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**FREE-FALLING PARTICLE MASS FLOW EQUATIONS FROM SOLAR
TOWER RECEIVERS COMPARISON**

Felipe Venancio Mitkiewicz Silva
Cristiana Brasil Maia

Instituto Politécnico da PUC Minas, Pontifícia Universidade Católica de Minas Gerais.
Avenida Dom José Gaspar, 500 Coração Eucarístico 30535-901 - Belo Horizonte – Minas Gerais – Brasil
felipemit@hotmail.com, cristiana@pucminas.br

***Abstract.** Climate change has been shaping the world weather patterns also influencing the economic status quo to shift to a more conscious human society. Renewable energy applications have become a trend around the world and the search for a more efficient and feasible technology to harvest energy has increased. The concentrated solar power application has been pointed out by renowned laboratories as a great alternative to fossil fuel power generation, due to that, a heliothermic plant based on the solar tower with free-falling particle receiver has been under development in Sandia National Laboratories. The potential efficiency and feasibility have been showing very promising as the research evolves. As the research in this area increases, it is important to have powerful and trustful tools to develop the knowledge that englobes it. One of the most relevant steps of this kind of research is modeling the phenomena. Therefore, this paper aims to compare the classical modeling particle mass flow equations (Beverloo and British Code) with state-of-the-art equation (Beverloo-Ho) to provide feedback about how they behave in relation of one another. The results confirmed the literature stands that classical equations do not fit very well to predict the particle mass flow in small apertures.*

***Keywords:** CSP, solar tower receiver, free-falling particle, mass flow estimation.*

1. INTRODUCTION

Climate change already affects and will continue to be affecting billions of people around the world with changing weather patterns, such as more intense droughts and rains. An Intergovernmental Panel on Climate Change (IPCC) report pointed out the relationship between global warming and greenhouse gases, as a product of human action, and climate imbalance. This imbalance includes the present and the future and consists of increases or reductions in the frequency, also intensity, of climatic events (Allen et al., 2018). This change in climate patterns directly impacts society's current living standards. Economy strategic sectors such as agriculture, livestock, transport and energy, dependents on those patterns, may be even more exposed to stress conditions as the demand for resources continues to grow, which corresponding to the trend of the increasing world population (World Water Assessment Programme (United Nations), 2018). Consequently, it is important to attenuate those impacts with assertive actions. The reduction of greenhouse gases is considered a viable solution, in the short to medium term, to mitigate the effects of global warming (United Nations, 2015).

In order to reduce emissions, a gradual substitution of power sources based on fossil fuels with renewable sources is required. Thus, among all renewable energy sources, an alternative to be considered is solar energy, due to its great abundance and availability. Regarding solar power applications, photovoltaic technologies stand out as an outcome of the recent investments in this technology over the years. However, the efficiencies of the modules tend to drop over time, up to 40%, due to effects such as soiling (Costa, Diniz, & Kazmerski, 2016). As a result of that, recently, solar power thermal energy has been highlighted as an option, mainly the concentrated solar power (CSP). Conventional CSP commercial plants can already deliver around 100 MWe (Fernández et al., 2019), they have an intermediate medium for the heat transfer process between solar radiation and the working fluid (usually a molten salt). However, the thermoelectric conversion efficiency and the installation costs are still factors considered as obstacles when compared to other types of power plants.

Therefore, since mid-2010, some renowned laboratories around the world have been investing in the development of technology improvement, aiming to turn CSP into a viable and more efficient alternative. Among all the laboratories, Sandia National Laboratories (SNL - United States) became noteworthy, due to its advanced level on the research on this subject.

Since then, Sandia has been researching the increase of the CSP efficiency. They have found that the usage of a better energy conversion power cycle could solve the issue and since then, they've been investing in the development of a promising solution, which uses a Brayton cycle with supercritical carbon dioxide (sCO₂) as the working fluid to transform

thermal energy into electricity (Ho, Peacock, et al., 2018). However, the application required higher temperatures than molten salt conventional CSP-ST plants work.

