variation of statistical moments and the bistable behavior.

Keywords: bistable flow, statistical analysis, probability density function

1. INTRODUCTION

The study of flow around circular cylinders is extremely important in engineering, and two cylinders side-by-side are usually used as a simplification for several practical applications, such as shell and tube heat exchangers, pipelines, transmission lines and in the core of nuclear reactors.

When two cylinders of equal diameter placed side-by-side are submitted to a transversal flow, the wake can present peculiar characteristics caused by the interaction of the wakes. The first characteristic observed is a gap flow deviated to the back of one or the other cylinder, this characteristic is present on $p/D < 2$, being D the diameter of the cylinder and p the distance between cylinders center. The second characteristic, consequence of the gap flow deviated, is the formation of two wakes, a narrow wake and a wide wake, presenting distinct velocities and fluctuations. In some cases, the configuration of wake, changes randomly and is called bistable flow. These characteristics are described by (de Paula and Möller, 2013a), (Alam et al., 2003), Afgan et al. (2011) and Zhou and Alam (2016).

Olinto et al. (2006) determined by means of wavelet analysis of the results of hot wire experiments the presence of a biased and bistable flow mode in in-line tube banks and compared to the flow on two cylinders placed side-by-side. (Olinto et al., 2009) Combining measurements together with flow visualizations in a water channel identified a phenomenon of random change in the flow mode, characterized by the presence of transversal components to the main flow that lead to the generation of a three dimensional behavior similar to the bistable flow that occurs in the flow on two tubes placed side by side, with the same scale to that of the channel.

De Paula and Möller, (2013b) studied the bistable phenomenon using a finite mixture model to generate the probability density function model. They applied wavelet transform as a filter for the turbulent velocity data and concluded that a doble well model was appropriate to describe the phenomenon.

According to Tennekes and Lumley, (1972) a time domain analysis consists of the calculation of the four first probability density functions (PDF) statistical moments, which consist of mean value, standard deviation, skewness, and kurtosis.

The objective of this work is to investigate the characteristics of an experimental signal from hot wire measurements of a bistable flow by means of its statistical quantities and probability density functions. The study aims the observation of the flow mode changes through the statistical analysis.

2. EXPERIMENTAL TECHNIQUE

The aerodynamic channel used in the experiments, is made of acrylic glass, with a rectangular test section of 0.146 m height, width of 0.193 m and 0.9 m length. The airflow is driven by a centrifugal fan with 1HP, and passes through two honeycombs and two screens, to reduce the turbulence intensity to about 1% of the free stream velocity in the test

section. Upstream the test section, placed in one of the sidewalls, a Pitot tube measures the reference velocity of the impinging flow, and this is considered as the average velocity in the channel. The aerodynamic channel is pictured by previous studies such as de Paula and Möller, (2013b).

The velocity and its fluctuations are measured by means of a DANTEC *StreamLine* constant hot-wire anemometry system, with a single hot wire probe (type DANTEC 55P11), with the wire perpendicular to the main flow. The probes were positioned at the internal tangent of the cylinders at 10 mm from the center position.

Data acquisition was performed with a 16-bit A/D board (National Instruments 9215-A) with USB interface, with a sampling frequency at 1000 Hz and a low pass filter at 300 Hz to avoid aliasing.

The cylinders diameter used in this experiment was D = 25.1 mm, with p/D = 1.26 and Reynolds number calculated based on the cylinders diameter and the free stream velocity was 1.44×10^4. Uncertainties associated with the mean velocity values are about $\pm$ 3% and for the instantaneous velocity values, about $\pm$ 5%.

2.1 Statistical Analysis

Tennekes and Lumley, (1972) define a time domain analysis as one consisting of the calculation of the four first probability density functions (PDF) statistical moments. Those moments consisting of mean value, standard deviation, skewness, and kurtosis.

The proposed analysis follows the idea of dividing the series in blocks as in the classical spectral analysis (Indrusiak et al., 2016), short enough to characterize an instant in time, but long enough to retain the statistical characteristics during the observed time interval. In this case, the mean value will be characterized as moving average.

