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Analysis of Parametrization Methods for Aerodynamic Profiles

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Abstract. *There is an incessant search for more efficient aerodynamic profiles in the literature. For this reason, some researchers have implemented optimization methods to find the ideal shape of airfoils. Many of these methods are based on known airfoil groups (NACA, Joukovsky and GOE), witch are defined by mathematical expressions. Also, they are used to reduce the optimization time of the parameterization, whose purpose is to obtain the aerodynamic profiles function with a minimum number of points. With fewer characteristic points of a profile, it is possible to reduce the number of parameters analyzed during optimization. Given the importance of parameterization applications, it is necessary to guarantee a good approximation of the generated profile function with the real one. In this paper, an analysis was made between two methods of parameterization (Bezier and PARSEC), to find which one represents the original airfoil most faithfully. A study was made on the number of ideal control points for a good representation of the aerodynamic profile, seeking a smaller geometric error between the real model and the one parameterized. Besides that, lift and pressure coefficients were compared through simulations in XFOIL. Finally, the two parameterizations presented reliable results. However, a better approximation was achieved with the Bezier method, which required fewer control points. Thus, the Bezier model could be considered the most suitable to be applied in these optimization processes.*

Keywords: *parameterization of airfoil, Bezier curve, PARSEC, NACA*

1. INTRODUCTION

The increasing global demand for renewable sources energy has been increasingly promoting the technological development of energy generators in order to reduce their manufacturing and operating costs. However, some of these sources are still not very well explored despite their great potential, such as ocean currents (Mitigation, 2011). The significant increase in the energy efficiency of these devices in recent years is drawing the attention of researchers and entrepreneurs and may become a viable alternative in the near future (Rostami and Armandei, 2017).

Most oscillating hydrofoil designed to extract energy from ocean currents found in literature use wing profiles of the NACA type (Campos *et al.*, 2019), to produce a combination of lift and drag force aiming at promoting a rotary motion for electricity generation. Herefore, it is essential to test several NACA geometries to verify which one yields the most efficient design.

Described in Abbott and Von Doenhoff (2012) book, the desired NACA profile can be achieved through a characteristic function that requires approximately one hundred points. However, the use of parameterization methods can reduce the number of points necessary to construct a similar geometry, with minimum relative error, in more than 90%.

The parameterization is a process that describes different geometries numerically controlled (Zhang *et al.*, 2018) and can be divided into constructive methods (e.g., spline) and deformative methods (e.g., free-form deformation) (Masters *et al.*, 2015). Every parameterization technique must obey three guidelines: minimize the control points, work with different geometries, and the parameters must be easy to formulate (Salunke *et al.*, 2014).

It is very important to compare the parameterized profiles with the original ones, which makes it possible to predict losses in the process. To ensure an even more accurate representation of the geometry generated by these methods, the analysis, and comparison of aerodynamic data are of utmost importance. This task was accomplished through the implementation of the open-source software XFOIL (Drela and Youngren, 2001), based on the panel method.

2. PARAMETERIZATION METHODS

There are several methods of parameterization, such as: PARSEC Method, Bezier parameterization, Sobieczky method, Modified Sobieczky method, Bezier-PARSEC Parameterization (BP 3333 parameterization and BP 3434 parameterization), Hicks-Henne method, and singular value decomposition method. All theses techniques can be divided in: constructive and deformative.

Constructive parameterization techniques are based on functions and equations that define the design of an airfoil. On the other hand, the deformative techniques adopts a known profile geometry and fits the new airfoil model (Masters *et al.*, 2015). In the article, two methods studied in the literature (Salunke *et al.*, 2014; Zhang *et al.*, 2018) are chosen: PARSEC (contructive) and Bezier (deformative). According to Salunke *et al.* (2014), PARSEC is an effective method for the parametrization of airfoils, and Bezier is the most popular technique in literature.

2.1 NACA 4-digit series

One of the primary airfoil families are NACA 4-Digit Series, were generated using analytical equations that describe the camber of the mean-line of the airfoil section as well as the section's thickness distribution along the length of the airfoil, Abbott and Von Doenhoff (2012).

