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ENC-2020-0792 QUANTIFICATION OF UNCERTAINTY IN TEMPERATURE MEASUREMENTS WITH THERMOCOUPLE AND ARDUINO® COMPATIBLE HARDWARE

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Abstract. *Temperature is a key aspect in the control of processes in engineering. The measurement, however, must take into account not only the accuracy of the sensor but also the implementation costs and uncertainty. Thermocouples are one of the most widely used temperature measurement methods combining low costs, high measurement ranges and relatively good accuracy. The acquisition system, however, is usually expensive. In this paper it was evaluated the uncertainty in the temperature measurements of a K-type thermocouple using low cost Arduino® compatible hardware as data acquisition system. This set was calibrated using a thermostatic distilled water bath at both freezing and boiling phase change points. The equations for uncertainty calculation were fully developed and the procedures described serve as reference for uncertainty assessment for thermocouples with other data acquisition systems. Although the calibration and limited resolution were the greatest contributors, the low variability of measurements shows the system has good stability and is a fine choice for industrial applications. The calculations are easy to implement as a routine for several measurements and guarantee traceability of results up to the international reference standard.*

Keywords: *uncertainty measurement, GUM, thermocouple, Arduino® microcontroller, MAX31855 module*

1. INTRODUCTION

Temperature measurement systems are widely used in industrial processes and research, with several different technologies available nowadays. Each of them have a determined operational principle, accuracy and precision, which strongly influences sensor and equipment costs. A high temperature accuracy is necessary when performing material characterization and process monitoring of very delicate applications, or when dealing with inverse problems in heat and mass transfer. However, there are certain industrial cases in which small temperature oscillations do not disturb the process and final product, dispensing the need for highly sensitive sensors. Yet, even in these scenarios, a minimum quality and reliability should be guaranteed.

Even in the most precise temperature instruments, readings may vary due to systematic and random errors. Although the first should be measured and corrected, there are many sources of errors difficult to determine or eliminate, such as those due to electromagnetic fields around electronic instruments or vibrations, among others. Therefore, according to principles of metrology a measurement cannot give the exact value of a measurand, but only estimate it within a certain range of quantified uncertainty given a determined confidence level (Vuolo, 1996).

The Guide for Expression of Uncertainty in Measurement-GUM (INMETRO, JCGM, 2008) provides a robust method for assessing uncertainty considering the different possible sources of errors during measurement. This procedure is important for various reasons. First, if the output results of a test do not come accompanied by a study of its reliability, this result cannot be trusted once there is no definition of how precise it was. Second, during the procedure it is possible to determine the uncertainty associated to each possible source of error, allowing operators to make any necessary adjustments to the measuring method or system. Finally, the expression of uncertainty guarantees traceability of all measures carried in different precision levels up to the international reference standard. The International Bureau for Weights and Measures defines through the SI the reference for each property to be measured (INMETRO, BIPM, 2012). During the 10th General Conference on Weights and Measures-CGPM in 1954 it was defined that the official temperature unit would be the Kelvin. Besides, the triple point of water would serve as a fixed reference for 273.15 K. The International Committee on Weights and Measures also accepted the Celsius as an official unit in 1990, meaning that 1 °C is equal to 1 K, which is the fraction 1/273.15 of the temperature for the triple point of water.

According to Ross-Pinnock and Maropoulos (2016), thermocouples are the most used invasive sensors due to low cost, wide measurement range and accuracies up to 0.1 K. Resistance-based temperature sensors (IPRT) are also used, since they are more resistant to shock, vibration and contamination than thermocouples and bear accuracies up to 0.01 K. But as a downside they are usually much more expensive, with platinum PT-100 being the most common representative. Despite having accuracies up to 30 mK, negative temperature coefficient thermistors (NTCs) are not as used as thermocouples because of their non-linear temperature-resistance curves. The most well known method of non-invasive temperature measurement is the IR radiation thermometry, which measures surface temperature in relation to its emissivity. The main disadvantages of the IR thermometry are the relatively low accuracy (about 1 K), high costs and sensitivity to the surface condition.

For this reasons, thermocouples are preferred in applications that require low cost sensors. In this work it was investigated the uncertainty of temperature measurements using a type K thermocouple with Arduino® compatible hardware and microcontroller as data acquisition system. This combination yields low cost and high potential for process automation. The detailed procedures and equations provide a methodology for assessing temperature uncertainty with thermocouples even for different acquisition systems. The data shows that calibration and resolution are the main sources of uncertainty in our case, while the variability around average was the smallest. The system was very stable in all measurements with standard deviations less than 1 °C and expanded uncertainties less than 3 °C. The stability was worse in low temperatures, but was still well within the maximum variation stated by the manufacturer. Finally, the method presented guarantees traceability of temperature results to the international reference standards, a good practice recommended by the GUM (INMETRO, JCGM, 2008) for all measurements in industrial and laboratorial applications.

2. UNCERTAINTY CALCULATION METHODOLOGY FOR THERMOCOUPLE MEASUREMENT SYSTEM

Thermocouples work according to the Seebeck effect, in which an electric potential difference (ddp) appear in a circuit formed by two different materials if their two junctions are subjected to different temperatures. This way, if one of the junctions is kept at a known constant temperature, regarded as the cold junction, the temperature of the other regarded hot junction may be estimated by correlating it with the analogic voltage signal of the circuit (Ripple and Garrity, 2006). The latter can be measured by a voltimeter or converted to digital data. The cold junction temperature is measured by a secondary temperature sensor which is usually a NTC or IPRT. Both thermocouple and cold junction sensor should be calibrated in order to correct systematic errors and characterize their precision through uncertainty assessment.