The mean value, or the first moment can find by the following definition, given by (Bendat and Piersol, 2010)

$$E[u(t)] = \int_{-\infty}^{\infty} u p(u) du = \mu_u \tag{1}$$

where E[*u(t)*] is the expected value of the variable *u(t)*, and *p(u)* designates the probability density function of the variable. The second moment or variance of *u(t)* is defined by the mean square value o *u(t)* about its mean value:

$$E[(u(t) - \mu_u)^2] = \int_{-\infty}^{\infty} (u - \mu_u)^2 p(u) du = \sigma_u^2. \tag{2}$$

The standard deviation of *u(t)* denoted by σ_u is by definition the positive square root of the variance and is measured in the same units as the mean value. It is no customary but, in the case of this study it was chosen to normalize the standard deviation by the mean value:

$$Std = \sigma_u / \mu_u \tag{3}$$

The third moment, is defined as:

$$E[(u(t) - \mu_u)^3] = \int_{-\infty}^{\infty} (u - \mu_u)^3 p(u) du = \mu_u^3 \tag{4}$$

it depends on the symmetry in *p(u)*. If *p(u)* is symmetric about the origin than $\mu_u^3 = 0$. The third moment is usually normalized by σ_u^3, which give the dimensionless measure of the asymmetry, called skewness:

$$S = \mu_u^3 / \sigma_u^3 . \tag{5}$$

The fourth moment, nondimensionalized by σ_u^4, is called kurtosis or flatness factor, it contains the information on the width of the density *p(u)*, and is defined by:

$$K = \mu_u^4 / \sigma_u^4 = \frac{1}{\sigma_u^4} \int_{-\infty}^{\infty} (u - \mu_u)^4 p(u) du \tag{6}$$

In the second method applied for the data analysis, the signal was filter by a discrete wavelet transform. A discrete wavelet transform (DWT) for a arbitrary time series x(t), can be defined, according to Indrusiak et al., 2016, as:

$$\widetilde{X_d}(j,k) = \sum_k x(t) \Psi_{j,k}(t) \qquad j, k \in \mathbb{Z} \tag{7}$$

and for each level *j* the part of the signal that contains the mean values, is given by the following equation:

$$\widetilde{X_d}(j,k) = \sum_k x(t) \phi_{j,k}(t) \qquad j, k \in \mathbb{Z} \tag{8}$$

The wavelet reconstruction of level 6 is then subtracted from the velocity signal, remaining only the fluctuations. For the local moments of the second analysis, the following quantities were defined. The local variance:

$$var = u_{fi}^2(t).$$ (9)

Local skewness:

$$skw = u_{fi}^3(t).$$ (10)

And local kurtosis:

$$krt = u_{fi}^4(t).$$ (11)

In Eqs. 9, 10 and 11, $u_i(t)$ represents the ith velocity value in the filtered time series. By this method it is possible to calculate the local quantities that are analog to the statistical moments.

3. RESULTS

The results of the statistical analysis of the time series for two cylinders side-by-side with diameter D = 25.1 mm, p/D= 1.26 and Reynolds number Re = was 1.44×10^4 are presented. As aforementioned, the statistical analysis is based on calculating the statistical moments which consist of, mean value, standard deviation which is the square root of the variance, skewness, and kurtosis. Skewness and kurtosis are normalized by the variance, and the variance is normalized by the local mean. The analysed data consists of a sample of eight seconds extracted from a longer velocity time series. The sample of eight seconds was selected in a way that the bistable characteristics of the signal where clearly identified. The data was processed according to two different methods, whose procedures and results are described below.

