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A NEW APPROACH FOR SOLVING RADIATIVE HEAT TRANSFER IN NON-PARTICIPATING MEDIA

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Abstract. *This paper has as main objective the presentation of an alternative methodology for the calculation of radiative exchanges between opaque, black or gray surfaces forming an enclosure in non-participating media. Instead of using the procedure for calculating radiative heat exchanges between surfaces based on the radiation-electrical network analogy, our methodology emphasizes the application of an energy balance to write three equations for each surface. The unknowns are the blackbody's emissive power, radiosity and heat transfer rate on the surface. Furthermore, this methodology is not restrict to a small number, in general up to three, radiative surfaces as is the case for the radiation-electric network analogy. Some applications are shown including, non radiative energy source and multimode heat transfer with more than three radiative surfaces.*

Keywords: *Multimode Heat Transfer, Radiation in Non-Partipating Media, Matricial representation*

1. INTRODUCTION

In the design of spaceships and stratospheric balloons, radiation heat transfer plays an important role. In general, it is desired to evaluate the maximum and minimum temperatures to which the present electronic components are subjected. Accordingly, it is important to analyze the radiative heat exchange in steady-state inside two enclosures — one composed by the external surfaces of the ship, the space and the surface of the celestial body; and another formed by the surfaces inside the box containing the electronic components. There is heat transfer by conduction between the enclosures, meaning that there is a coupling and the equations representing the radiative exchange inside the enclosures must be solved simultaneously. One approach for solving radiative heat exchanges in such situation is to consider an enclosure, formed by n isothermal gray surfaces with a non-participating medium. Each surface is subjected to a specified temperature (T_i) or heat transfer rate (q_i). The radiation network analogy proposed by Oppenheim (1956) can be used if the number of surfaces is small. The methodology allows the calculation of the radiosity of each surface (J_i); from this information the respective temperature or surface heat flow can be calculated. For situations in which conduction and convection are present, with one or more enclosures, an iterative procedure is expected. In problems with a larger number of surfaces, this methodology is not practical. For these cases, the use of the so-called direct method, which involves the construction and solution of the radiosity matrix (J_i), is more convenient. Each line of the matrix corresponds to a surface and must be written according to the unknown to be determined, which may be the temperature (T_i) or the heat flow (q_i) on the surface in question. In total, there are $2n$ equations and unknowns. Siegel and Howell (1992) provide an excellent review of other alternative methods that can also be used, some of which are based on Poljak (1935) and Gebhart (1961). According to the authors, Sparrow (1963) demonstrated the equivalence between these methods. Another methodology that can be used, based on Gebhart (1961), consists of calculating the fraction (G_{ji}) of radiation emitted by the surface A_j that reaches A_i and is absorbed by It, including the direct portion and the other resultant of multiple reflections (Siegel and Howell, 1992). The result is a linear system with N equations and unknowns for each surface i. Therefore, it is necessary to solve a linear system $n \times n$, n times, one for each surface. It is noteworthy, however, that the coefficients are independent of the surface and, therefore, the matrix needs to be inverted only once.

All these methods are arranged to directly solve for T_i or q_i. In multi-mode heat transfer problems, with one or more enclosures, the limitations of the two commonly used methodologies are evident, particularly when the heat fluxes on the surfaces depend on the temperature of a fluid, solid surface or enclosure neighborhood. Subsequently, this paper has as main objective the presentation of an alternative methodology for the calculation of radiative exchanges between opaque, black or gray surfaces forming an enclosure. Our methodology emphasizes the application of an energy balance to write three equations for each surface. Through the solution of the resultant $3n \times 3n$ linear system the unknowns, blackbody's emissive power, radiosity and heat transfer, can be directly calculated.

Additionally, the paper includes the detailed presentation of the methodology, to serve as a supplementary material to heat transfer courses. In order to reinforce this pedagogical facet, application examples are presented.