Therefore, by replacing conventional solar tower systems through the development of a new heat transfer mechanism, Sandia overcame the old thermal limitations. Consequently, the CSP solar tower receivers have become able to run at higher temperatures. The tower working fluids were substituted from nitrate salt (above 600 °C it becomes chemically unstable) to ceramic particles. Moreover, they developed a heat transfer mechanism of a free-falling particle receiver that uses these sintered bauxite particles, then, the particles absorb the sun's radiation heat directly, without a solid medium. Consequently, they reduced the thermal stress impact, outcome of high temperatures combined with the solar radiation transience on the receiver (Ho, 2016).

In addition, the Brayton power cycle was implemented combined with supercritical carbon dioxide as the working fluid, due to its high conversion from thermal to electrical energy, which results in a more efficient power plant cycle. Therefore, along with the research, new equipment had to be designed to fit the system configuration, such as turbines and compressors, which were smaller than the vapor power cycle equipment due to the thermophysical fluid properties of the sCO₂ (Ho, Carlson, et al., 2018).

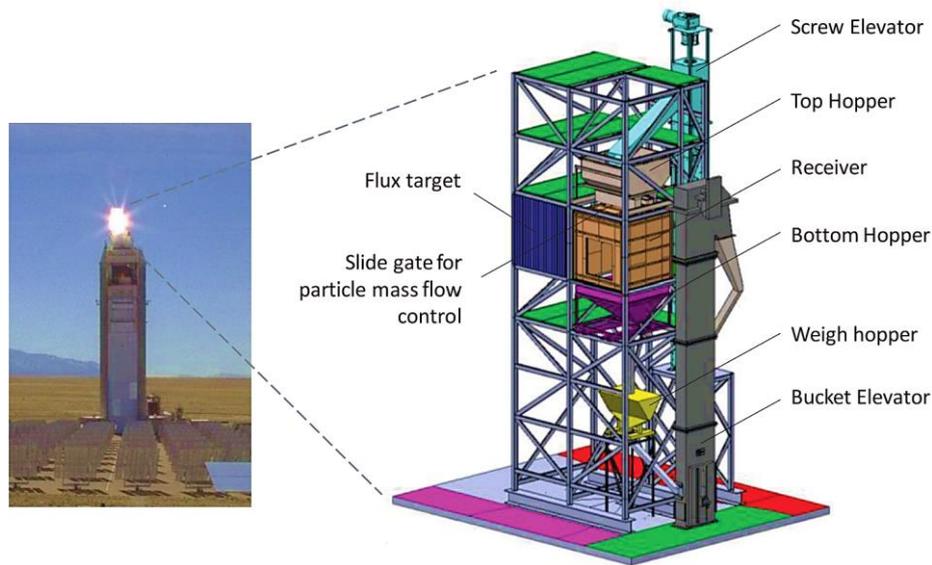


Figure 1 - On-sun testing of Sandia's 1 MWt Particle Test Loop on top of the 61 m (200 ft) solar tower with over 200 heliostats (each 37 m²) at the National Solar Thermal Test Facility (Ho et al., 2019).

Finally, as the sintered bauxite absorb the direct solar radiation, it is then transferred to the power cycle working fluid (sCO₂) through a heat exchanger. Again, as the other cycle elements, the heat exchanger also had to be designed and the state of the art is consisted by a shell and plate printed circuit heat exchanger (Albrecht & Ho, 2017)(Fernández-Torrijos, Albrecht, & Ho, 2018).