For the first statistical analysis the signal was processed as follow. First the signal was scanned by a window of 256 points, with an overlapping of 64 points, creating in total, 125 dyadic blocks of signal. This window can be adjusted according to need, size of the signal and other specific characteristics. For each block a probability density function (PDF) was generated, and the statistical moments of each PDF block were than calculated. Through this analysis it is possible to associate each wake mode to their statistical characteristics providing general characterization of the bistable phenomenon. Figure 1 illustrates this process of scanning the signal, for three different situations, first with the window of 256 points located in a random part of the large wake (Fig. 1.a). Secondly the window comprehends part of the large wake and part of the narrow wake (Fig. 1.b). And at last with the window in a random part of the narrow wake (Fig. 1.c).

The PDFs created by the method can be approximated by an asymmetric t-Student distribution (as was already studied in de Paula and Moller, (2013a). Figure 2 shows the PDF's corresponding to the data blocks represented and extracted in Fig.1. When the signal is at the large wake the PDF concentrates in the lower values of velocity, the distribution has positive asymmetry Fig. 2a). When the signal comprehends a mode change, meaning, part of it is in the large wake, and part in the narrow wake, the PDF presents two curves, being the combination of two t-Student distributions, Fig. 2b). And when the signal is in the narrow wake, the distribution is narrowed and concentrates in the higher values, presenting a negative skewness, as showed in Fig. 2c).
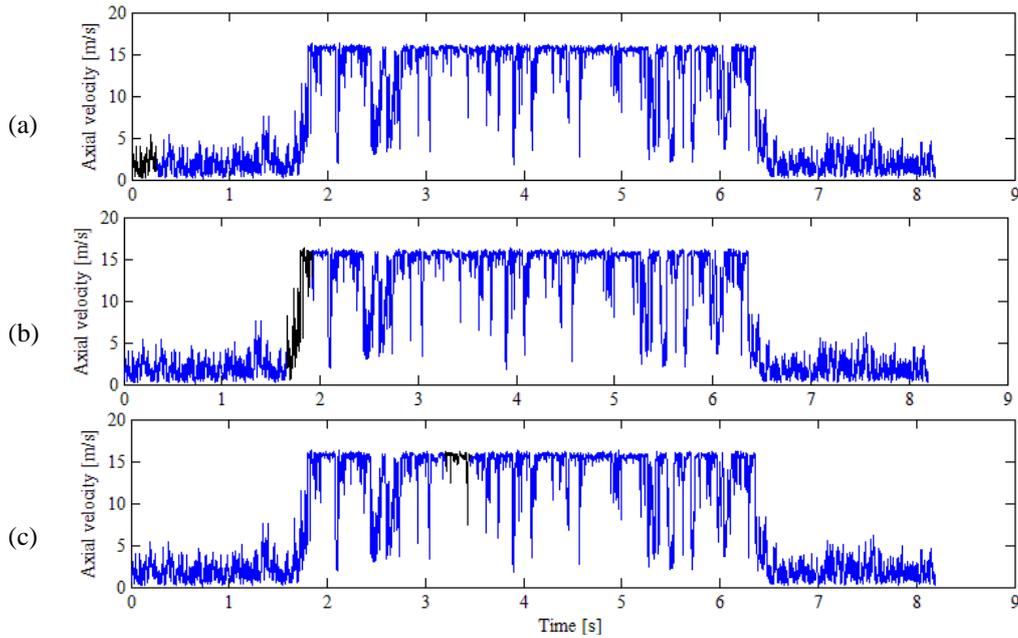
Figure 1 - Schematic representation of the velocity time series (in blue) and the blocks of 256 points (in black) placed in different sections of the data: a) large wake, b) the two modes simultaneously, c) narrow wake.
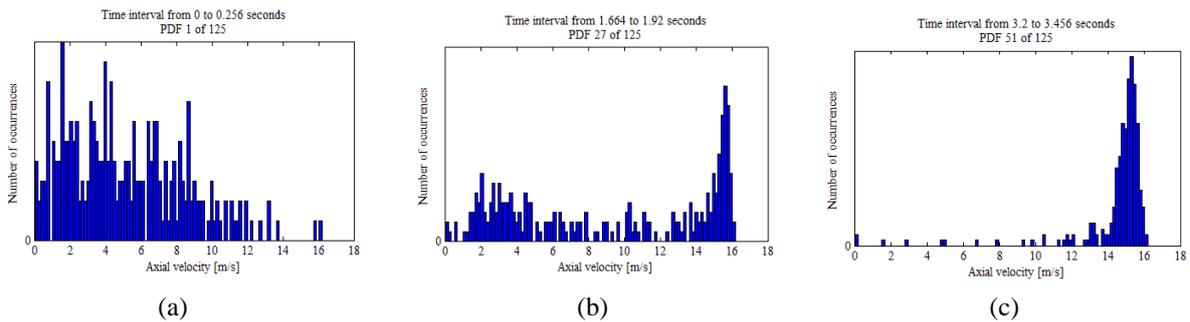