Sârbu and Beniugă (2018) performed calibration and uncertainty measurements of type K thermocouples considering a reference type S thermocouple already calibrated. In his work, ddp was directly measured via calibrated multimeters, so that the measurement uncertainty calculated would relate only to sensor related issues such as material inhomogeneity and nonlinearities. But this procedure would not be applicable for systems which already convert the voltage signals of cold junction probe and thermocouple to digital data and calculate the corresponding temperature according to thermocouple type. In such cases, the measured uncertainty will also account for errors pertaining the whole system, such as possible measuring errors of the inbuilt voltimeter, or during analogic to digital conversion, and even the sensitivity to electromagnetic fields, which depends on how internal components were manufactured. The term related to cold junction sensor should also be considered separately. These terms were described by Manso (2013), whose values for each uncertainty parcel were defined arbitrarily or according to catalogues. In addition, a further correction might be necessary after thermocouple calibration because the temperature is determined after multiplying the ddp by the constant of the two materials of the circuit. For type K, this value is about 41.276 μV/°C (Maxim Integrated, 2015), but it might vary due to slight changes in wire chemical composition or manufacturing method.

Oliveira et. al. (2020) studied uncertainty measurements with thermocouples for comparison with the obtained errors between an experiment and analytic solution for unidimensional heat conduction. They expressed temperature uncertainty as in Eq. (1). An Agilent 34970 board was employed for data acquisition, which is a very precise system, and all measurements were performed in a temperature controlled environment. That is not always the case for many industrial applications, and therefore the uncertainty related to cold junction compensation should also be expressed, as in Eq. (2), where ΔT is the combined uncertainty, $\overline{\Delta T}$ is the uncertainty due to variability around mean, ΔR is the uncertainty due to finite resolution of the system, $\Delta ICJF$ is the uncertainty due to cold junction compensation and ΔIC is the uncertainty due to thermocouple and system calibration. Considering the GUM methodology, Eq. (3) describes a general mathematical model for calculating the uncertainty of a measurand, where Y is the output and $X_1, X_2, \dots, X_N$ are the input variables.

$$\Delta T = \overline{\Delta T} + \Delta R + \Delta IC \quad (1)$$

$$\Delta T = \overline{\Delta T} + \Delta R + \Delta ICJF + \Delta IC \quad (2)$$

$$Y = f\ (X_1, X_2, \dots, X_N) \quad (3)$$

As long as there are no correlations between the uncertainty sources, the combined standard uncertainty $u_c(y)$ can be calculated through the law of uncertainty propagation shown in Eq. (4), where $u(x_i)$ is the variance of each input variable X_i and the result of the partial derivative for each variable is called the sensitivity coefficient c_{x_i}. The variance is calculated for each variable depending on its evaluation type, which can be A or B, and the percentage contribution of the *i*-th variable results from the division of the product of squares in Eq. (4) by the combined standard uncertainty. A detailed explanation can be found in the GUM (INMETRO, JCGM, 2008).

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 \cdot u^2(x_i) \quad (4)$$

The guide recommends that the expanded uncertainty *U(y)*, calculated as per Eq. (5), should be described within a confidence interval which is dependent on the measurement coverage factor *K*. Equation (6) shows how to determine the effective degrees of freedom ν_{eff} through the Welch-Satterthwaite equation, where ν_i are the degrees of freedom of each input variable. The former will determine the value of *K* and the correspondent confidence interval.

$$U(y) = K \cdot u_c(y) \quad (5)$$

$$\nu_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{(u_i(x_i) \cdot c_{x_i})^4}{\nu_i}} \quad (6)$$

3. THERMOCOUPLE AND ARDUINO® COMPATIBLE HARDWARE AND SOFTWARE

Arduino® compatible hardware was chosen due to price and robustness, since the measuring site is prone to contamination by dust. It consists of an ATMEGA 2560 microcontroller and a module with chip MAX31855 to read thermocouple voltage. The latter does the analogic to digital conversion with maximum sampling rate around 10 Hz, depending on software and hardware setup, and sends 14-bit data to SPI ports. Readings are lively shown and registered to a notebook via USB serial communication at 5 Hz in this work. The module has an inbuilt thermistor for cold junction temperature measurement that allows 0.06 °C resolution, while for thermocouple it is 0.25 °C (Maxim Integrated, 2015). A type K thermocouple was employed, which is made from wires of Nickel-Chromium and Nickel-Aluminum alloys. Due to differences in material composition, the actual sensitivity constant might be different from the value pre-defined in the MAX31855 software library. It is a much cheaper acquisition system, all costing less than US$25, which is the upside against the high maximum error stated in catalogue of ±2 °C to ±6 °C depending on temperature interval. Although it is not adequate for precise temperature monitoring, it is enough for some less sensitive applications (Vasconcelos Neto et. al., 2018).