The technology development requires several steps before reaching a commercial scale, thus, it is common to develop mathematical models and smaller experimental models in order to study the phenomena and consequently deliver the project with efficiency and safety. One of the main phenomena of the CSP-ST with particle central receiver (PCR) is the heat transfer from the heliostats field to the particle on the receiver. This is a complex phenomenon that involves a mixture of particles and air flowing downward due to gravitational forces while it absorbs the thermal energy. The model has to account for not only the heat dynamic but also viscosity features. However, the classical equations that describe the particle mass flow rate englobe only the particle motion without considering the heat. Those equations were more focused on the geometry of the involved parts aiming to predict flow patterns such as funnel flow or mass flow. As the CSP-ST PCR demands a more precise equation, one that includes heat, recently some research has been showing some advances in describing the phenomenon, such as the one developed by Ho at Sandia National Laboratories. Thus, this paper aims to compare the classical equations with the most recent equation on the prediction of the mass flow rate of an CSP-ST PCR application.

2. PARTICLE MASS FLOW RATE MATHEMATICAL MODELLING

One of the CSP research projects' fundamental stages is prototyping. Through the phenomenon modeling and experiments, it is possible to study and make inferences in the system without the need to have it on a larger scale. Thus, all sorts of hypotheses can be tested with lower costs and with time reduction. The following section presents the modelling equations to estimate the particle mass flow on solar tower receiver applications based on free-falling particles.

2.1 Particle mass flow equations

Firstly, is important to mention that due to the large number of parameters influencing the particle mass flow rate, it is impossible to predict its behavior precisely using a global equation. There are several equations in the literature which have been developed empirically based on a specific application. Therefore, on a silo case, the outlet particle mass flow rate is majority based on the outlet size D and the particles diameter, d (Janda, Zuriguel, & Maza, 2012). A first order global approach would be the classical Beverloo's equation, as presented in Eq. (1) (Beverloo, Leniger, & van de Velde, 1961):

$$W = 35\rho_b\sqrt{g}(D_o - 1,4d)^{2,5} \quad (1)$$

Where, W is the particle mass flow rate [g/min], ρ_b is the particle bulk density above the aperture [g/cm³], g is the gravitational constant [cm/min²], D_o is the diameter of the circular orifice [cm], d is the average screen size of particles [cm].

Another reference is the equation provided by the British Materials Handling Board, as displayed on Eq. (2), for rectangular outlet openings ($L > 3D$) (Woodcock & Mason, 1988):

$$W = 1.03\rho_b\sqrt{g}(L - kd)(D - kd)^{1,5}K\theta \quad (2)$$

Where W is the mass flow rate [kg/s], L is the outlet aperture length [m], k is the factor dependent of the particle shape [$k = 1.6$ for spherical and $k = 2.5$ for non-spherical particles], D is the outlet width [m], θ is the hopper half angle [°] and $K\theta$ is factor that accounts the hopper inclination, 1 when $\theta \geq 45^\circ$, for lower angles, it can be calculated by Eq. (3):

$$K\theta = (\tan\theta)^{-0,35} \quad (3)$$

2.2 Particle mass flow equations for CSP application

The discrepancies between the particle mass flow phenomenon modeling from Beverloo's equation, Eq.(1), and the experiments of heated particles flowing through a silo, led the authors to develop their own equation considering the static and dynamic characteristics of the model (Ho et al., 2015). It displayed on Eq. (4) and was based on Beverloo *et al.* (1961) and Janda *et al.* (2012):

$$W = C_1\rho_b\sqrt{g}(D - C_2d)^{n+0,5} \quad (4)$$

Where W is the particle mass flow rate [kg/min/m for 2D (sheet flow)], C_1 is the dimensionless constant related to material properties [62], C_2 is the geometrical factor accounting for the effective outpouring section being smaller than the aperture [1.4] and n is 1 for 2D [= 2 for 3D in the original Beverloo equation].