Figure 2 - Corresponding PDFs for data in Fig. 1. a) PDF corresponding to the large wake, b) PDF corresponding to the two wakes simultaneously and c) PDF corresponding to the narrow wake.

After all the blocks are computed, the statistical moments of each block are calculated. The results can be observed in Fig. 3. Figure 3.a) shows the velocity time series of a bistable signal, for p/D = 1.26, for D = 25.1 mm. The bistable characteristics of the signal are identified by the alternation between the two velocity levels, small velocity values corresponding to the large near wake and the higher velocity values corresponding to the narrow near wake. The pseudo-frequency of the mode change being of 0.122 Hz and only two mode changes are observed in the time period. The mean values of each PDF mode are represented in Fig. 3. b).

Figure 3.c) shows the variation of normalized standard deviation for each of the 125 blocks of the signal. It is possible to associate the behaviour of the standard deviation with each wake mode. When the signal is in the large wake, from block 1 to 26 and 100 to 125, corresponding to the small velocity, the standard deviation has values in the range of 0.45 to 0.85. Those values reduce when the signal changes mode, from large wake into the narrow wake, corresponding to PDF blocks from 27 to 99, or the higher velocity values. Then the standard deviation values became lower, in the range from 0.46 to 0.02. The moment the wake changes mode is preceded by an increase in the standard deviation values, corresponding to the peaks found in blocks 26 and 100. The signal also has some mode changes attempts that caused the smaller peaks in block 39, 61 and 84.

Figure 3.d) shows the variation of the skewness for each PDF block. It is possible to associate the skewness behaviour to the standard deviation behaviour and to the signal mode changes. When the signal is in the large wake skewness values are positive, (blocks 1 to 26) coinciding with the higher values of standard deviation. When the mode changes to the narrow wake, the skewness values became negative, (block 27 to 98) coinciding with the lower values of

standard deviation. The instant the signal changes from on wake to the other is marked by a peak in the skewness values, in block 24 and 100. In the skewness peaks are also found in blocks 39, 61 and 84, that would correspond to the instant when the signal tries to change mode but failed.