4. METHODOLOGY

4.1 Uncertainty of temperature measured by thermocouple and module

Equation (7) results from the application of Eq. (4) to the mathematical model of Eq. (2). The latter is linear because the variables are not correlated, and so the sensitivity coefficients resultant from the partial derivatives are all equal to 1, as shown in Eq. (8).

$$u_c^2(\Delta T) = \left(\frac{\partial \Delta T}{\partial \overline{\Delta T}}\right)^2 \cdot u^2(\overline{\Delta T}) + \left(\frac{\partial \Delta T}{\partial \Delta R}\right)^2 \cdot u^2(\Delta R) + \left(\frac{\partial \Delta T}{\partial \Delta ICJF}\right)^2 \cdot u^2(\Delta ICJF) + \left(\frac{\partial \Delta T}{\partial \Delta IC}\right)^2 \cdot u^2(\Delta IC) \quad (7)$$

$$c_{\overline{\Delta T}} = \frac{\partial \Delta T}{\partial \overline{\Delta T}} = 1; \quad c_{\Delta R} = \frac{\partial \Delta T}{\partial \Delta R} = 1; \quad c_{\Delta ICJF} = \frac{\partial \Delta T}{\partial \Delta ICJF} = 1; \quad c_{\Delta IC} = \frac{\partial \Delta T}{\partial \Delta IC} = 1 \tag{8}$$

The cold junction temperature must be verified against a calibrated instrument, such as a bulb thermometer, in a constant temperature environment. Therefore, the parcel $\Delta ICJF$ can be calculated applying Eq. (4) to the mathematical model of Eq. (9), where $\overline{\Delta Tjf}$ is the uncertainty related to cold junction temperature variability around mean, ΔRjf is the uncertainty due to resolution of cold junction temperature measurement through the inbuilt NTC, and $\Delta ICbt"$ comes from the calibrated bulb thermometer, which has resolution of 0.1 °C. This yields Eq. (10) and Eq. (11).

$$\Delta ICJF = \overline{\Delta Tjf} + \Delta Rjf + \Delta ICbt" \tag{9}$$

$$u_c^2(\Delta ICJF) = \left(\frac{\Delta ICJF}{\partial \overline{\Delta Tjf}}\right)^2 \cdot u^2(\overline{\Delta Tjf}) + \left(\frac{\Delta ICJF}{\partial \Delta Rjf}\right)^2 \cdot u^2(\Delta Rjf) + \left(\frac{\Delta ICJF}{\partial \Delta ICbt"}\right)^2 \cdot u^2(\Delta ICbt") \tag{10}$$

$$c_{\overline{\Delta Tjf}} = \frac{\partial \Delta ICJF}{\partial \overline{\Delta Tjf}} = 1; \quad c_{\Delta Rjf} = \frac{\partial \Delta ICJF}{\partial \Delta Rjf} = 1; \quad c_{\Delta ICbt} = \frac{\partial \Delta ICJF}{\partial \Delta ICbt"} = 1; \tag{11}$$

In case the thermometer is not calibrated, its uncertainty can be estimated if the operator has enough knowledge of this measuring system (Vuolo, 1996), establishing a factor recommended between 0.5 to 2 to be multiplied by the resolution *Rbt* of the instrument. In this work, the broadest factor was adopted because the thermometer has not been calibrated in the recent years.

4.2 Calibration of thermocouple and system

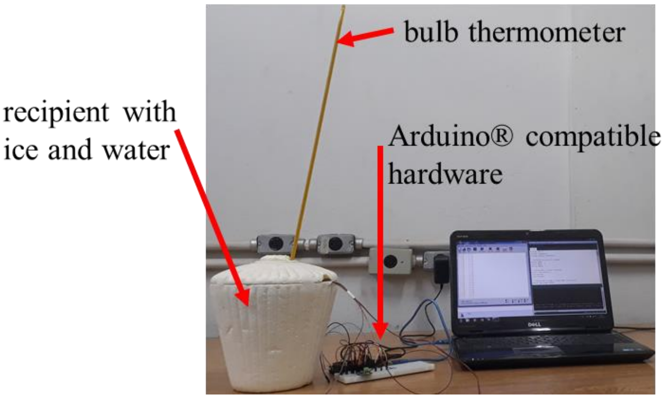

Figure 1. Calibration of thermocouple and system by fixed method point, using mix of ice and distilled water and bulb thermometer as reference.

The calibration of thermocouple and system can be performed by the fixed point method, which uses phase change temperatures of substances such as distilled water. Figure 1 provides a scheme of the procedure. In order to determine ΔIC, it is first necessary to quantify the maximum uncertainty for temperature measured in water during phase change, which is done by using the system to probe temperature at a mix of water and ice, and then at boiling water. This way, the uncertainty of the measuring system is considered as the highest value obtained from applying Eq. (4) to the mathematical models of Eq. (12) and Eq. (13). These same procedure and models were carried out by Oliveira et. al. (2020). If the measurements are not performed at 20 °C and 101325 Pa, for which the triple point is well known, then the actual temperature must be measured by a secondary bulb thermometer, for which uncertainty will be $\Delta ICbt$. This results in Eq. (14) up to Eq. (17). Although this other instrument has a resolution of 0.2 °C, it is possible to use 0.1 °C through interpolation, since it is analogic and has a clear scale.