Finally, due to the silo small aperture dimension and the differences between numerical and experimental values of heated free-falling ceramic particles, a more precise equation were developed to describe the phenomenon (Ho et al., 2019), as can be seen on Eq. (5).

$$W = [C_1\rho_b\sqrt{g}(D - C_2d)^{n+1/2} + C_3D^x](C_4D(T_{amb}/T))^y \quad (5)$$

Where, W is the particle mass flow rate [kg/min/m for 2D (sheet flow)], C_1 is the dimensionless constant related to material properties [38.8], g is the gravitational constant [9.81 m/s²], D is aperture size [m], C_2 is the geometrical factor accounting for the effective outpouring section being smaller than the aperture [8.9], n is 1 for 2D [= 2 for 3D in the original Beverloo equation], C_3 is the factor to account for flow around the edge of the slide gate [1.9e8], x is the exponent to account for exponential growth as the slide gate opens [3.4], C_4 is the factor to account for elevated temperatures and greater particle/wall friction [2.0], T_{amb} is the reference temperature [283 K], T is the particle temperature [K] and y is the exponent to account for elevated temperatures and greater particle/wall friction [0.4].

3. RESULTS

Indeed, there are some equations for modeling the particles mass flow rate, however, each one has its own peculiarities and limitations, as presented on the previous section. The application defines what equation best fit in each situation as they were developed empirically. For example, the Beverloo, Eq. (1), is considered the most generic equation and it

doesn't represent well the phenomenon in cases where the mass flow pass through small orifices and also when it includes thermal applications, because it does not take into account thermal energy and other features on its correlation, such as silo geometry.

The same analogy can be assumed for the British Code of Practice as presented in Eq. (2). The equation is generally used for rough order-of-magnitude assessment of the discharge rate of coarse, free falling materials, from circular and rectangular outlets. In addition of the content from Beverloo, it considers the silo geometry, a relevant feature that can express the type of flow inside the silo, such as "mass flow" or "funnel flow". The type of flow defines whether the particles accumulate on the silo sides or not. Moreover, the equation offers a refinement regarding the particle shape by defining the geometry of them, not only their dimension as in Beverloo. Nevertheless, thermal energy is not added on the model.

While most of the previous studies based on Beverloo have concentrated on agricultural processes Ho *et al.* (2015), Eq. (4) was developed as an attempt for the usage of alumina-silica ceramic particles, used on CSP experiments on SNL. It was done by determining the appropriate dimensionless constants C_1 and C_2 . The equation covered particles sizes from 280 to 697 micron. However, it does not account the thermal energy as well.

Thus, later on, the authors derived a modified Beverloo-Ho equation, Eq. (5), in a study for a particle mass-flow control system assembled and installed onto Sandia's 1 MWt Particle Test Loop (SPTL) on top of a 61 m tower with a 6 MWt north heliostat field.

The SPTL was developed to run the CSP-ST closed Brayton recompression cycle with sCO₂ experiments. As described by Ho *et al.* (2019) and displayed on Figure 1, the SPTL was consisted of a hopper that releases particles through a 2 x 2 x 2 m cubical cavity receiver with a north-facing aperture (1 m x 1 m) before being collected and recirculated back to the top using a screw elevator capable of transporting the particles up to 10 kg/s. The control system was a linear actuator able to define the hopper opening aperture and it was installed just beneath the hopper. Finally, the modified equation, differently from the others approach, considered the particles temperature, also its thermal effects regarding a greater particle/wall friction. Figure 2 presents the numerical values of the particle mass flow rate per aperture dimension obtained and validated by Ho *et al.* (2019).