2. MODEL PRESENTATION

In this section, the fundamental equations for radiative heat transfer in non-participating media as well as their matricial representation and numerical model are presented. The limitations of the numerical model are also discussed.

2.1 Model Simplifications

Any mathematical-physical model is built upon simplifying assumptions that must be taken in order to get a problem solution. The main simplifications and limitations of the proposed methodology for radiative heat transfer in non-participating media are:

1. All radiant surfaces must have uniform properties along its area;
2. All radiant surfaces must be opaque;
3. All radiant surfaces must be gray or black in the problem operating wavelenght bandwidth;
4. There is no participating media among the radiant surfaces;
5. The problem is in steady state;
6. All radiant surfaces must be diffuse, that is, the thermal radiation emission and reflection do not depend on viewing direction; and
7. All radiant surfaces have unique heat transfer parameters to each one of them.

2.2 Fundamentals of Thermal Radiation Heat Exchange

In this subsection, some basic concepts for calculating thermal radiation heat exchange between non-black surfaces in non-participating media are reviewed.

2.2.1 Radiant Surface

A radiant surface is defined as a surface that exchanges heat through radiation, although it is a simple and general concept, it brings constant misunderstandings on behalf of the undergraduates. A common mistake is to think of a radiant surface as a flat plate with two sides.

2.2.2 Radiant Enclosure

A radiant enclosure is formed by a group of radiant surfaces that exchanges heat only by radiation among them.

2.2.3 View factor

The fraction of the radiative energy that leaves a radiant surface i and reache a radiant surface j can be evaluated based on the view factor F_{ij}, which defined by Bergman *et al.* (2011):

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{cos\, \theta_i \, cos\, \theta_j}{\pi\, R^2} dA_i dA_j \tag{1}$$

where:

- F_{ij} is the view factor from the i surface to the j surface;
- A_i is the area of the i surface and A_j of the j surface;
- θ_i is the angle from the normal line from the i surface to the j differential surface;
- θ_j is the angle from the normal line from the j surface to the i differential surface;
- R is the distance between the i differential surface and the j differential surface; and
- dA_i and dA_j are the differential surfaces areas.

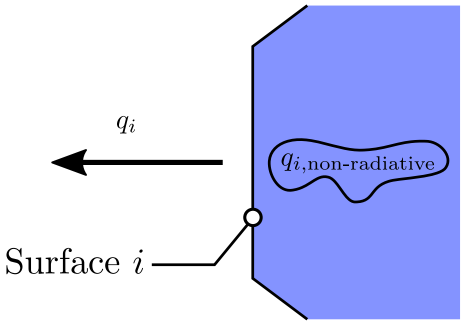

Figure 1. Non-radiative energy in a radiant surface.

2.2.4 Net Radiative Heat Rate

The net radiative heat rate on each radiant surface can be evaluated taking into account the emissivity of the surface, its black body emissive power, its radiosity, the radiosity of the others surfaces of the radiative enclosure as well as the view factor among them (Bergman *et al.*, 2011):

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i}, \tag{2}$$

and also as:

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}, \tag{3}$$

where:

- q_i is the radiative heat transfer rate of the i^{th} surface;
- ε_i is the emissivity of the i^{th} surface;
- E_{bi} is the blackbody's emissive power of the i^{th} surface; and
- J_i is the radiosity of the i^{th} surface.

2.2.5 Energy Balance

To close the energy balance on the radiation surface i (or a group of them), it is necessary to include the non-radiative energy exchanges, which are not considered in Equations (2) and (3). As the model deals with stationary problems, the net radiative heat rate must be balanced by other energy supply or sink, henceforth called $q_{i,\text{non-radiative}}$, to the radiant surface i (as presented in Figure 1):

$$q_i = q_{i,\text{non-radiative}}. \tag{4}$$

In this work, two types of non-radiative heat transfer will be considered to the i surface: convection and/or heat source/sink, so that:

$$q_{i,\text{non-radiative}} = \underbrace{q_{i,\text{source/sink}}}_{\text{source or sink}} + \underbrace{hA(T_\infty - T_i)}_{\text{convection}}. \tag{5}$$

Based on the energy balance Equations (2), (3) and (4), one can see that for each radiant surface there might be three unknowns.

