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EXPERIMENTAL DETERMINATION OF THE DISCHARGE COEFFICIENT OF A FLOODGATE AND THE SURFACE ROUGHNESS SCALE IN A PHYSICAL MODEL OF A SMALL HYDROELECTRIC POWER PLANT

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Abstract: *One of the ways to observe the physical phenomena that happen in a hydroelectric plant reservoir, such as sediment transportation, is the analysis of a reduced scale model, on which the variables are more easily controlled, being able to represent with assertiveness the real case. On the Hydraulic Research Centre (CPH) of the Federal University of Minas Gerais (UFMG) there is a physical model of the Small Hydroelectric Power Plant (SHPP) Salto do Paraopeba on the geometric scale of 1:40. The aim of this paper is to experimentally determine the discharge coefficient of the aforementioned SHPP and the surface roughness scale between the model and the prototype. Four equations found in the literature are used to determine the discharge coefficient and the geometrics, kinematics and dynamics similarities theory to determine the surface roughness scale. The results have shown good consonance with the estimated methods for the discharge coefficient. Furthermore, a scale of 2:3 for the Manning coefficient of the floodgate was reached.*

Keywords: *Hydroelectric plant, physical model, floodgate, discharge coefficient, surface roughness.*

1. INTRODUCTION

Open-channel flows are characterized by having a free surface open to the atmosphere, having a great number of practical applications in engineering. Its complexity resides on the vast combination of factors that define the channel, since the free surface can vary on time and space, and, as a result, the flow depth, the flow rate and the bed slope are interdependent physical quantities.

In open-channels, as opposed to confined environments such as pipes, the flow rate is controlled by partially blocking the channel, which can be done by an obstacle that forces the flow to go over or under it. In the present case, there is a floodgate or sluice gate called an underflow gate, as the fluid flows under the obstacle, through an adjustable opening.

According to Çengel (2007), the discharge from an underflow gate is called a free outflow if the fluid coming out of the gate is in direct contact with the atmosphere, unlike the drowned outflow, when the discharged liquid returns and influences the behavior of the fluid going through the floodgate.

The frictional effects can be calculated by modifying the discharge velocity relation of a free jet manipulating Bernoulli's equation and continuity equations, using a discharge coefficient. The discharge coefficient equals 1 when the flow is ideal, which means that there is no loss due to friction. However, for a real flow the discharge coefficient is lower than 1.

Henry (1950) executed a considerable experimental research, on which he achieved a curve based on data gathered from tests that estimated the discharge coefficients of floodgates on free and drowned outflows.

Henderson (1966) said that most of the discharge coefficient values of a free outflow in a vertical sluice gate varies between 0.5 and 0.6. Henderson (1966) estimated the value of the contraction coefficient as equal to 0.61 for the free outflow condition and came to the conclusion that the coefficient of energy loss due to friction changes according to the contraction coefficient.

The experiments conducted by Sepuvela et al. (2009) in submerged flows utilizing high precision measurements and sophisticated software showed mean errors lower than 10%. The method that presented the most significant error was Swamee's (1992) and that can be due to the numerical method used to fit data to the curve.

Habibzadeh et al. (2011) estimated a mean value for the friction factor of 0.062, knowing that, for a specific value of a gate opening, there will be a maximum friction factor value.

The contraction coefficient and the friction factor values justify the small variation of the results for the discharge coefficient, as stated by Shayan and Farhoudi (2013).

Dabral et al. (2014) obtained discharge coefficient values for a vertical gate that varied between 0.7 and 0.8, with a flow rate of 40 to 100L/min and for a radial gate, obtained values that varied between 0.75 and 0.89 with a flow rate of 60 to 120L/min, showing the relation, in both cases, of the flow rate and the height of the floodgate.

Similarity, dimensional analysis and modelling theories allow tests executed in a lab, using a physical model, to be replicated to real scales. A physical model is a representation of the physical system, which can be used to predict the behavior of said system regarding an intended aspect. The real physical system, to which estimates are made, is called prototype. In light of that, the Hydraulic Research Centre (CPH) of the Federal University of Minas Gerais (UFMG) built a physical model on the scale of 1:40 of the Small Hydroelectric Power Plant (SHPP) Salto do Paraopeba, located in Belo Vale, State of Minas Gerais (MG), Brazil. The present paper shows the analysis of an experiment that was conducted on this physical model with the purpose of determining the discharge coefficient of the floodgate and compare it to the values obtained through experimental equations from different researchers. Moreover, the different variables of the model and the prototype were determined through geometric, kinematic and dynamic similarities involved, highlighting the surface roughness scale.