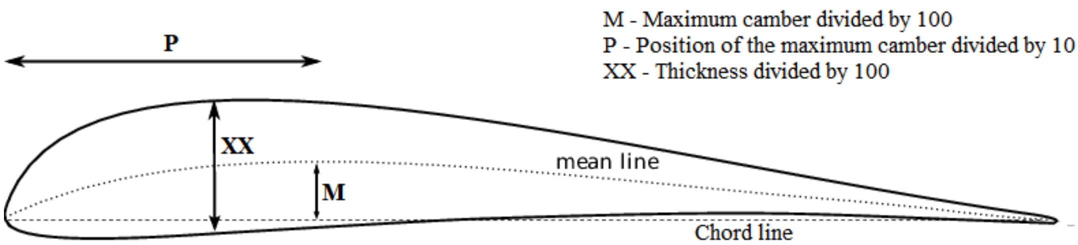

Figure 1: Representation of a NACA 4-Digit Series - NACA MPXX, adapted from Ayton (2016).

In Figure 1,the geometric parameters of a 4-Digit Series NACA MPXX airfoil type are illustrated. The value of camber (M), position of camber (P), and profile thickness are all defined as a percentage of the chord, which facilitates the profile design. In this way, it is possible to generate any 4-digit NACA airfoil, using the following geometric references,

Camber Line:

$$y_c = \frac{M}{P^2}(2Px - x^2) \qquad (0 \leq x < P) \tag{1}$$

$$y_c = \frac{M}{1-P^2}(1 - 2P + 2Px - x^2) \qquad (P \leq x \leq 1) \tag{2}$$

Thickness:

$$y_t = \frac{T}{0.2}(a_0 x^{0.5} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4) \tag{3}$$

$$a_0 = 0.2969 \quad a_1 = -0.126 \quad a_2 = -0.3516 \quad a_3 = 0.2843 \quad a_4 = -0.1015$$

Upper Surface:

$$x_u = x_c - y_t \sin\theta \tag{4}$$
$$y_u = y_c + y_t \cos\theta \tag{5}$$

Lower Surface:

$$x_l = x_c + y_t \sin\theta \tag{6}$$
$$y_l = y_c - y_t \cos\theta \tag{7}$$

where, $\theta = \arctan\left(\frac{\mathrm{d}y_c}{\mathrm{d}x}\right)$.

Another important feature when generating an airfoil is the abscissa distribution of point, which can provide a better Leading and Trailing edges resolution, and superior results. It is an important aspect to be evaluated and can provide different results depending on the method chosen. Therefore, in this paper, two different spacing methods are proposed: linear and cosine, which are presented respectively in Eq. 8 and Eq. 9.

$$x = t \qquad (0 \leq t \leq 1), \tag{8}$$

$$x = \frac{1}{2}(1 - \cos\beta) \qquad (0 \leq \beta \leq \pi). \tag{9}$$

Figure 2 presents the point distribution throughout an airfoil for both spacing methods adopted. As can be seen, the cosine method provides a finer point distribution over the leading and trailing edges of the airfoil, while the linear method provides a constant point distribution.

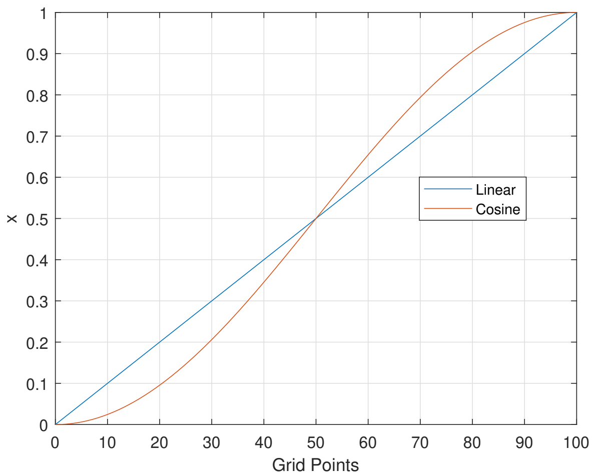

Figure 2: Comparison between cosine and linear spacing

2.2 Bezier curve

Bezier curves are a special subset of B-Spline and provides a easy and precise way to produce an airfoil surface with a few control points. The possibility of generating complex curves with a reduced number of points is of great importance when considering optimization methods, mainly in aerodynamic profiles. A Bezier curve can be mathematically defined (Mohebbi, 2014) by,