$$\Delta T_{0\,°C} = \overline{\Delta T_{0\,°C}} + \Delta R + \Delta ICbt \tag{12}$$

$$\Delta T_{100\,°C} = \overline{\Delta T_{100\,°C}} + \Delta R + \Delta ICbt \tag{13}$$

$$u_c^2(\Delta T_{0\,°C}) = \left(\frac{\partial \Delta T_{0\,°C}}{\partial \overline{\Delta T_{0\,°C}}}\right)^2 \cdot u^2(\overline{\Delta T_{0\,°C}}) + \left(\frac{\partial \Delta T_{0\,°C}}{\partial \Delta R}\right)^2 \cdot u^2(\Delta R) + \left(\frac{\partial \Delta T_{0\,°C}}{\partial \Delta ICbt}\right)^2 \cdot u^2(\Delta ICbt) \tag{14}$$

$$c_{\overline{T_{0\,°C}}} = \frac{\partial T_{0\,°C}}{\partial \overline{T_{0\,°C}}} = 1; \quad c_{\Delta Rjf} = \frac{\partial T_{0\,°C}}{\partial \Delta R} = 1; \quad c_{\Delta ICbt} = \frac{\partial T_{0\,°C}}{\partial \Delta ICbt} = 1; \tag{15}$$

$$u_c^2(\Delta T_{100\,°C}) = \left(\frac{\partial \Delta T_{100\,°C}}{\partial \overline{\Delta T_{100\,°C}}}\right)^2 \cdot u^2(\overline{\Delta T_{100\,°C}}) + \left(\frac{\partial \Delta T_{100\,°C}}{\partial \Delta R}\right)^2 \cdot u^2(\Delta R) + \left(\frac{\partial \Delta T_{100\,°C}}{\partial \Delta ICbt}\right)^2 \cdot u^2(\Delta ICbt) \tag{16}$$

$$c_{\overline{\Delta T_{100\,°C}}} = \frac{\partial \Delta T_{100\,°C}}{\partial \overline{\Delta T_{100\,°C}}} = 1; \quad c_{\Delta Rjf} = \frac{\partial \Delta T_{100\,°C}}{\partial \Delta R} = 1; \quad c_{\Delta ICbt} = \frac{\partial \Delta T_{100\,°C}}{\partial \Delta ICbt} = 1; \tag{17}$$

4.3 Uncertainty of corrected temperature

Temperature correction accounts for off-set errors and discrepancies in the material sensitivity constant. It is done through linear regression of collected data, which in this case consists only of two points. Thus, a linear equation can be simply calculated through the coordinates $(x_0;\, y_0)$ and $(x_1;\, y_1)$ that correspond to values obtained by thermocouple system and bulb thermometer, for which the points $(Tbt_{0\,°C},\ T_{0\,°C})$ and $(Tbt_{100\,°C},\ T_{100\,°C})$ are shown in Fig. 2.

Solving the linear system of Eq. (18) yields Eq. (19), from which the corrected temperature Tc is determined in Eq. (20) according to the temperature indicated by measuring system. After substituting the average values for each variable, Eq. (21) is obtained. The uncertainty of Tc is determined through application of Eq. (4) to Eq. (20), taking into consideration that uncertainty can never be subtracted, yielding Eq. (22) and Eq. (23). In this case, the partial derivatives will be different from 1 and they are determined by the average values of each variable, as shown in Eq. (24) to Eq. (26). The uncertainties $\Delta Tbt_{0\,°C}$ and $\Delta Tbt_{100\,°C}$ are estimated after applying Eq. (4) again, as described in Eq. (27) to Eq. (32).

$$y_0 = a \cdot x_0 + b \quad ; \quad y_1 = a \cdot x_1 + b \tag{18}$$

$$a = \frac{(y_1 - y_0)}{(x_1 - x_0)} = \frac{(Tbt_{100\,°C} - Tbt_{0\,°C})}{(T_{100\,°C} - T_{0\,°C})} \quad ; \ b = y_1 - a \cdot x_1 = \left(Tbt_{100\,°C} - \frac{(Tbt_{100\,°C} - Tbt_{0\,°C})}{(T_{100\,°C} - T_{0\,°C})} \cdot T_{100\,°C}\right) \tag{19}$$

$$Tc = \frac{(Tbt_{100\,°C} - Tbt_{0\,°C})}{(T_{100\,°C} - T_{0\,°C})} \cdot T + \left(Tbt_{100\,°C} - \frac{(Tbt_{100\,°C} - Tbt_{0\,°C})}{(T_{100\,°C} - T_{0\,°C})} \cdot T_{100\,°C}\right) \tag{20}$$

$$Tc = 1.01991 \cdot T - 1.30713 \tag{21}$$

$$\Delta Tc = \frac{(\Delta Tbt_{100\,°C} + \Delta Tbt_{0\,°C})}{(\Delta T_{100\,°C} + \Delta T_{0\,°C})} \cdot \Delta T + \frac{(\Delta Tbt_{100\,°C} + \Delta Tbt_{0\,°C})}{(\Delta T_{100\,°C} + \Delta T_{0\,°C})} \cdot \Delta T_{100\,°C} + \Delta Tbt_{100\,°C} \tag{22}$$

$$\begin{aligned} u_c^2(\Delta Tc) = {} & \left(\frac{\partial \Delta Tc}{\partial \Delta Tbt_{100\,°C}}\right)^2 \cdot u^2(\Delta Tbt_{100\,°C}) + \left(\frac{\partial \Delta Tc}{\partial \Delta Tbt_{0\,°C}}\right)^2 \cdot u^2(\Delta Tbt_{0\,°C}) + \left(\frac{\partial \Delta Tc}{\partial \Delta T_{100\,°C}}\right)^2 \cdot u^2(\Delta T_{100\,°C}) \\ & + \left(\frac{\partial \Delta Tc}{\partial \Delta T_{0\,°C}}\right)^2 \cdot u^2(\Delta T_{0\,°C}) + \left(\frac{\partial \Delta Tc}{\partial \Delta T}\right)^2 \cdot u^2(\Delta T) \end{aligned} \tag{23}$$