2. METHODOLOGY

The mathematical modelling to determine the discharge coefficient, the experimental procedures conducted on the physical model and the similarity theories involved in the determination of the variables from the physical model and the prototype (power plant), highlighting the surface roughness scale, are described below.

2.1 Mathematical Modelling of the discharge coefficient

According to Porto (2006), floodgates are hydraulic structures designed to control fluid flow rate, generally water, in channels, levees, dams, etc. They control the fluvial flow characteristics upstream and torrential downstream. Its operation consists of lifting a movable plate that serves as a bulkhead to the water, allowing the manipulation of the orifice opening and control the discharge (Figure 1).

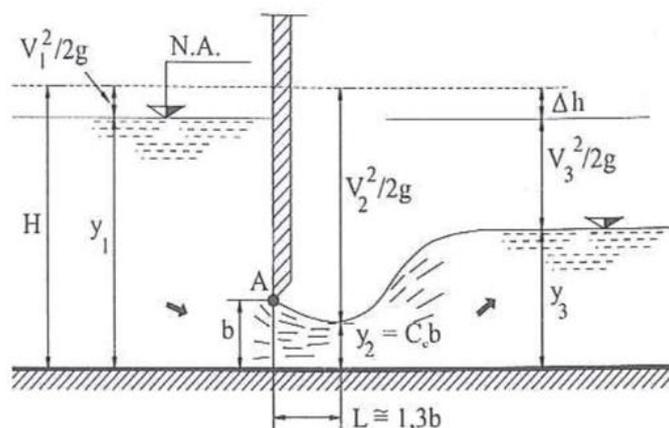


Figure 1. Vertical floodgate. Font: Porto (2006).

Analyzing the phenomenon of a fluid going through an orifice, it is possible to notice that the velocity vectors of the particles are not all the same, and also the area of the orifice, despite being constant when the opening is fixed, generates fluid contraction. However, to calculate the flow rate, which depends on the aforementioned variables, a discharge coefficient for the purpose of correction must be considered. Therefore, this coefficient needs to be measured with values between zero and one, with the last being the ideal case where losses are negligible.

2.1.1 Torricelli's Equation

According to Porto (2006), Eq. (1) is known as the Torricelli's theorem which provides the theoretical velocity V_t of a jet through an orifice.

$$V_t = \sqrt{2gH} \quad (1)$$

Where H is the head of the water upstream of the floodgate and g is the acceleration due to gravity. Nonetheless, due to the existence of energy loss when passing through the orifice, the real velocity is slightly smaller than the theoretical velocity. Thus, the relation between the real velocity V and the theoretical velocity is called velocity coefficient C_v , given by Eq. (2).

$$C_v = \frac{V}{V_t} \quad (2)$$

From Eq. (1) and Eq. (2), the real velocity is given by Eq. (3).

$$V = C_v \sqrt{2gH} \quad (3)$$

Knowing that the real volumetric flow rate Q is given by the product of the contracted section area A_c and the real velocity, from Eq. (3) derives Eq. (4).

$$Q = A_c C_v \sqrt{2gH} \quad (4)$$

The relation between the contracted section area A_c and the orifice area A is known as the contraction coefficient C_c , given by Eq. (5).

$$C_c = \frac{A_c}{A} \quad (5)$$

From Eq. (5) and Eq. (4), Eq. (6) is obtained.

$$Q = C_c C_v A \sqrt{2gH} \quad (6)$$

The product of the contraction coefficient C_c and the velocity coefficient C_v is defined as the discharge coefficient C_d . Therefore, the volumetric flow rate is given by Eq. (7).

$$Q = C_d A \sqrt{2gH} \quad (7)$$

In a floodgate, the head of the water upstream becomes the height of the column of water and the flow can be treated as bidimensional using the orifices law, which gives Eq. (8).

$$Q = C_d A \sqrt{2gy_1} \quad (8)$$

Where y_1 is the height of the column of water upstream of the floodgate. The area through which the water flows can be calculated by the product of the floodgate width L_{fg} and the height of the water level downstream y_2 , leading to Eq. (9).

$$Q = C_d \sqrt{2g} L_{fg} y_1^{0.5} y_2 \quad (9)$$

2.1.2 Rajaratnam and Subramanya's experimental equation (1967)

Rajaratnam and Subramanya (1967) reached Eq. (10) revising and investigating the results obtained by Henry (1950). Figure 2 displays Henry's curve (1950).