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t), \tag{10}$$

where,

$$J_{n,i}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-1}. \tag{11}$$

To construct the airfoil surface two Bezier curves are needed: one for the upper surface and another one for the lower surface. The equations that describe NACA 4-Digit allow the airfoil coordinates to be obtained,

$$[P(t)] = [J(t)]\,[B]\,. \tag{12}$$

If the number of the chosen points on the airfoil surface is m and the degree of Bezier curve is n, then Eq. 12 can be re-write as,

$$\begin{aligned} [P(t)]_{n\text{x}2} &= [J(t)]_{m\text{x}(n+1)}\,[B]_{(n+1)\text{x}2} \\ [B]_{(n+1)\text{x}2} &= [J(t)]^{-1}_{m\text{x}(n+1)}\,[P(t)]_{m\text{x}2} \\ [J(t)]^T_{(n+1)\text{x}m}\,[P(t)]_{m\text{x}2} &= [J(t)]^T_{(n+1)\text{x}m}\,[J(t)]_{m\text{x}(n+1)}\,[B]_{(n+1)\text{x}2} \end{aligned} \tag{13}$$

Finally, Eq. 14 can be rearranged into,

$$[B]_{(n+1)\text{x}2} = \left[[J(t)]^T_{(n+1)\text{x}m}\,[J(t)]_{m\text{x}(n+1)}\right]^{-1} [J(t)]^T_{(n+1)\text{x}m}\,[P(t)]_{m\text{x}2} \tag{14}$$

Thus, Equation 14 provides us the control points of the required surface, enabling the use of Eq. 10 to generate the airfoil geometry (Mohebbi, 2014).

2.3 Parametric Section method (PARSEC)

Parameterization with PARSEC attempts to recreate any type of airfoil geometry, avoiding big losses during the process. The method was developed by Sobieczky (1999), requiring eleven geometric parameters for its correct use. These parameters are shown in Fig. 3.

Using geometry parameters could be a safe way to maintain aerodynamics characteristics such as lift and pressure coefficients. However, it is not a suitable method when trying to, successfully, optimize a process, as the Bezier Curve does, due to the requirement of a large number of parameters to generate the airfoil.

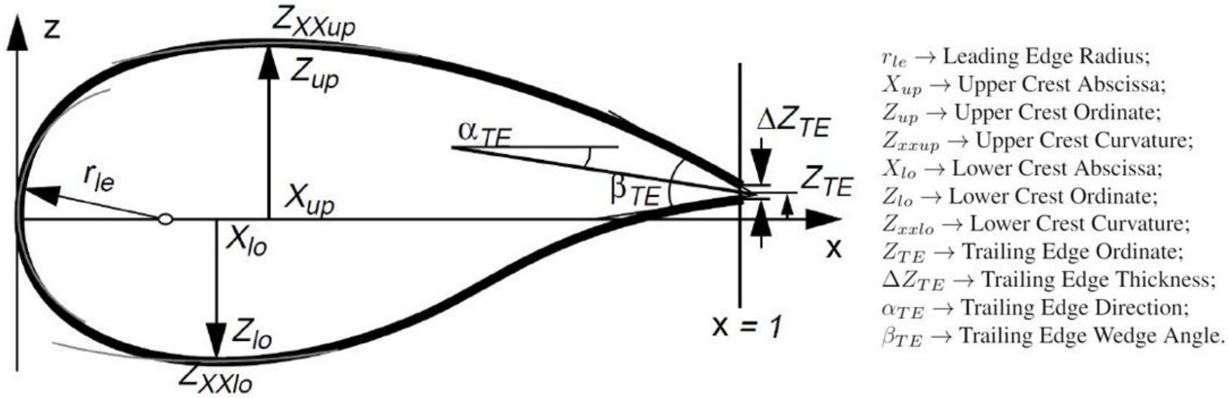

Figure 3: PARSEC parameters for an airfoil representation, adapted from Sobieczky (1999).

The airfoil generated by PARSEC parameterization, are defined by two polynomial equations as,

$$y_e\left(x_e\right) = \sum_{k=1}^{6} \alpha_{ek} \mathbf{x}_e^{k-\frac{1}{2}} \tag{15}$$

$$y_i\left(x_i\right) = \sum_{k=1}^{6} \alpha_{ik} \mathbf{x}_i^{k-\frac{1}{2}} \tag{16}$$

The constants α_{ek} and α_{ik} can be found analytically using the original geometric parameters from the airfoil and the limits known for the trailing and leading edges ($\alpha_{i1} = -\alpha_{e1}$ for symmetrical airfoils).