$$c_{\Delta Tbt_{100\,°C}} = \frac{\partial \Delta Tc}{\partial \Delta Tbt_{100\,°C}} = \frac{\Delta T + \Delta T_{100\,°C}}{\Delta T_{100\,°C} + \Delta T_{0\,°C}} + 1; \quad c_{\Delta Tbt_{0\,°C}} = \frac{\partial \Delta Tc}{\partial \Delta Tbt_{0\,°C}} = \frac{\Delta T + \Delta T_{100\,°C}}{\Delta T_{100\,°C} + \Delta T_{0\,°C}}; \tag{24}$$

$$c_{\Delta T_{0\,°C}} = \frac{\partial \Delta Tc}{\partial \Delta T_{0\,°C}} = \frac{(\Delta Tbt_{100\,°C} + \Delta Tbt_{0\,°C}) \cdot (-\Delta T - \Delta T_{100\,°C})}{(\Delta T_{100\,°C} + \Delta T_{0\,°C})^2}; \quad c_{\Delta T} = \frac{\partial \Delta Tc}{\partial \Delta T} = \frac{\Delta Tbt_{100\,°C} + \Delta Tbt_{0\,°C}}{\Delta T_{100\,°C} + \Delta T_{0\,°C}}; \tag{25}$$

$$c_{\Delta T_{100\,°C}} = \frac{\partial \Delta Tc}{\partial \Delta T_{100\,°C}} = \frac{(\Delta Tbt_{0\,°C} + \Delta Tbt_{100\,°C}) \cdot (\Delta T_{0\,°C} - \Delta T)}{(\Delta T_{100\,°C} + \Delta T_{0\,°C})^2}; \tag{26}$$

$$\Delta Tbt_{0\,°C} = \overline{\Delta Tbt_{0\,°C}} + \Delta Rbt + \Delta ICbt \tag{27}$$

$$u_c^2(\Delta Tbt_{0\,°C}) = \left(\frac{\partial \Delta Tbt_{0\,°C}}{\partial \overline{\Delta Tbt_{0\,°C}}}\right)^2 \cdot u^2(\overline{\Delta Tbt_{0\,°C}}) + \left(\frac{\partial \Delta Tbt_{0\,°C}}{\partial \Delta Rbt}\right)^2 \cdot u^2(\Delta Rbt) + \left(\frac{\partial \Delta Tbt_{0\,°C}}{\partial \Delta ICbt}\right)^2 \cdot u^2(\Delta ICbt) \tag{28}$$

$$c_{\overline{\Delta Tbt_{0\,°C}}} = \frac{\partial \Delta Tbt_{0\,°C}}{\partial \overline{\Delta Tbt_{0\,°C}}} = 1; \quad c_{\Delta Rjf} = \frac{\partial \Delta Tbt_{0\,°C}}{\partial \Delta Rbt} = 1; \quad c_{\Delta ICbt} = \frac{\partial \Delta Tbt_{0\,°C}}{\partial \Delta ICbt} = 1; \tag{29}$$

$$\Delta Tbt_{100\,°C} = \overline{\Delta Tbt_{100\,°C}} + \Delta Rbt + \Delta ICbt \tag{30}$$

$$u_c^2(\Delta Tbt_{100°C}) = \left(\frac{\partial \Delta Tbt_{100°C}}{\partial \overline{\Delta Tbt_{100°C}}}\right)^2 \cdot u^2\left(\overline{\Delta Tbt_{100°C}}\right) + \left(\frac{\partial \Delta Tbt_{100°C}}{\partial \Delta Rbt}\right)^2 \cdot u^2(\Delta Rbt) + \left(\frac{\Delta Tbt_{100°C}}{\partial \Delta ICbt}\right)^2 \cdot u^2(\Delta ICbt) \tag{31}$$

$$c_{\overline{\Delta Tbt_{100\,°C}}} = \frac{\partial \Delta Tbt_{100\,°C}}{\partial \overline{\Delta Tbt_{100\,°C}}} = 1; \quad c_{\Delta Rjf} = \frac{\partial \Delta Tbt_{100\,°C}}{\partial \Delta Rbt} = 1; \quad c_{\Delta ICbt} = \frac{\partial \Delta Tbt_{100\,°C}}{\partial \Delta ICbt} = 1; \tag{32}$$

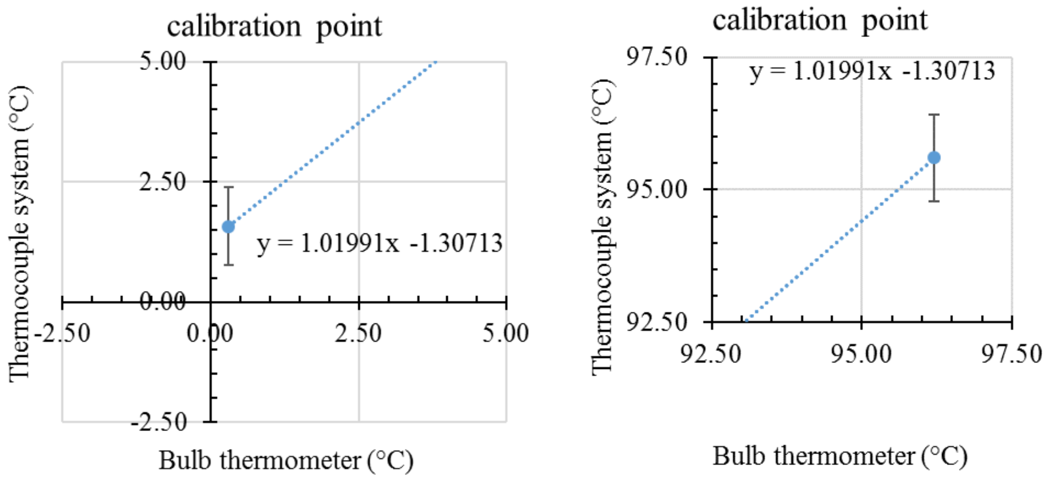

Figure 2. Temperature measurements during calibration in water mix, where bars represent the expanded uncertainty calculated for $T_{0\,°C}$ and $T_{100\,°C}$ with 95.45 % confidence interval and 9 effective degrees of freedom.