2.3 System Assembly

The entire system energy balance is represented in the following linear system:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{6}$$

where $\mathbf{A}$ is the coefficients matrix, $\mathbf{x}$ is the variable vector and $\mathbf{b}$ is the independent vector. In the following, the balance equations will be transferred to the matricial representation. In order to do this, the first step is to evaluate the number of radiant surfaces. As seen before, each radiant surface will have 3 unknowns, thus the order of the linear system will be 3 times the number of the radiant surfaces. In a problem with n radiant surfaces, the matricial representation must have:

- A $3n \times 3n$ matrix (the $\mathbf{A}$ matrix);
- A $3n$ variable vector (the $\mathbf{x}$ vector); and
- A $3n$ independent vector (the $\mathbf{b}$ vector).

1. Variable vector $\mathbf{x}$ The first set of elements of the variable vector will be the radiosities of each surface, the second set will be the emissive powers, and, the last one, the net radiative heat power. It is recommended, but not mandatory, to order these set of variables by the surface numbers.
2. Matrix $\mathbf{A}$ and independent vector $\mathbf{b}$ The $3n \times 3n$ matrix and the $3n$ independent vector elements are going to be filled with Equations (2), (3) and (4) or an overall energy balance in a group of radiant surfaces, that will be applied to the n radiant surfaces as:
   (a) n times the Equation (2) rewritten as:
   $$\frac{1}{(1-\varepsilon_i)/\varepsilon_i A_i} J_i - \frac{1}{(1-\varepsilon_i)/\varepsilon_i A_i} E_{bi} + q_i = 0, \quad (7)$$
   if a radiant surface is a blackbody, the Equations 2 or 7 degenerate to:
   $$J_i - E_{bi} = 0; \quad (8)$$
   (b) n times the Equation (3) rewritten as:
   $$\sum_{j=1}^{N} \frac{1}{(A_i F_{ij})^{-1}} J_i - \sum_{j=1}^{N} \frac{1}{(A_i F_{ij})^{-1}} J_j - q_i = 0; \quad (9)$$
   (c) n_1 times Equation (4), where there is n_1 radiant surfaces non-radiative energy heat flux; in this paper, there are prescribed sink/source, adiabatic surface or convective heat transfer;
   (d) n_2 times Equation:
   $$E_{bi} = \sigma T_i^4, \quad (10)$$
   where there are n_2 radiant surfaces with prescribed temperatures, in order to make the linear system solvable:
   $$n_1 + n_2 = n. \quad (11)$$

3. RESULTS

In this section two examples will be presented using the proposed methodology. The equations to be applied, will be presented in the algebraic form, then values will be substituted. After that, all equations that composes the linear system will be numerically presented and the results will be shown.

3.1 Long Air Duct

Consider the long-evacuated duct, as shown in Figure 2, with a square cross section of side $1\,m$. Surface 1 is black and is at $T_1 = 300\,K$, the remaining surfaces 2, 3 and 4 have an emissivity of $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.5$. Surface 3 is at $T_3 = 400\,K$ while the back of surface 2 is exposed to convection with a convective heat transfer coefficient of $h = 10\,Wm^{-2}K^{-1}$. The fluid temperature is at $T_\infty = 350\,K$. The view factors are $F_{13} = F_{24} = 0.64$; $F_{12} = F_{14} = F_{32} = F_{34} = 0.18$ and all of the 4 surfaces areas per length unit are equal to $1\,m$. The methodology is applied to calculate the temperature T_2 and the heat flux in each surface considering the duct with $1\,m$ of length.