3. METHODOLOGY

In this article, PARSEC and Bezier methods were implemented in the MATLAB software together with the XFOIL code. XFOIL was of great importance to evaluate the most effective method since it allowed obtaining the characteristic curves from the profiles (i.e., pressure and lift coefficient). Drela (1989) developed XFOIL using panel methods, linear vorticity, and boundary layer functions taking into account friction due to drag and flow transition. This formulation allows studies with compressible, incompressible, viscous, and non-viscous fluids. At last, after establishing some fluid parameters an algorithms based on parameterization methods were developed and tested.

3.1 Fluid parameters

For the simulations, a viscous, rotational, and incompressible fluid was considered passing around an airfoil. The wing profile was chosen based on previous studies of oscillating hydrofoils (Kinsey *et al.*, 2011; Campos *et al.*, 2019), which is presented in Tab. 1, together with the Reynolds number, the Mach number and the fluid characteristics adopted.

Table 1: Fluid and foil characteristics

Parameter	**Reynolds**	**Mach**	**Fluid**	**Kinematic viscosity**	**Profile**
Value	$4.9 \cdot 10^5$	0.006	Water	$4.2 \times 10^{-6}\ m^2/s$	NACA0015

3.2 Bezier algorithm

The Bezier algorithm is based on equations in Section 2.2 With the standard profile determined in a Cartesian points, through the NACA 4-Digit code, it is possible to estimate the position of the control points. After, the number of control points is chosen and the calculations of the Bezier function are performed. In Fig. 4, the program flowchart is shown.

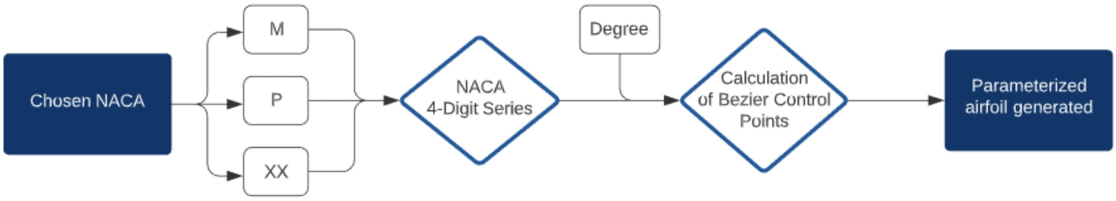

Figure 4: Bezier flowchart

3.3 PARSEC algorithm

The PARSEC algorithm uses the equations defined in Section 2.3 As in the case of Bezier, Cartesian points are also used to direct the method. From this geometry, the eleven variables that govern the PARSEC equations can be surveyed. The scheme with each phase of the algorithm are presented in Fig. 5.

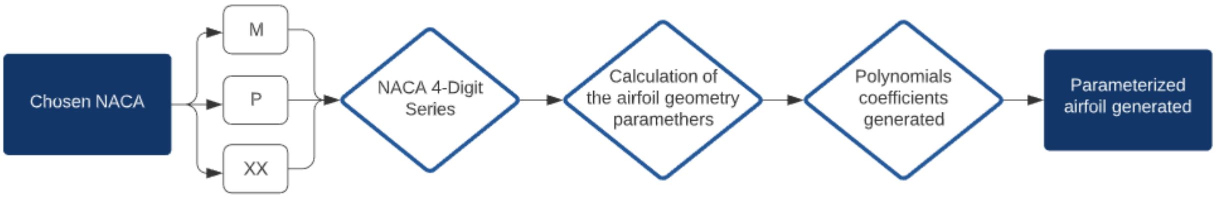

Figure 5: PARSEC flowchart

3.4 XFOIL parameters

The XFOIL© code was implemented following the steps presented by Mauclère (2009), for the purpose of evaluate the graphs of the lift and pressure coefficients, in other words, evaluating aerodynamic similarities, Figure 6.

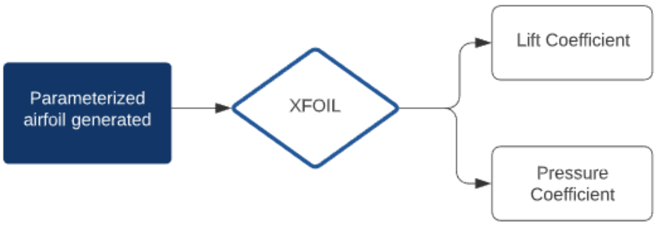

Figure 6: XFOIL flowchart

4. RESULTS AND DISCUSSION

4.1 Model validation

In order to verify the accuracy of the parameterization methods implemented in this article, the data obtained by Derksen and Rogalsky (2009) was used to validate the algorithms. Derksen and Rogalsky (2009) obtained in its simulations with the interactive Bezier curve, an average error equal to 1.06×10^{-4}, for NACA0008-34.