   Figure 3.e) shows the variation in kurtosis for each PDF block. Kurtosis presents almost constant values, around 2 and 3.2, when the flow is in the large wake. Around the PDF blocks that include the mode change some peaks on the values start to appear. When the mode changes to the narrow wake, from block 29 to 100, kurtosis presents high peaks, reaching values like 28 in block 52, intercalating with lower values, that may be related to the flow trying to change mode again. Around block 100 the values of kurtosis tend to reduce back to the range of values of 5 to 2.
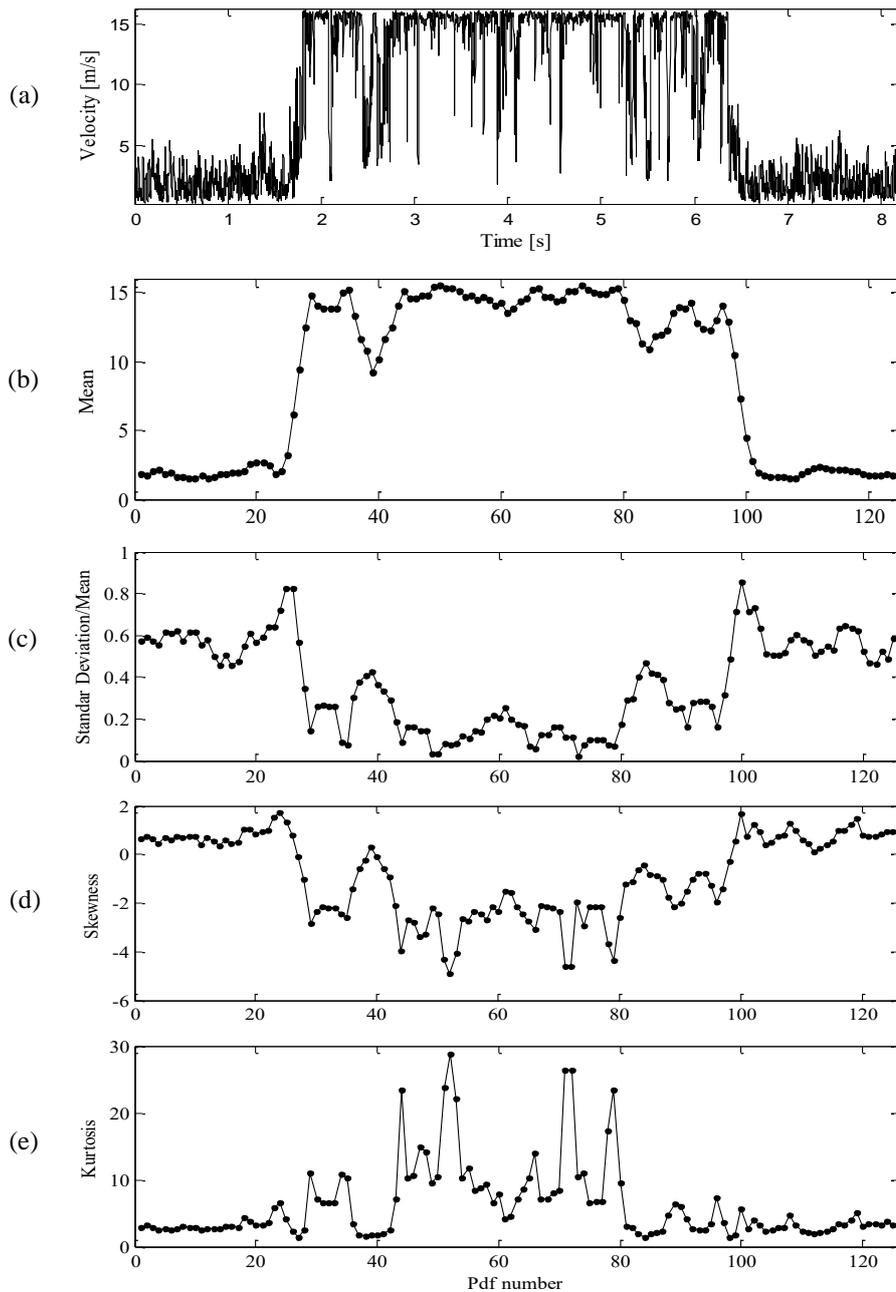

Figure 3 – Velocity signal and the variation of statistic moments for each PDF block for D = 25.1 mm, p/D = 1.26, Re = 1.44x10⁴. a) Velocity signal, b) mean values, c) standard deviation normalized by the local mean, d) Skewness and e) Kurtosis.