5. RESULTS

Table 1 shows the data obtained with this measuring system in an industrial application for controlling polymer temperature during deposition of coatings through directed heat. The temperature T was measured for 7 s and the methodology was applied to calculate the correct temperature Tc based upon calibration. Each variance was calculated according to evaluation type and sample distribution, as shown in Tab. 2. Degrees of freedom for type A evaluation depend on the number of points n used for calculating the mean, such that $DOF = n - 1$. Meanwhile for type B evaluations it depends on the distribution assumed for that variable. Again, the GUM (INMETRO, JCGM, 2008) shows how each parameter is determined. The number of points during calibration should be high, showing that the water phase mix is in equilibrium. Fewer points should suffice for other measurements, depending on how stable the system is.

Table 1. Mean, standard deviation, standard and expanded uncertainty for the main variables.

	Thermometer (°C)			**Thermocouple system (°C)**				
	$Tbt_{0\,°C}$	$Tbt_{100\,°C}$	Tbt	$T_{0\,°C}$	$T_{100\,°C}$	Tjf	T	Tc
average	0.3	96.2	22.3	1.58	95.60	21.98	76.79	77.01
std. dev.	0.1	0.3	0.42	0.80	0.41	0.11	0.38	-
max - min	0.2	0.6	0.6	6.00	2.50	0.75	1.50	
n	3	3	1	1198	1198	1180	35	-
v	2	2	-	1197	1197	1179	34	13
uc	0.23805	0.28868	-	0.35195	0.35139	0.00309	0.50642	1.23687
K	4.30	3.18	-	2.32	2.32	-	2.16	2.16

Figure 3 shows the contributions of each source in the different models, from which it is clear that the variabilities around the average are the smallest sources of uncertainty for this system. The first reason is the high number of points n sampled, since the standard uncertainty is inversely proportional to the root square of n for type A variables. The second is the low standard deviations, which are less than 1 °C, thanks to the many points and low spread of values shown by the difference between maximum and minimum in Tab. 1. Thus, this system is stable enough for industrial applications where variations of a few degrees are acceptable. The only exception is $T_{0\,°C}$, for which the measuring system seems more unstable. Still, it is in accordance with the maximum variability stated in the catalogue for the MAX31855 module.

Table 2. Characteristics and values of each uncertainty parameter in all equations.

Variable /	Type	Probability distribution	Degrees of freedom	$u(x_i)$	$u(x_i)$ (°C)	c_{x_i}	Contrib. (%) [Eq.]	K	$U(Y)$ (°C)
ΔT	B	Normal	17	= result of Eq. (7)	0.50642	0.99301	16.53 [23]	2.16	1.10
$\overline{\Delta T}$	A	Normal	34	$= \frac{s_{\overline{\Delta T}}}{\sqrt{n}}$, where $s_{\overline{\Delta T}}$ is the standard deviation around $\overline{\Delta T}$	0.06358	1	1.58 [7]		
ΔR	B	rectangular	∞	$= \frac{R}{2\sqrt{3}}$, where R is the resolution of digital instrument	0.28868	1 1 1	32.49 [7] 67.27 [14] 67.49 [16]		
$\Delta ICJF$	B	Normal	1	= result of Eq. (10)	0.21264	1	17.63 [7]	13.97	2.98
ΔIC	B	Normal	9	= highest between $\Delta T_{0\,°C}$ and $\Delta T_{100\,°C}$	0.35195	1	48.30 [7]	2.32	0.82
$\overline{\Delta Tjf}$	A	Normal	1179	$= \frac{s_{\overline{\Delta Tjf}}}{\sqrt{n}}$, where $s_{\overline{\Delta Tjf}}$ is the standard deviation around $\overline{\Delta Tjf}$	0.00309	1	0.02 [9]		
ΔRjf	B	rectangular	∞	$= \frac{Rjf}{2\sqrt{3}}$, where Rjf is the resolution of digital instrument	0.07217	1	11.52 [9]		
$\Delta ICbt"$	B	t-Student	1	$= 2 \cdot Rbt"$, where Rbt is the resolution of analogic instrument	0.20000	1	88.46 [9]		
$\Delta T_{0\,°C}$	B	Normal	9	= result of Eq. (14)	0.35195	-1.76161	25.13 [23]	2.32	0.82
$\Delta T_{100\,°C}$	B	Normal	9	= result of Eq. (16)	0.35139	-0.76860	4.77 [23]	2.32	0.82
$\overline{\Delta T_{0\,°C}}$	A	Normal	1197	$= \frac{s_{\overline{\Delta T_{0\,°C}}}}{\sqrt{n}}$, where $s_{\overline{\Delta T_{0\,°C}}}$ is the standard deviation around $\overline{\Delta T_{0\,°C}}$	0.02320	1	0.43 [14]		
$\overline{\Delta T_{100\,°C}}$	A	Normal	1197	$= \frac{s_{\overline{\Delta T_{100\,°C}}}}{\sqrt{n}}$, where $s_{\overline{\Delta T_{100\,°C}}}$ is the standard deviation around $\overline{\Delta T_{100}}$	0.01195	1	0.12 [16]		
$\Delta ICbt$	B	t-Student	1	$= 2 \cdot Rbt$, where Rbt is the resolution of analogic instrument	0.20000	1	32.29 [14] 32.39 [16] 70.59 [28] 48.00 [31]		
$\Delta Tbt_{0\,°C}$	B	t-Student	2	= result of Eq. (28)	0.23805	1.77401	11.66 [23]	4.30	1.03
$\overline{\Delta Tbt_{0\,°C}}$	A	t-Student	2	$= \frac{s_{\overline{\Delta Tbt_{0\,°C}}}}{\sqrt{n}}$, where $s_{\overline{\Delta Tbt_{0\,°C}}}$ is the standard deviation around $\overline{\Delta Tbt_{0\,°C}}$	0.05774	1	5.88 [28]		
$\Delta Tbt_{100\,°C}$	B	t-Student	3	= result of Eq. (31)	0.28868	2.77401	41.92 [23]	3.18	0.92
$\overline{\Delta Tbt_{100\,°C}}$	A	t-Student	2	$= \frac{s_{\overline{\Delta Tbt_{100\,°C}}}}{\sqrt{n}}$, where $s_{\overline{\Delta Tbt_{100\,°C}}}$ is the standard deviation around $\overline{\Delta Tbt_{100\,°C}}$	0.17321	1	36.00 [31]		
ΔRbt	B	rectangular	∞	$= \frac{Rbt}{2\sqrt{3}}$, where Rbt is the resolution of bulb thermomether	0.11547	1	23.53 [28] 16.00 [31]		
ΔTc	B	t-Student	13	= result of Eq. (23)	1.23687			2.16	2.68