In Figure 7 the number of control points for the average error for the Bezier algorithm are presented, with the code developed in this article. Thus, it is concluded that from 15 control points the Bezier algorithm obtain the value found by Derksen and Rogalsky (2009). On the other hand, the algorithm based on PARSEC reached a lower error value (9.8910×10^{-5}) than found by Derksen and Rogalsky (2009), showing its effectiveness. Thus, validating both proposed algorithms.

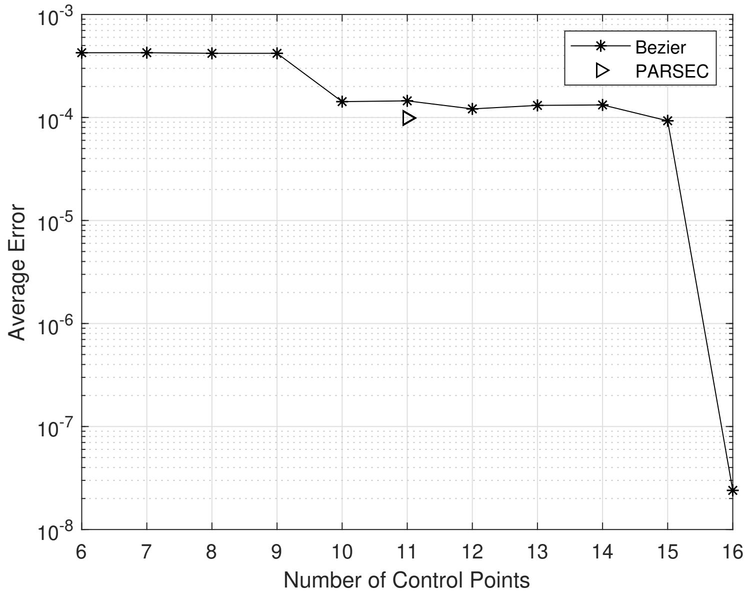

Figure 7: Bezier average error.

4.2 Analysis of the parameterization for NACA0015

Some simulations were carried out to verify which method has the least geometrical and hydrodynamic error. The comparisons were made between the original NACA0015 profile and the Bezier and PARSEC parameterization methods. The average errors obtained for each method are presented in Tab. 2, where the PARSEC method is compared with the Bezier method for six different numbers (6 to 11) of control points (CP). Also, two spacing curves, cosine and linear, were assessed for both methods.

Table 2: Comparison between Bezier and PARSEC methods.

	$\bar{\delta}_G$		$\bar{\delta}_{C_l}$		$\bar{\delta}_{C_p}$	
	Linear	**Cosine**	**Linear**	**Cosine**	**Linear**	**Cosine**
Bezier 6 CP	2.9226×10^{-3}	1.4302×10^{-4}	0.0237	0.0048	0.0583	0.0057
Bezier 7 CP	1.9984×10^{-3}	1.2626×10^{-4}	0.0122	0.0051	0.0479	0.0043
Bezier 8 CP	1.4095×10^{-3}	5.6360×10^{-5}	0.1512	0.0014	0.0424	0.0039
Bezier 9 CP	1.0146×10^{-3}	1.1328×10^{-5}	0.0113	0.0009	0.0373	0.0005
Bezier 10 CP	7.4313×10^{-4}	8.1430×10^{-6}	0.0263	0.0007	0.0358	0.0008
Bezier 11 CP	5.5225×10^{-4}	7.8232×10^{-7}	0.0198	0.0001	0.0324	0.0002
PARSEC	3.5854×10^{-5}	6.2496×10^{-5}	0.0196	0.0036	0.0017	0.0024

As expected, it is observed that the greater the control points number for the Bezier method, the lower the absolute error obtained. Also, the cosine spacing curve proved to be the most efficient, with the lower absolute errors in any of the cases where the Bezier method was used. However, when comparing with the PARSEC method, it is concluded that with 8 CP, both methods generate similar geometric and aerodynamic errors. Therefore, if the control points number used in the Bezier method is equivalent to the number of parameters in the PARSEC method (i.e., 11), the Bezier method will present higher accuracy.