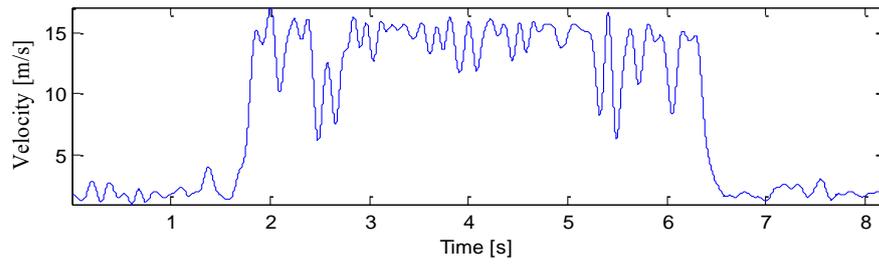Figure 4 – wavelet detail of level 6 (0 to 7.81 Hz).
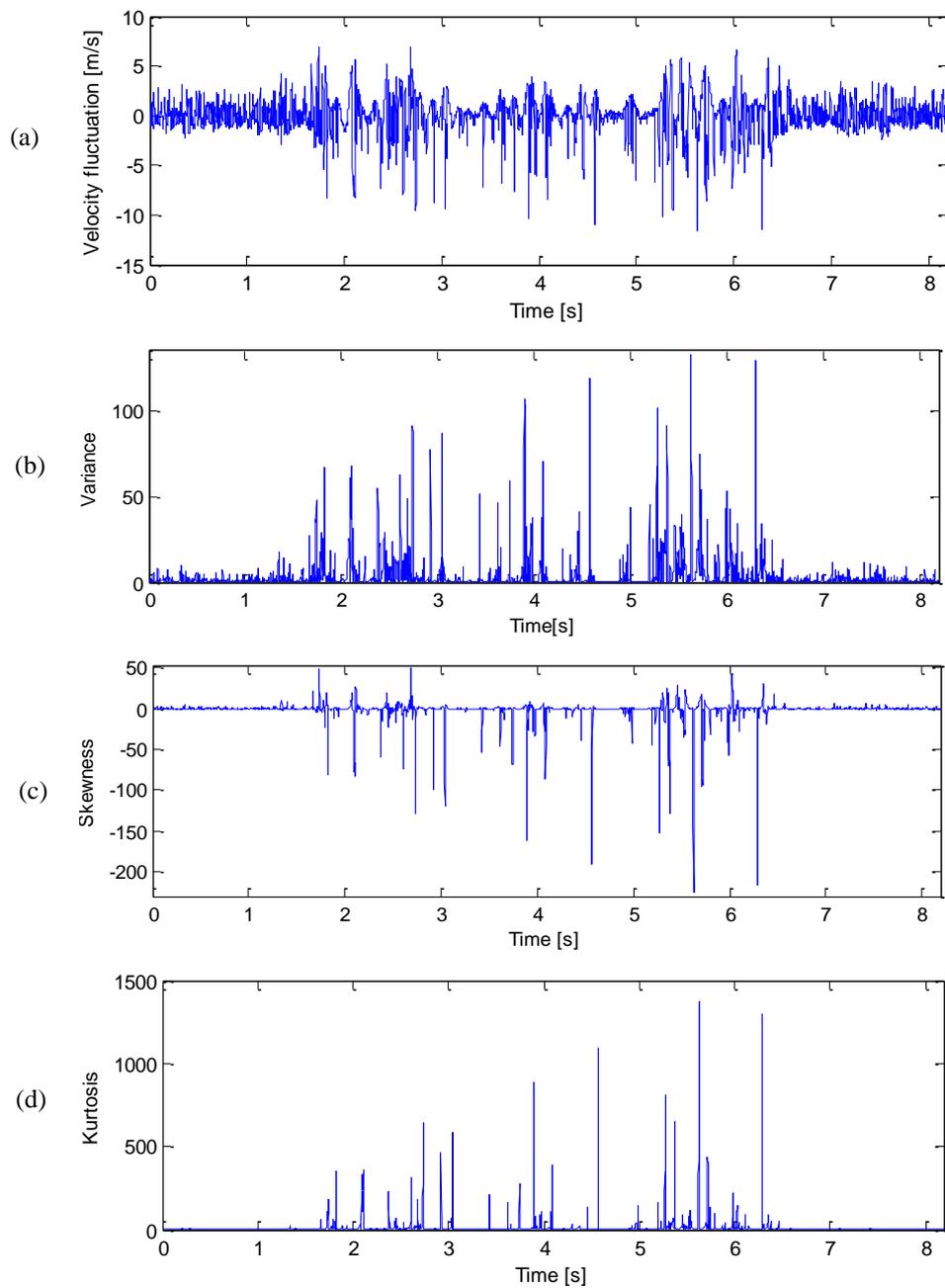
(a)