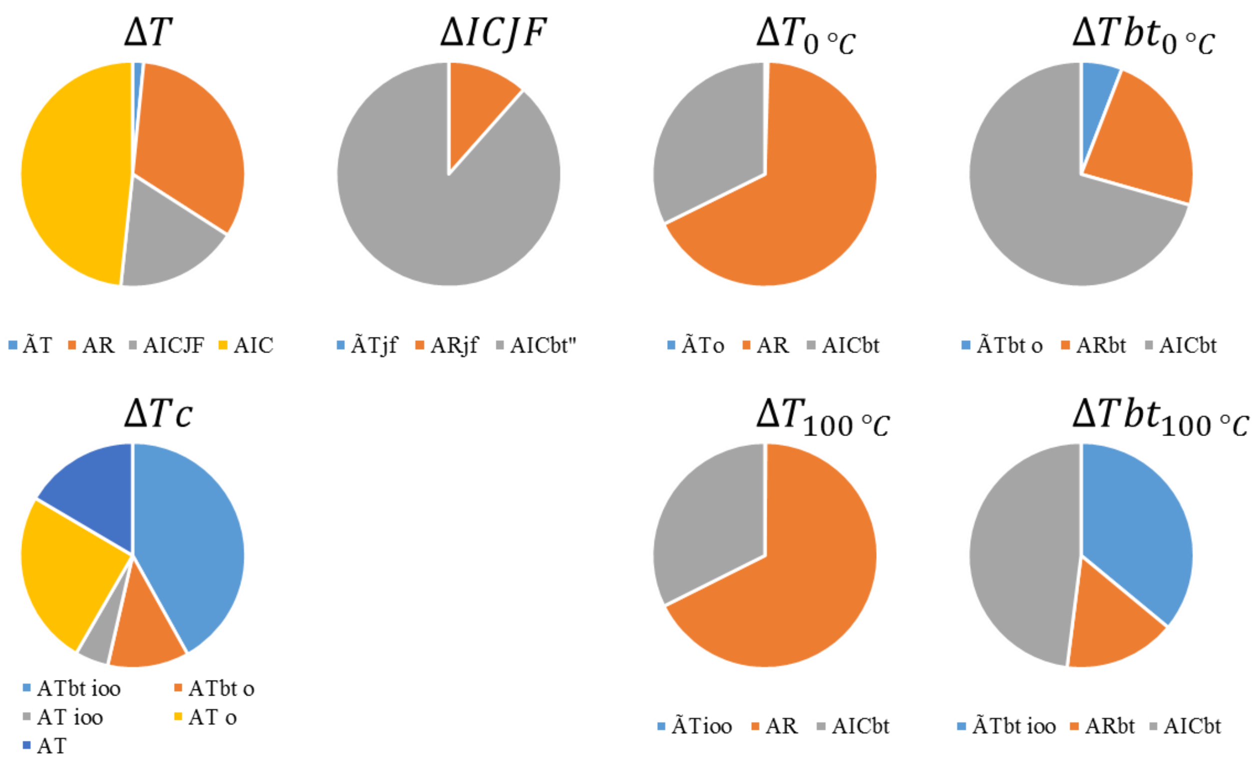

Figure 3. Percentage contribution of each source in different uncertainty models.