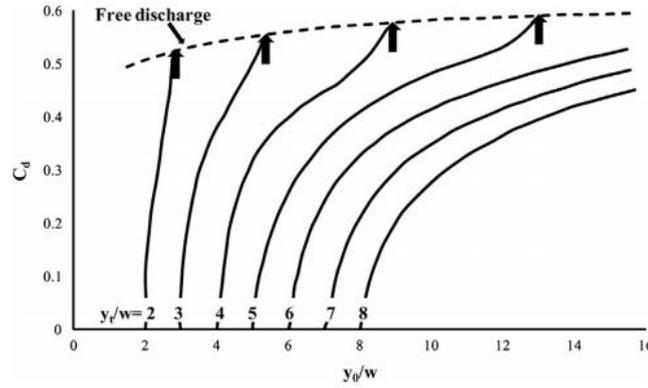


Figure 2. Henry's curve to determine the Discharge Coefficient of floodgates. Henry (1950).

$$C_d = \frac{0,611}{\sqrt{1 - 0,611^2 \left(\frac{y_2}{y_1}\right)^2}} \quad (10)$$

2.1.3 Nago's experimental equation (1978)

Nago (1978), during his studies about the influence of gate shapes on discharge coefficients, obtained Eq. (11).

$$C_d = 0,6 \exp(-0,3 \frac{y_2}{y_1}) \quad (11)$$

2.1.4 Swamee's experimental equation (1992)

Swamee (1992) reached Eq. (12) for the discharge coefficient of a free outflow by using a nonlinear regression on Henry's curve (1950).

$$C_d = 0,611 \left(\frac{y_1 - y_2}{y_1 + 15y_2}\right)^{0,072} \quad (12)$$

Swamee's (1992) methodology can be used to solve common problems in floodgates operations, such as the one presented on this paper, of predicting the discharge coefficient for a free outflow.

2.2 Experimental procedures

With the intention of simulating methods to provide control over siltation processes, as well as the maintenance of its hydraulic structures, the CPH, in partnership with the Energetic Company of Minas Gerais (CEMIG), built a non-distorted, reduced physical model of the SHPP Salto Paraopeba, encompassing on the prototype a total length of 240 m with a geometric scale of 1:40 (Figure 3). The plant was built in the 1960s and has a water reservoir, that historically suffers from continuous action of siltation processes in its operation (Campos et al., 2017).



Figure 3. Physical Model / SHPP Salto Paraopeba.

A floodgate was installed in the physical model representing a prototype with a 7.00 m gap over 9.20 m height, with the goal of complementing the capacity of the spillway. Figure 4 shows the reservoir located in the SHPP Salto Paraopeba in March 2010. The sluice gate was allocated on the model on the right of the dam, replacing the mechanical room.



Figure 4. SPHH Salto Paraopeba.

Experimental procedures executed in the physical model are registered below.

- The width of the floodgate, $L_{fg}=23.2\text{ cm}$, was measured using a scale;
- The pump was activated and the water flow was established in the channel;
- The floodgate was opened and adjusted in a way that the water level in the dam was as high as possible, avoiding, however, overflowing in the spillway;
- Water level readings were made downstream y_2 and upstream y_1 of the sluice gate, as shown in Figures 5 and 6. The flow rate Q established by the adjustment of the frequency inverter that triggers the pump was also registered;
- Two other operation points were established, with different values of the flow rate Q and, consequently, new floodgate openings and water levels downstream y_2 . The water level upstream y_1 was kept constant for all measured points.