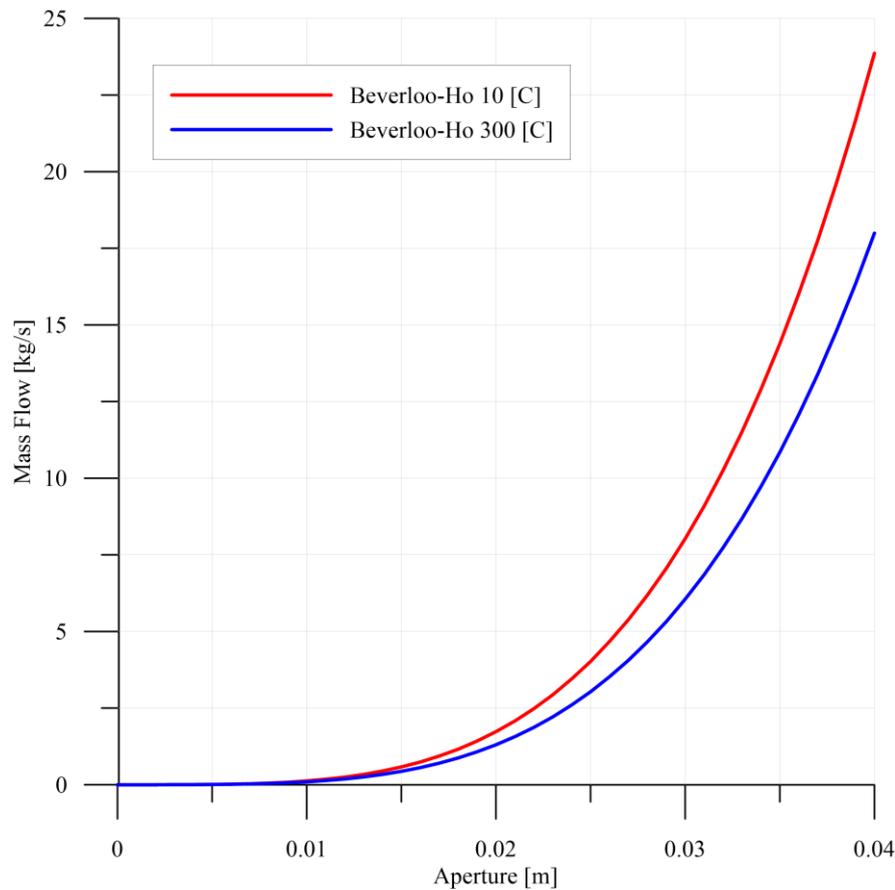


Figure 2 - Particle mass flow rate vs. aperture opening for different particle inlet temperatures. The lines represent the predictions using modified Beverloo-Ho equation (adapted from Ho *et al.*, 2019).

The experiment verified the mass flow behavior varying the aperture dimension and with two different particle temperatures, 10 °C and 300 °C. As can be seen from Figure 2, the higher the temperature, the larger the aperture has to be in order to achieve the same mass flow rate. It is also noted the parabolic mass flow curves pattern, beyond 3 cm, the particle mass flow escalates rapidly.

Therefore, as mentioned by the literature, the first order approach, as Beverloo and British, had to deviate from those more precise equations on small aperture dimensions. Figure 3 presents a comparison between the other equation's behavior with Beverloo-Ho.

As Figure 3 describes, from 0 to 0.06 m, the mass flow rate obtained by Beverloo, British Code also Ho *et al.* (2015) equations deviated from Beverloo-Ho and all have higher values for small apertures, while Beverloo-Ho has a more gradual pattern. Although Beverloo-Ho has an exponential pattern, close to 0.06 m, almost all curves intersect, but Ho *et al.* (2015). The same graph presents the Beverloo-Ho behavior through temperature and aperture variation. It was noted a right offset from the lowest temperature to the highest. The pattern suggests that, the higher the temperature, the larger the aperture must be for the same mass flow rate. It can be observed by the curves of Beverloo-ho from 10 °C to 775 °C. This last one was calculated in order to verify the curve pattern based on a CSP-ST sCO₂ application which is being developed by SNL. It is important to mention that the Beverloo-Ho's equation is also influenced by the ambient temperature. Thus, through Fig. 3 is possible to verify the correlation between the silo aperture, the particle mass flow rate and the temperatures involved on the phenomenon, as different outlet apertures with different temperatures will lead to different mass flow rate. For example, if the ambient temperature is set the same for all cases and the silo aperture is defined as 0.06 m for all cases as well, when the temperature varies from 10°C, 300°C and 775°C the mass flow will also vary, from 115 kg/s, 82 kg/s and 62 kg/s, respectively.