The graphs also indicate that the factor of 2 adopted to calculate the standard uncertainty for calibration of bulb thermometers was very conservative because this variable accounts for the highest source among the models for $\Delta Tbt_{0\,°C}$, $\Delta Tbt_{100\,°C}$ and $\Delta ICJF$. Regarding the calibration of the measuring system, the uncertainty due to resolution represents almost 75 %, which happened specifically due to the very low variabilities around the average for each measurement. Although this could indicate the need for a system with better sensitivity for temperature variations, this is not necessary for the applications discussed in this work.

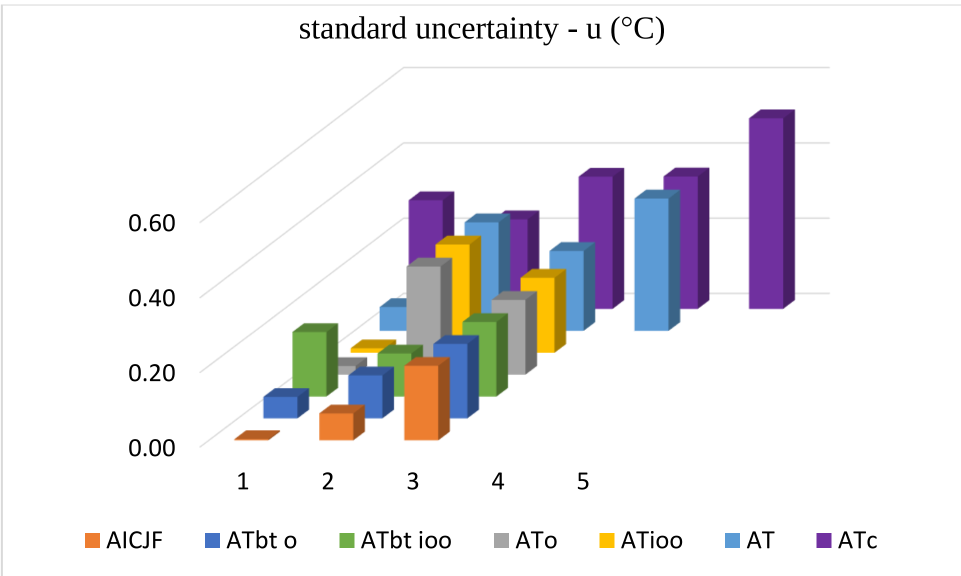

Figure 4. Absolute values for standard uncertainties relative to each source for the various models. The numbers in x-axis refer to the parcels in each model of Fig. 3.

Table 1 shows that the corrected temperature is very close to the normal measurement, with less than 0.5 °C of difference. Indeed, due to the linear model and the higher error obtained during calibration for 0 °C, the temperatures will be very close for temperatures closer to 100 °C. This is expected, because the linearity of K thermocouples is about 99 %

in this range (Omega Engineering, 2015). That does not mean the correction is unimportant, especially because there is a great change in the uncertainty from T to Tc, which increases more than twice. This happens due to contributions from all uncertainties related to calibration procedures, in which it is possible to see a great contribution from $Tbt_{100\,°C}$, due to a higher coefficient of sensitivity. This is a circumstantial consequence of the mathematical model for the linear regression in this case, since the standard uncertainty of this variable is smaller than T, $T_{0\,°C}$, and $T_{100\,°C}$.

Finally, the measurement result can be expressed as shown in Eq. (33), in accordance with the NBR ISO/IEC 17025:2017 (ABNT, ISO/IEC, 2017). The measurement, now fully characterized, gives a clear idea of the precision of the measuring system. As stated before, this precision is enough for applications that allow small variations in process temperature. Although the main examples cited were industries, this measuring system can also be safely employed in many research applications for manufacturing processes such as machining and welding. In such cases, there is often the need to analyze temperatures that differ much more than 3 °C when changing process parameters, and the measurement with Arduino® microcontroller and hardware would be much cheaper than the commonly used Agilent boards.

$$Tc = 77.01 \pm 2.68\,°C\ with\ K = 2.16\ and\ 95\%\ confidence\ interval \tag{21}$$

6. CONCLUSIONS

The procedures for uncertainty assessment are usually very time consuming, which is the main reason why most works do not approach this domain. Nevertheless, once the methodology is stablished, it can be easily turned into a computational routine for calculation with any similar data through use of software like Excel® or Matlab®. Above all, it reveals the main sources of uncertainty, which in this case related to calibration and resolution of the instruments. This reveals that the temperature variations are not much greater than the resolution of the system, and since the main application is for processes tolerant to variations, the measuring system is adequate. Also, this work shows how the different variables may interact in more complex models such as in the calculation of ΔTc, for which uncertainty was much higher than it was before correction.

The equations described are applicable for any other measurements performed with thermocouples and other data acquisition systems in industrial and laboratory applications. The same mathematical models may be used to estimate the uncertainty through the Monte Carlo method, although the procedure for determining each standard uncertainty is different in this second methodology. The precision of about 3 °C indicates a very cost effective solution when comparing the US$20 Arduino® board and module with an Agilent board that may cost a few hundred dollars, or even more, thus making it an attractive solution for research applications as well.

The final calculations give not only a clear idea about the precision of the whole measuring system, but it guarantees its traceability in order to make comparisons against other measurements conducted in the same conditions and also in different measuring systems. That is of utmost importance because as discussed in the GUM (INMETRO, JCGM, 2008), there is a worldwide tendency of applying these uncertainty measuring routines in all future works to evaluate the repeatability and reproducibility of results either in industrial or in research applications.

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9. RESPONSIBILITY NOTICE

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