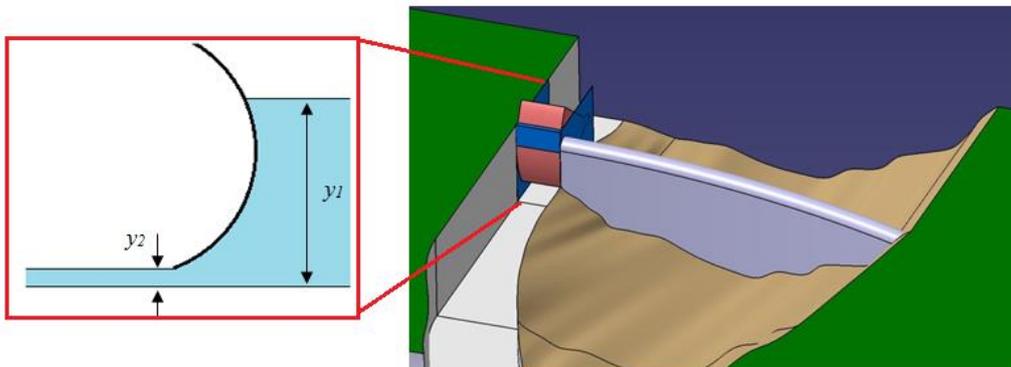


Figure 5. Schematic representation of the floodgate and the dam.

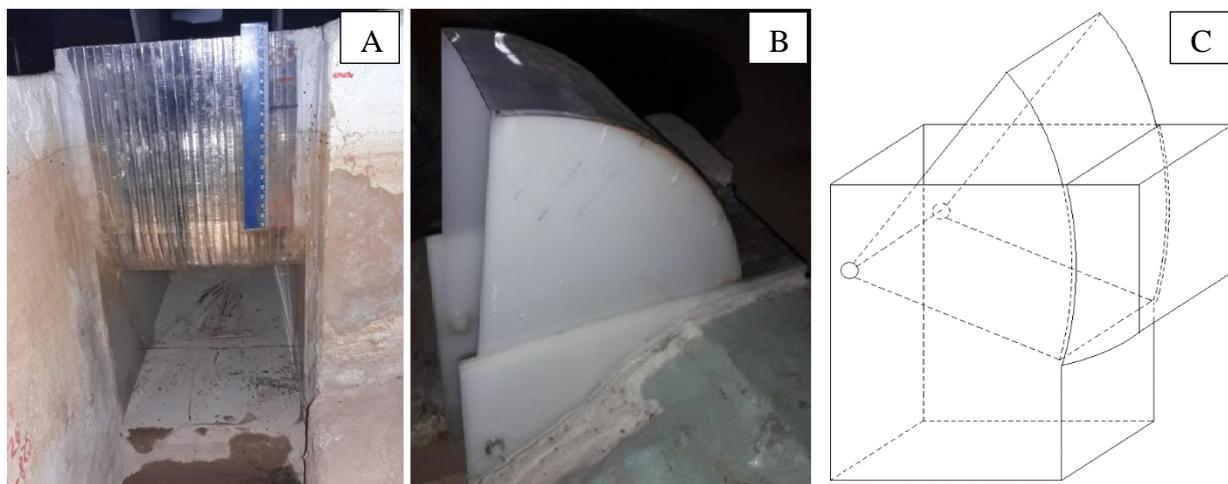


Figure 6. (A) Upstream view of the floodgate, (B) side view of the floodgate and (C) schematic representation of the floodgate.

2.3 Similarity theory

To ensure the similarity between flows of the model and the prototype, some conditions have to be satisfied, whether they are conditions of geometric, kinematic or dynamic similarity (Fox, 2011).

Geometric similarity requires that both systems have the same shape and that every linear dimension of the model is related to the corresponding dimension of the prototype by a constant scale factor.

Kinematic similarity, however, imposes that the velocities in corresponding points must have the same direction and sense, being different only by a constant scale factor. Kinematic similarity demands that the flow regimes must be the same for the model and the prototype. The current lines of two kinematic similar flows are also related by a constant scale factor. Thus, since current lines are formed by solid boundaries, kinematic similar flows must also be geometrically similar.

Dynamic theory is reached when two flows have such distributions of force that their identical types are parallel, relating in magnitude by a constant scale factor in all of the corresponding points. Once the dynamic similarity is achieved, data measured from a flow in the physical model can be related to the conditions of the prototype.

Kinematic similarity requires geometric similarity, and, on the other hand, kinematic similarity is a necessary requirement, though not sufficient to ensure dynamic similarity (Potter, 2004).