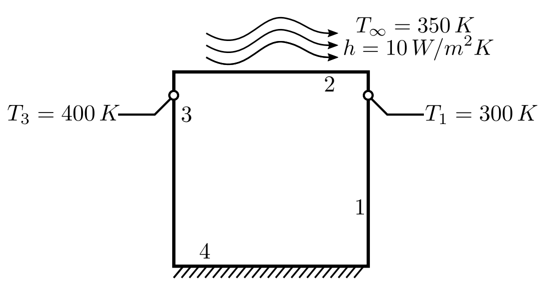

Figure 2. Multimode heat transfer - Convection and Radiation.

Applying the Equation (7) to the radiant surface 1:

$$\frac{1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \cdot J_1 - \frac{1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \cdot E_{b1} + q_1 = 0; \tag{12}$$

since the radiant surface is a blackbody, the equation is substituted by:

$$J_1 - E_{b1} = 0. \tag{13}$$

Equation (7) applied to the radiant surface 2 gives:

$$\frac{1}{(1-\varepsilon_2)/\varepsilon_2 A_2} \cdot J_2 - \frac{1}{(1-\varepsilon_2)/\varepsilon_2 A_2} \cdot E_{b2} + q_2 = 0;$$

substituting the numerical values it becomes:

$$J_2 - E_{b2} + q_2 = 0. \tag{14}$$

Doing the same procedure with the surfaces 3 and 4:
To the surface 3:

$$J_3 - E_{b3} + q_3 = 0. \tag{15}$$

To the surface 4:

$$J_4 - E_{b4} + q_4 = 0. \tag{16}$$

Now, it is time to go to the second set of $n = 4$ equations. Applying the Equation (9) to the surface 1 gives:

$$\sum_{j=1}^{N} \frac{1}{(A_1 F_{1j})^{-1}} J_1 - \sum_{j=1}^{N} \frac{1}{(A_1 F_{1j})^{-1}} J_j - q_1 = 0; \tag{17}$$

or

$$\left(\frac{1}{(A_1 F_{12})^{-1}} + \frac{1}{(A_1 F_{13})^{-1}} + \frac{1}{(A_1 F_{14})^{-1}} \right) J_1 - \frac{1}{(A_1 F_{12})^{-1}} J_2 - \frac{1}{(A_1 F_{13})^{-1}} J_3 - \frac{1}{(A_1 F_{14})^{-1}} J_4 - q_1 = 0; \tag{18}$$

substituting the numerical values it becomes:

$$J_1 - 0.18\, J_2 - 0.64\, J_3 - 0.18\, J_4 - q_1 = 0. \tag{19}$$

Doing the same procedure with the remaining surfaces:
To the surface 2:

$$-0.18\, J_1 + J_2 - 0.18\, J_3 - 0.64\, J_4 - q_2 = 0. \tag{20}$$

To the surface 3:

$$-0.64\, J_1 - 0.18\, J_2 + J_3 - 0.18\, J_4 - q_3 = 0. \tag{21}$$

To the surface 4:

$$-0.18\, J_1 - 0.64\, J_2 - 0.18\, J_3 + J_4 - q_4 = 0. \tag{22}$$

After writing the first and the second set of $n = 4$ equations, the last set is to be formulated. To do so, each one of the 4 equations are going to be presented separately.
To the surface 1:

$$E_{b1} = \sigma T_1^4 = 459.30. \tag{23}$$

To the surface 3:

$$E_{b3} = \sigma T_3^4 = 1451.62. \tag{24}$$

To the surface 4:

$$q_4 = 0. \tag{25}$$

To the surface 2, there is:

$$q_2 = h\,A_2(T_\infty - T_2). \tag{26}$$

Note that this equation has a temperature term raised to the power of 1 which makes the system non-linear. To get rid off this problem, a linearizing factor (β) is defined as:

$$\beta_2 = \frac{1}{\sigma T^3_{2\,\text{solution}}}, \tag{27}$$

and the linearized equation reads

$$(h\,\beta_2)\,E_{b2} + q_2 = h\,T_\infty. \tag{28}$$

Substituting the numerical values it becomes:

$$(10\,\beta_2)\,E_{b2} + q_2 = 3500. \tag{29}$$

The whole linearized system of equations reads:

$$\begin{cases} J_1 - E_{b1} = 0 \\ J_2 - E_{b2} + q_2 = 0 \\ J_3 - E_{b3} + q_3 = 0 \\ J_4 - E_{b4} + q_4 = 0 \\ J_1 - 0.18\,J_2 - 0.64\,J_3 - 0.18\,J_4 - q_1 = 0 \\ -0.18\,J_1 + J_2 - 0.18\,J_3 - 0.64\,J_4 - q_2 = 0 \\ -0.64\,J_1 - 0.18\,J_2 + J_3 - 0.18\,J_4 - q_3 = 0 \\ -0.18\,J_1 - 0.64\,J_2 - 0.18\,J_3 + J_4 - q_4 = 0 \\ E_{b1} = 459.30 \\ (10\,\beta_2)\,E_{b2} + q_2 = 3500 \\ E_{b3} = 1451.62 \\ q_4 = 0 \end{cases} \tag{30}$$

The system is solved iteratively. The initial step is to guess a value for β:

$$\beta_2 = \frac{1}{\sigma 380^3} = 0.321393\,W^{-1}m^2K. \tag{31}$$

The system is solved and a new β_2 value is calculated. The procedure is repeated until the convergence as shown in the Figure 3. The unknown temperature of the surface 2 converges to $T_2 = 346.86\,K$.

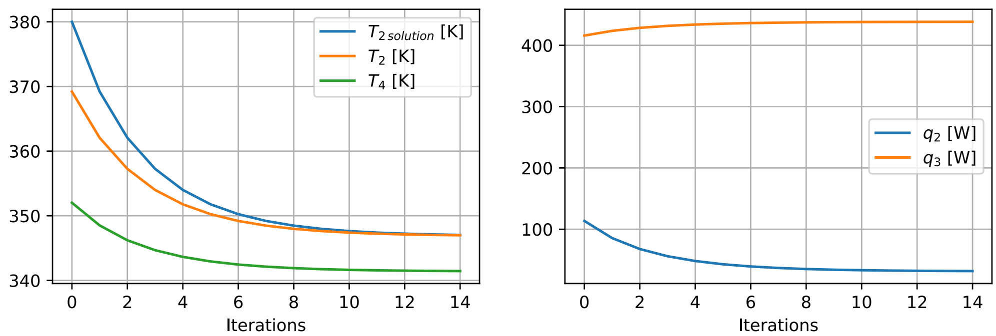

Figure 3. Evolution of the unknown temperature and radiative powers

3.2 Hot Oil Tube

Consider a hot pressurized oil duct, made of two concentric tubes, as depicted in Figure 4. The inner one is fulfilled with hot pressurized oil, with a temperature of $T_1 = 500\,K$ and the outer one separates the internal air from the external air with a thin layer of metal. There is a neighbourhood that envolves the outer tube in a temperature of $T_4 = 300\,K$. The inner surface area per length unit is equal to $1\,m$ and the outer one is equal to $3\,m$, their emissivities are $\varepsilon_1 = 1$ and $\varepsilon_2 = \varepsilon_3 = 0.8$. The proposed method is applied to calculate the temperature of the outer tube.

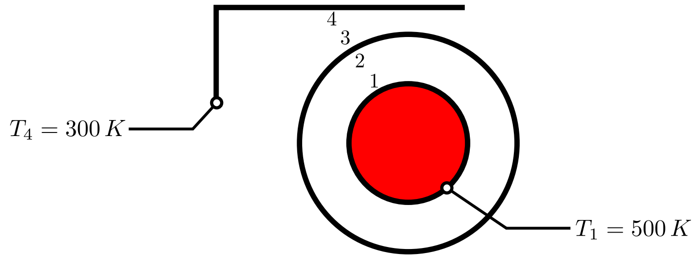

Figure 4. Pressurized hot oil tube.