Figure 8 shows the location of each control point used to determine the Bezier 8 CP geometry with a cosine curve and the original NACA0015 profile. Because it is not possible to visually distinguish the original geometry from the geometry obtained using the Bezier method, Fig. 9 presents their geometric differences for the upper and lower surfaces.differences for the upper and lower surfaces.

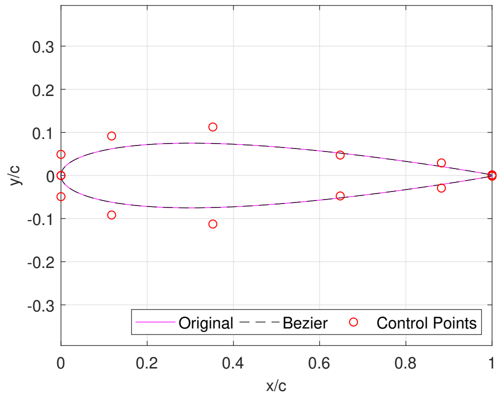

Figure 8: Airfoil NACA 4-digit and Bezier comparison (cosine spacing).

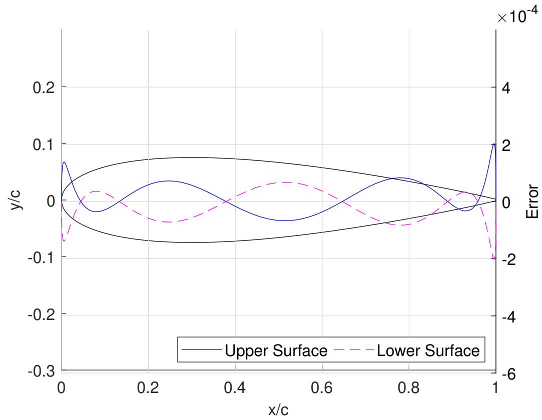

Figure 9: Error distribution comparison for Bezier (cosine spacing).

The Bezier curves yield in an average absolute error of 5.6360×10^{-5} whose higher values are located in the leading and trailing edge, as illustrated in Fig. 9.

The aerodynamic coefficients comparison is presented in Fig. 10 and Fig. 11, where one can notice an excellent agreement between both results. To guarantee the statement, the mean absolute error for the lift coefficient ($\bar{\delta}_{C_l}$) was 0.0014 and the pressure coefficient ($\bar{\delta}_{C_p}$) was 0.0039.

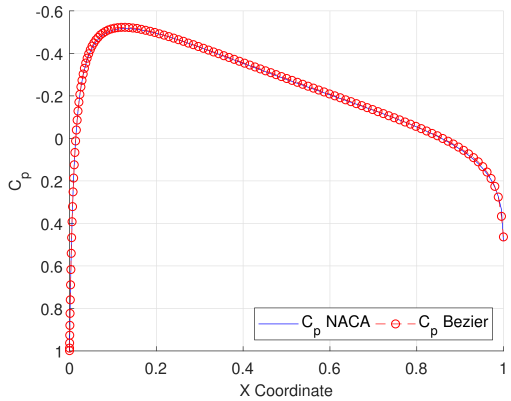

Figure 10: Pressure coefficient comparison for Bezier - Lower Surface.

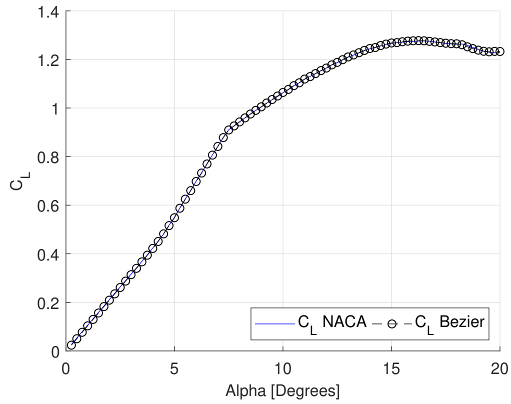

Figure 11: Lift coefficient comparison for Bezier.

Figure 12 shows the PARSEC geometry superimposed on the original geometry. One more time, because it is not possible to visually distinguish the original and the PARSEC geometry, Fig. 13 presents their geometric differences for the upper and lower surfaces. The average absolute error obtained was 6.2496×10^{-5}, whose higher values are located in the leading edge.