(b)

(c)

(d)

Figure 5 – Velocity fluctuation after filtering the signal using the wavelet reconstruction on level 6 and
instantaneous variation of statistical moments. a) filtered signal, b) local variance, c) local skewness and d)
local kurtosis.

For the second method of analysis, a discrete wavelet reconstruction of level 6 of the signal, was used as a representation of the instantaneous mean of the bistable signal. This level of reconstruction corresponds to the frequency band from 0 to 7.81 Hz, and the wavelet of choice was a Db20. Figure 4 shows the reconstruction of the level 6 detail. The velocity signal used in this analysis is the same one represented in Fig. 1. By filtering the velocity signal using this mobile mean, represented by the discrete wavelet reconstruction, causes only the velocity fluctuations to remain at the end for the following analysis.

The velocity fluctuations that remained after subtracting the wavelet are represented in Fig. 5.a). Variance, skewness and kurtosis were calculated instantaneously with the porpoise of understanding how those values behaved locally in each stage of the flow. Figure 5.b) represents the local variance, Fig. 5.c) shows the local skewness (normalized by the local variance) and Fig. 5.d) presents the local kurtosis (normalized by the local variance).

As is possible to observe, the resulting local values, especially for local skewness and local kurtosis, present essentially the same characteristics as the statistical moments presented before. When the flow is in the narrow wake, local skewness, Fig. 5.c) presents mainly negative values with higher amplitudes, while when the flow is in the large wake those values are positive, and with smaller amplitude. Local kurtosis, Fig. 5.d) present positive and high amplitude values when the flow is found in the narrow wake. All the values are zero for the large wake. Local variance results, Fig. 5.b) present positive values for the whole data as expected, with small amplitudes for the large wake and higher amplitudes for the narrow wake.

4. CONCLUSIONS

This paper presents a statistical analysis of a bistable flow signal. The data was separated in blocks and each block was then converted in a PDF. By the analysis of the PDF is possible to identify that when the flow is in the large wake, which has the lower velocities, the flows presents higher velocities fluctuations, consequently higher standard deviation values and a positive skewness. When the flow changes mode, and is part in each wake, the PDF presents a combination of both distributions. And at last when the flow is in the narrow wake, which has the higher velocity values, the distributions presents small standard deviation, meaning that the flow presents smaller fluctuations, consequently this distribution presents negative skewness.

The statistical moments for each PDF block were calculated. By the results is possible to associate the behavior of each bistable wake mode, to its statistical characteristics. A more profound analysis may still be required, but it is possible to observe in general that, skewness and kurtosis present distinguished behaviors according to the mode wake they represent. Skewness present positive values for the large wake, which corresponds to the lower velocity values, those values also tend to vary around an almost constant value in the range of 0.45 to 0.85. When in the narrow wake mode, which corresponds to the higher velocity levels, the skewness is negative and due to some failed attempts of mode changing, those values present peaks and are not as constant as those for the large wake.

When the flow is in the large wake, kurtosis present small values, in the range around 2 and 3.2, and those values are almost constant with only small variations. However when the flow mode is the narrow wake, kurtosis values are no longer constant around a range of values, but show higher values, from 10 to 50 and intense oscillation between those values, this being cause probably by the attempts of mode changing observed in the narrow wake. The same pattern of behavior can be observed for local skewness and local kurtosis.

This paper demonstrated that is possible to characterize the bistable phenomenon through its wake modes statistical characteristics. Each wake mode presents unique features, and those can be represented and measured by statistical methods.

5. ACKNOWLEDGEMENTS

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