2.3.1 Dimensionless groups

Phenomena analysis involving fluid flows, where heat transfer can be neglected, must take into consideration variables such as pressure (p), characteristic length (L) or section diameter (D), viscosity (μ), surface tension (σ), gravitational acceleration (g), specific mass (ρ) and velocity (V).

By using these variables, it is possible to form independent dimensionless groups such as the Reynolds (Re), Froude (Fr), and Weber (We) numbers. The relation between these variables and the dimensionless groups, as well as the relation between the forces they represent, is expressed in Table 1.

Parameter	Definition	Qualitative relation of effects
Reynolds Number	$Re = \frac{D \cdot V \cdot \rho}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$
Froude Number	$Fr = \frac{V}{\sqrt{L \cdot g}}$	$\frac{\text{Inertia}}{\text{Gravity}}$
Weber Number	$We = \frac{\rho \cdot V^2 \cdot D}{\sigma}$	$\frac{\text{Inertia}}{\text{Surface tension}}$

A complete similarity between two flows (prototype and physical model) is achieved when there is an equality in every dimensionless group involved. Nevertheless, it stands out the fact that this situation becomes, if not physically impossible, financially impracticable, once complete similarity can not be reached in scales different than the unitary for the same fluid (Fox, 2011).

2.3.2 Dynamic similarity in open-channel flows

In open-channel flows, the force of gravity and obviously the inertial forces are the most relevant ones. On these cases, the Froude number becomes the main dimensionless parameter in order to achieve dynamic similarity between the model and the prototype. Reynolds similarity is then put on the back burner, with the sole intention of maintaining the flow regime, laminar or turbulent, between the model and the prototype.

Therefore, the goal is that the Froude number of the prototype must be the same as the Froude number of the model (Baptista, 2003), hence the equality given by Eq. (13).

$$Fr_p = Fr_m \quad \text{or} \quad \frac{V_p}{\sqrt{L_p \cdot g}} = \frac{V_m}{\sqrt{L_m \cdot g}} \quad (13)$$

Where the subscribed “p” refers to the prototype and “m” refers to the model. By this equality, geometric, velocity, time, acceleration, flow rate and slope scales can be established according to Table 2, where λ is the geometric scale relation, which is 1/40 in the analyzed SHPP.

In order to determine the friction factor f and the Reynolds number, Eq. (14) to (17) are used.

$$\frac{1}{\sqrt{f}} = -2 \log \frac{k}{12R_h} \quad (14)$$

$$Re = \frac{VL}{\nu} \quad (15)$$

$$R_h = \frac{A_{wet}}{P_{wet}} \quad (16)$$

$$L = 4R_h \quad (17)$$

Where ν is the kinematic viscosity of water, k is the roughness coefficient (in meters), R_h is the hydraulic radius (in meters), A_{wet} is the wetted area and P_{wet} is the wetted perimeter.

Table 2. Obtained scales through Froude similarity.

Parameter	Scale
Geometric	$\frac{L_m}{L_p} = \lambda$
Velocity	$\frac{V_m}{V_p} = \lambda^{\frac{1}{2}}$
Time	$\frac{t_m}{t_p} = \lambda^{\frac{1}{2}}$
Acceleration	$\frac{a_m}{a_p} = 1$
Flow rate	$\frac{Q_m}{Q_p} = \lambda^{\frac{5}{2}}$
Slope	$\frac{I_m}{I_p} = 1$

The roughness coefficient in function of the Manning coefficient n is given by Eq. (18). This equation is valid for a fully rough turbulent flow regime, with Reynolds number greater than 10^6 .

$$k = (25,6 \cdot n)^6 \quad (18)$$

3. RESULTS

The results gathered from the application of the models to determine the discharge coefficient as well as the values of the variables involved in the physical problem, together with the surface roughness scale are showed below.

3.1 Discharge coefficient

All data gathered experimentally in the physical model is displayed on Table 3.

Table 3. Experimental data.