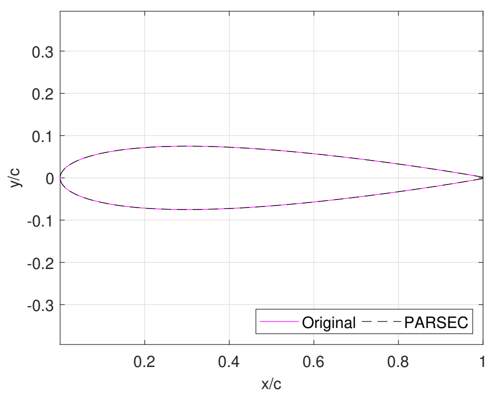

Figure 12: Airfoil NACA 4-digit and PARSEC comparison (cosine spacing).

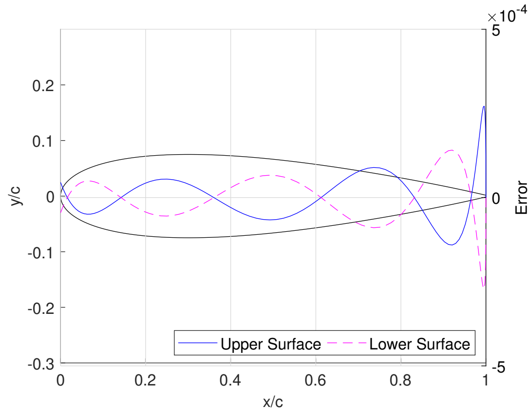

Figure 13: Error distribution comparison for PARSEC (cosine spacing).

Once again, the aerodynamic coefficients comparison is presented in Fig. 14 and Fig. 15, and, one more time, an excellent agreement between both results was achieved, with a mean absolute error for the lift coefficient ($\bar{\delta}_{C_l}$) and the pressure coefficient ($\bar{\delta}_{C_p}$) of 0.0036 and 0.0024, respectively.

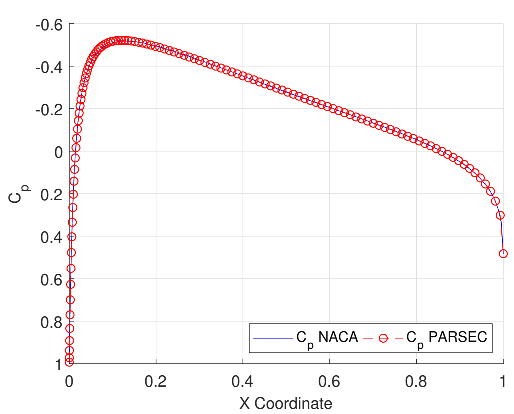

Figure 14: Pressure coefficient comparison for PARSEC - Lower Surface.

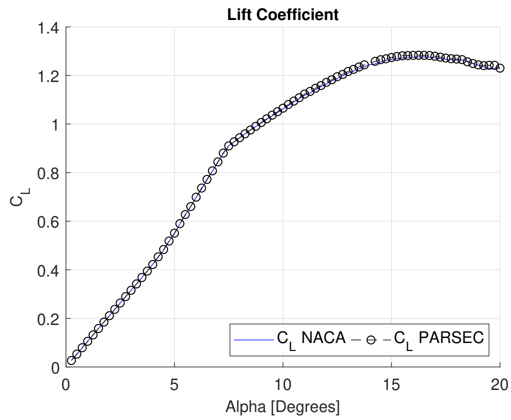

Figure 15: Lift coefficient comparison for PARSEC.

5. CONCLUSION

Parameterization methods are commonly used to simplify functions or curves. In this article, two methods were presented based on different solution strategies, with one being constructive (PARSEC) and the other deformative (Bezier). In both cases, geometric and aerodynamic errors were verified and compared against each other. The aerodynamic analysis was possible using the XFOIL code in conjunction with the parameterization algorithms. The algorithms were validated with literature data and then applied for a NACA0015 airfoil.

Several tests were performed to verify the performance of the parameterization algorithms. As a result, it was observed that the PARSEC method using a cosine curve had similar results with the Bezier 8 CP method, also using a cosine curve. However, for higher control points number, the Bezier method showed smaller mean absolute errors in comparison to the PARSEC method. Therefore, it is possible to state that the Bezier method is a better option for future optimization studies.

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