Applying the Equation (7) to the radiant surface 1:

$$\frac{1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \cdot J_1 - \frac{1}{(1-\varepsilon_1)/\varepsilon_1 A_1} \cdot E_{b1} + q_1 = 0;$$

as the radiant surface is a blackbody, the equation is substituted by:

$$J_1 - E_{b1} = 0. \tag{32}$$

The neighbourhood (radiant surface number 4) behaves as blackbody surface because of its large area compared to the other surfaces, therefore:

$$J_4 - E_{b4} = 0. \tag{33}$$

Applying the Equation (2) to the radiant surface 2:

$$12\,J_2 - 12\,E_{b2} + q_2 = 0. \tag{34}$$

Applying the Equation (2) to the radiant surface 3:

$$12\,J_3 - 12\,E_{b3} + q_3 = 0. \tag{35}$$

Applying the Equation (3) to all surfaces:
Surface 1:

$$J_1 - J_2 + q_1 = 0. \tag{36}$$

Surface 2:

$$-J_1 + J_2 + q_2 = 0. \tag{37}$$

Surface 3:

$$3\,J_3 - 3\,J_4 + q_3 = 0. \tag{38}$$

Surface 4:

$$-3\,J_3 + 3\,J_4 + q_4 = 0. \tag{39}$$

The first and the second set of $n = 4$ equations are filled. Now it is time to assembly the last $n = 4$ equations to complete the system of equations. Applying the Equation (10) to surfaces with prescribed temperature, it gets for surface 1:

$$E_{b1} = \sigma\, T_1^4 = 3543.98, \tag{40}$$

and for surface 4:

$$E_{b4} = \sigma T_4^4 = 459.30. \tag{41}$$

The energy balance for outer tube wall relates the net radiative heat flux at surfaces 2 and 3. Since there is no energy source/sink in the outer tube wall, the net radiative heat flux of surface 2 must balance with surface 3:

$$q_2 + q_3 = 0. \tag{42}$$

Additionally, for sake of simplicity and because the outer tube wall is thin, it is assumed that $T_2 = T_3$, so:

$$E_{b2} - E_{b3} = 0. \tag{43}$$

It is important to emphasize that this simplification could be replaced by a heat conduction model for the tube wall. To sumarize, the linear system is shown:

$$\begin{cases} J_1 - E_{b1} = 0 \\ 12\,J_2 - 12\,E_{b2} + q_2 = 0 \\ 12\,J_3 - 12\,E_{b3} + q_3 = 0 \\ J_4 - E_{b4} = 0 \\ J_1 - J_2 + q_1 = 0 \\ -J_1 + J_2 + q_2 = 0 \\ J_3 - J_4 + q_3 = 0 \\ -J_3 + J_4 + q_4 = 0 \\ E_{b1} = 3543.98 \\ E_{b4} = 459.30 \\ E_{b2} - E_{b3} = 0 \\ q_2 + q_3 = 0 \end{cases} \tag{44}$$

The solution of this linear system is presented in Table 1.

$T_1\,[K]$	$T_2\,[K]$	$T_3\,[K]$	$T_4\,[K]$	$q_1\,[W]$	$q_2\,[W]$	$q_3\,[W]$	$q_4\,[W]$
500	390.32	390.32	300	2056.45	-2056.45	2056.45	-2056.45

Table 1. Solution of the linear system of the hot oil tube problem.

4. CONCLUSIONS

An alternative methodology — to the common used radiant-electric network analogy — for the calculation of radiative exchanges between opaque, black or gray surfaces forming an enclosure in non-participating media was presented and applied. The numerical issue related to the non-linearity, that may occur, was also discussed and successfully applied. The main adavantages of the model are a) its general formulation for multi-mode heat transfer problems and b) its ability of handling problems with large number of radiant surfaces or radiant enclosures.

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