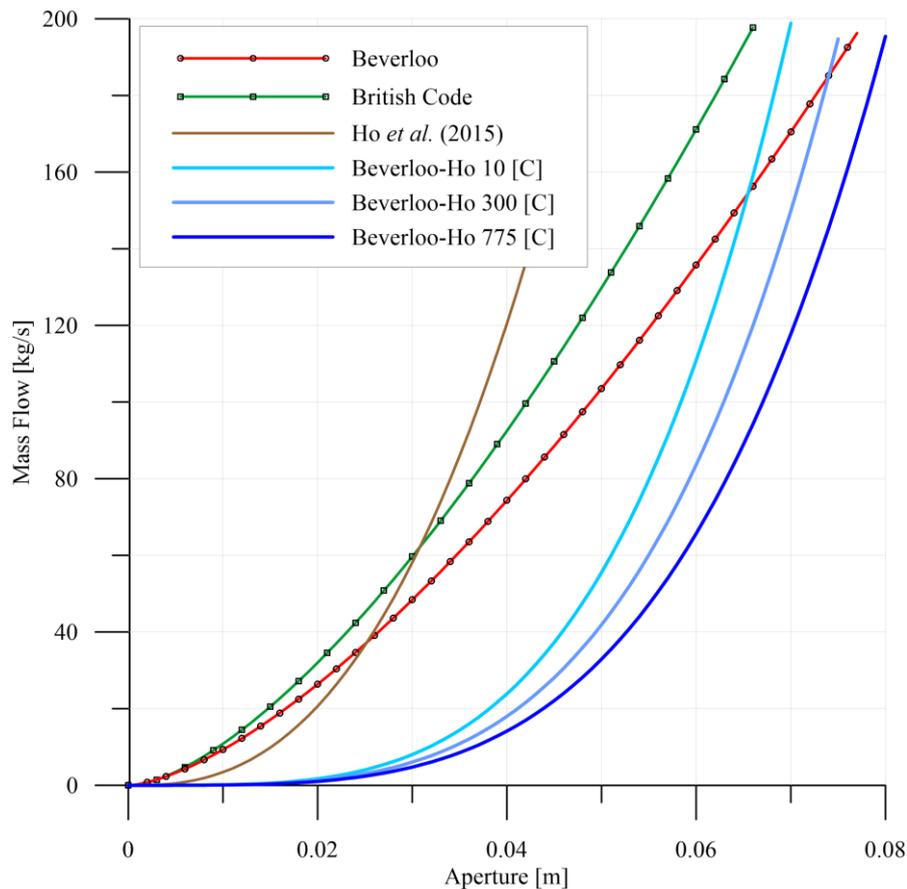


Figure 3 - Comparative of particle mass flow equations per aperture dimensions

However, as Beverloo and British Code curves does not account the thermal energy, they are not affected by the variation of it, only by the silo outlet aperture. Finally, another aspect to be highlighted is the proximity of both curves, Beverloo and British Code. Although the last one accounts features of the hopper geometry, they do not differ expressively from each other. However, the larger the aperture, the more the curves start to set apart from each other.