Measurement	Q [m ³ /h]	Q [m ³ /s]	Floodgate opening [m]	y ₁ [m]	y ₂ [m]	y ₁ / y ₂
1	0	0	0	0.309	0	-
2	70	0.01944	0.050	0.309	0.050	6.180
3	100	0.02778	0.081	0.309	0.081	3.815
4	150	0.04167	0.116	0.309	0.116	2.664

It stands out that on the first measurement the floodgate was completely closed and, consequently, the flow rate was zero, restating that the water head was kept constant, that is, $y_1 = 0.309\text{ m}$. The height of the layer downstream was made the same as the floodgate opening. It is noted that the measurements 1 to 4 were obtained with a sequential opening of the floodgate.

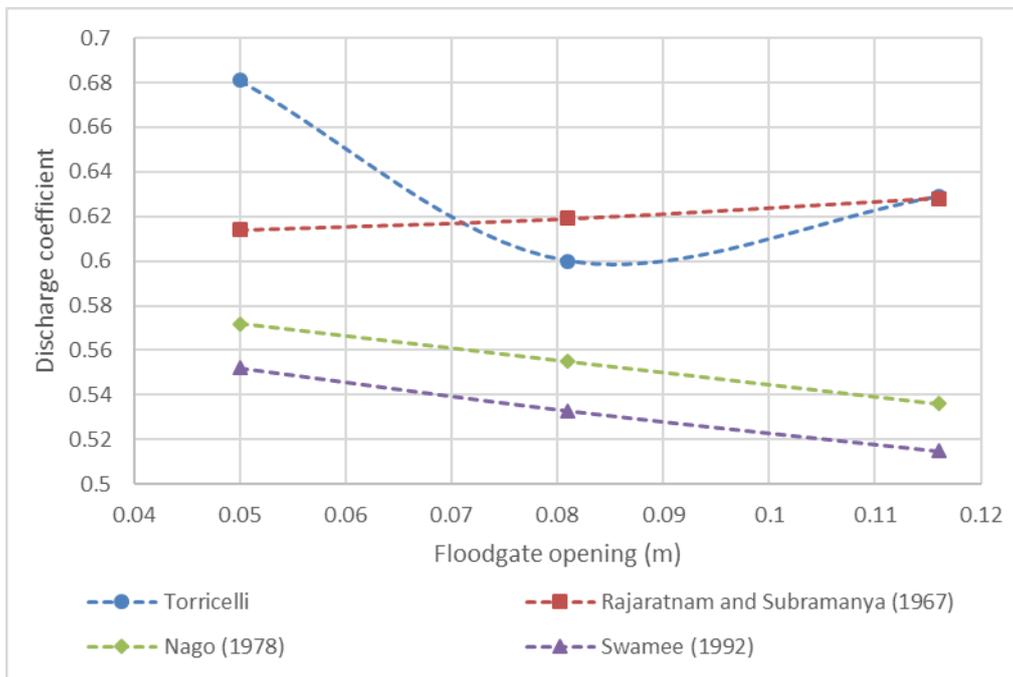
Table 4 displays the results for the calculations of the discharge coefficient utilizing all four equations from the aforementioned literature. Graph 1 shows the curves gotten from the equations.

Rajaratnam and Subramanya's equation (1967) was the closest one to the results achieved experimentally via the formula of the orifices law (Torricelli), and the one that showed the largest discrepancy regarding the results was Swamee's equation (1992). This behavior detected in the comparison of the results gathered by Swamee's equation (1992) had already been observed before by Sepulveda et al. (2009), Shayan et al. (2014) and Bagheri et al. (2010). The explanation for this phenomenon is based on the difference between the original data gathered by Henry (1950) and the approximation adopted by Swamee (1992), that according to Sepulveda et al. (2009), gets good results for values of $y_1/y_2 > 5$, even affirming that most of the errors were consequences of the adjustment made.

Table 4. Results for the discharge coefficient.

Measurement	Torricelli	Rajaratnam and Subramanya (1967)	Nago (1978)	Swamee (1992)
1	-	0.611	0.600	0.611
2	0.681	0.614	0.572	0.552
3	0.600	0.619	0.555	0.533
4	0.629	0.628	0.536	0.515
Average	0.637	0.618	0.566	0.553

Regarding the experimental methods used by Rajaratnam and Subramanya (1967) and Nago (1978), both showed good results due to their closer approximation to the classical method. What the results show is that the Rajaratnam and Subramanya's curve (1967) is better approximated by a line from the experimentally obtained results.



Graph 1. Curves gotten from the equations.