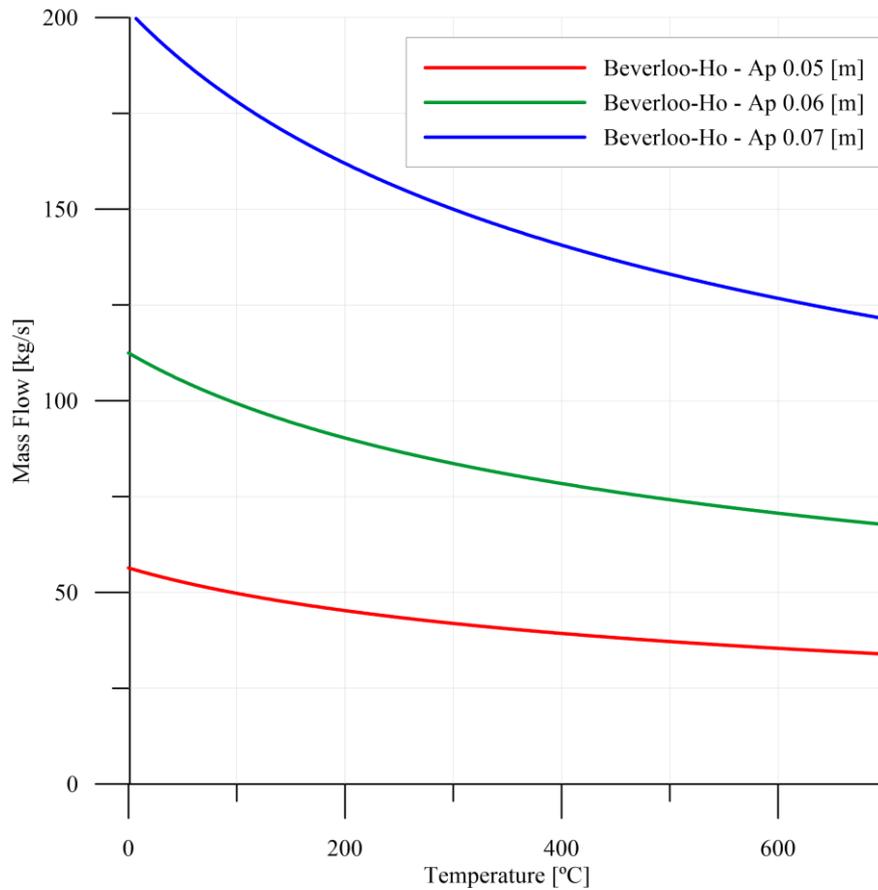


Figure 4 – Comparative mass flow rate by Beverloo-Ho per Temperature with defined aperture dimension

At last, but not least, Figure 4 presents the relation between temperature and mass flow rate. It was set different outlet apertures (0.05, 0.06 and 0.07 m) and then varied the particles temperature. It can be noted that the higher the outlet aperture is, the more affected the mass flow rate is from temperature variation. This can be easily seen by the comparison of 0.05 with 0.07 m outlet aperture, which the mass flow rate differences based on temperature variance were 22 kg/s and 80 kg/s, respectively. It can be seen that the first one is almost linear while the last one has an exponential decay function pattern.

4. CONCLUSIONS

In order to turn CSP into a viable and more efficient alternative to fossil fuels energy sources, research is required. Sandia National Laboratories research has pointed out that a heliothermic plant composed of a Brayton power cycle using supercritical carbon dioxide as the working fluid, powered by a solar tower receiver based on the free-falling particle (ceramic), as a very promising alternative to transform thermal energy to electricity. An assertive research investment requires precise models of each part of the technology for its sustainable development. Regarding the solar tower receiver, the available global particle mass flow equations were insufficient to describe the phenomenon, since they didn't account for some relevant features related with thermal energy, consequently, failing on small apertures conditions, as mentioned by the literature. Therefore, a comparison between the state of the art (Beverloo-Ho) and the classical equations (Beverloo and British Code) was performed to measure how much the classical ones deviate from the first one.

The literature predictions were confirmed, at small aperture dimensions the classical mass flow curve pattern deviated up to 40 kg/s from the Beverloo-Ho. However, around 0.06 m aperture, all the equations seems to merge in a short range, then, right after the Beverloo-Ho exponential pattern standouts.

Finally, the free-falling particle mass flow rate from solar tower receivers (CSP) is an specific application, which involves a large variety of parameters, such the dynamic interactions between particles and walls (thermal and frictional), the geometries of the particles and the hopper, particle thermophysical properties, the type of flow, the outlet features, etc. Thus, it is imperative to emphasize the importance of the alliance between the experiments and modelling for this type of application in order to have an assertive research. Therefore, is mandatory to verify before modelling if the selected equations can represent faithfully the phenomenon to be studied.

5. ACKNOWLEDGEMENTS

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