3.2 Determination of the problem variables and the surface roughness scale

From the geometric relation and the measurements taken on the physical model for each of the four positions of the floodgate, the areas referring to each material on which the water flowed was obtained. During the water flow through the floodgate on physical model, the fluid gets in touch with both side walls, the base, the fixed bulkhead (that has the function of facilitating the fluid's abduction) and the mobile bulkhead (regulator of the volumetric flow rate), all made of acrylic. However, on the prototype, the walls, the fixed bulkhead and the base are made of concrete and the mobile bulkhead is made of steel. Thus, consulting Porto (2006) and Tavares et al. (2015), values for the Manning coefficient of

the above-mentioned materials were adopted. Weighted arithmetic mean was applied to the areas that form the prototype's floodgate in order to get a Manning coefficient with bigger assertiveness.

Table 5 displays the parameters involved in the problem that were determined for the model and for the prototype. The high values for Re confirmed that it is a fully rough turbulent flow.

Table 6 shows the parameters related to losses in the fluid flow and the scales obtained between model and prototype.

Table 5. Values of the problem parameters for the model and the prototype.

Position	Floodgate opening (m)		Flow area (m ²)		Flow rate (m ³ /s)		Hydraulic radius (m)		Reynolds	
	Model	Prototype	Model	Prototype	Model	Prototype	Model	Prototype	Model	Prototype
1	0.0	0.0	0.0	0.0	0.0	0	-	-	-	-
2	0.050	2.0	0.012	18.7	0.0194	196	0.0350	1.40	2.3E+05	5.9E+07
3	0.081	3.2	0.019	30.3	0.0278	281	0.0479	1.91	2.8E+05	7.1E+07
4	0.116	4.6	0.027	43.4	0.0417	422	0.0582	2.33	3.6E+05	9.1E+07

Table 6. Surface roughness scales.

Position	Manning Coefficient			Roughness Coefficient (m)			Friction Factor		
	Model	Prototype	Proportion	Model	Prototype	Proportion	Model	Prototype	Proportion
1	-	-	-	-	-	-	-	-	-
2	9.0E-03	1.4E-02	0.66	1.5E-04	1.7E-03	0.09	0.021	0.016	1.34
3	9.0E-03	1.4E-02	0.66	1.5E-04	1.8E-03	0.08	0.019	0.015	1.31
4	9.0E-03	1.4E-02	0.65	1.5E-04	1.9E-03	0.08	0.019	0.014	1.29
Average	9.0E-03	1.4E-02	0.66	1.5E-04	1.8E-03	0.08	0.020	0.015	1.31

From the Manning coefficient results, it is concluded that the average proportion was 0.66, that is, a scale of 2:3 from the model to the prototype, differing from the geometric scale of 1:40. However, for the roughness coefficient, which represents the mean height of the wave crest of the fluid, this proportion was 0.08, meaning a scale of 2:25. Lastly, the proportion of the friction factor was 1.31, that is a scale of 13:10.

4. CONCLUSIONS

The study of an open-channel flow with a floodgate is important because it provides a tool to predict the behavior and sizing of dams. In this paper, the results achieved from classical techniques and empirical methods were compared, using the same test data. The results demonstrated good consonance with the methods estimated for the discharge coefficient. The expected effect of the height variation of the floodgate on the discharge coefficient was efficiently showed in the experiment.

It is noted that for each value of the floodgate opening, there is a coherent value for the discharge coefficient, that varies inversely proportional to the increase of the floodgate opening, that is, the bigger the floodgate opening, the lesser is the loss due to friction, which was already expected. In addition, there is a maximum value for the discharge coefficient to a certain value of the floodgate opening.

Rajaratnam and Subramanya's equation (1967) was the one that showed better approximation to the equation known as the orifices law, which is obtained utilizing the energy, mass and momentum conservation equations. The rest of the equations, based on empirical formulations, presented acceptable results in comparison with the same law.

Factors that could have contributed for maximization of errors are related to the precision of the readings in the reservoir's ruler and the fact that the main references adopt the rectangular profile of sluice gates in order to obtain their discharge coefficients.

Finally, surface roughness scales between the model and the prototype were achieved. For the Manning coefficient the scale was 2:3, for the roughness coefficient the scale was 2:25 and for the friction factor the scale was 13:10.

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