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A PROPOSAL FOR OPTIMIZATION OF TESLA TURBINES ROTORS

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Abstract. *This paper examines the use of rotor discs with modified profiles proposed in order to increase the efficiency and power generated in multi-disc turbines, also known as Tesla turbine. For this analysis, an analytical method is used to predict the efficiency and power produced by this type of device in conjunction with PIKAIA's evolutionary algorithm in order to obtain a rotor configuration that minimizes the power variation produced along the radius of the turbine rotor. The results indicate that a theoretical increase of up to 11.2% in the power generated by the turbine from the adoption of rough discs in the rotor.*

Keywords: *Tesla turbine, rotor optimization, bulk-parameter method.*

1. INTRODUCTION

The multiple turbine disks (MDT), also known as Tesla turbine, is a device whose rotor consists of several disks without blades positioned parallel to the long axis at small distances. It was invented by Nikola Tesla, who described his operation and some of its possible applications in a series of patents published between 1910 and 1920. Spite of low commercial interest in conventional turbines, according to Cairns, 2001, and Lampart, 2009, multiple disks turbines can be competitive especially in applications that require, among other reasons, low power, low maintenance cost and high versatility. The Fig. (1) shows schematically the operation of a typical multiple disks turbine.

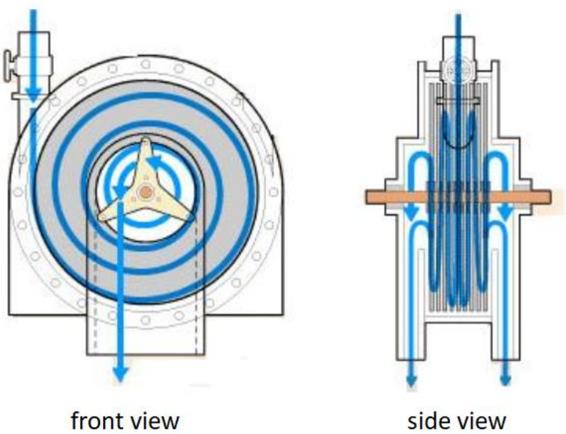

Figure 1. Schematic drawing of a multiple disk turbine (MDT) in operation showing the fluid trajectory.

As shown in the Fig. (1) in a MDT fluid is injected in the periphery of the rotor discs, after being accelerated by a nozzle inlet. In the rotor, the fluid flows through the disks in a spiral path until discharged by holes located near to the machine axis. Unlike a conventional turbine, the power output occurs exclusively due to shear stress between the working fluid and the surface of the rotor disks. Therefore, the variation in static pressure of the fluid in a MDT occurs entirely in the injector nozzle.

There are several recent studies that present possible applications for multi-disc turbines. Cairns *et al*., 2001, showed that this type of equipment can be competitive in applications that require low power and cost. Lampart *et al*., 2011, investigated the use of multiple disc turbines in low power (20 kW) organic thermal cycles, which has shown to be quite promising for certain operating parameters. Batista, 2009, and Schmidt, 2002, analysed the application of MDTs for microgeneration of electric energy from biomass (firewood), aiming, among other applications, the use in remote locations. Carey, 2010, evaluated the use of MDTs for power generation from solar energy using thermal steam power cycles, where the results showed that it is possible to achieve efficiencies in the order of 70% for needs that demand low power. In their study, Deam *et al*., 2010, demonstrated that small turbines that operate according to the MDT principle will always have greater efficiencies than conventional turbines. These characteristics and possible applications of multi-disc turbines are due to the fact that it is a simple construction equipment, capable of operating in adverse conditions, such as, for example, with fluids susceptible to phase change, impurities or very viscous.

Various efforts have also been made in order to develop theoretical and semi-empirical models to assist the design and analysis of MDT's. The first related studies refer to the 1960s with the analytical methods proposed by Rice, 1965, who obtained several equations for calculating the power of pumps and compressors that work according to the principle of MDT's. Felsch et al., 1981, developed expressions that describe the behavior of fluids considered incompressible in flow in laminar and non-isothermal flow inside a MDT. Couto *et al*. 2006, presented a calculation method based on existing equations for flows over the surface of a free disk. Carey, 2010, perfected the analysis described by Rice, 1965, by using the concept of friction factor, thus obtaining simpler equations valid for the laminar flow regime. Emran et al., 2011, obtained equations for the analytical calculation of the power for different flow regimes and number of nozzles present in the MDT. Guha, *et al.*, 2010, obtained semi-empirical equations from the analysis of the main forces acting on the fluid flowing through the rotor. Tahil, 1999, obtained equations for the performance of MDT through considerations of potential flow in cylindrical coordinates.

In this study, was analysed a MDT supplied with compressed air that has been studied experimentally by Rice, 1965. The rotor is shaped by 11 flat disks, with external radius of 88.9 mm and the exhaust holes located 33.5 mm from the shaft centerline. A turbina na bancada de testes e o seu rotor são mostrados na Fig. 2 e Fig. 3, respectivamente:

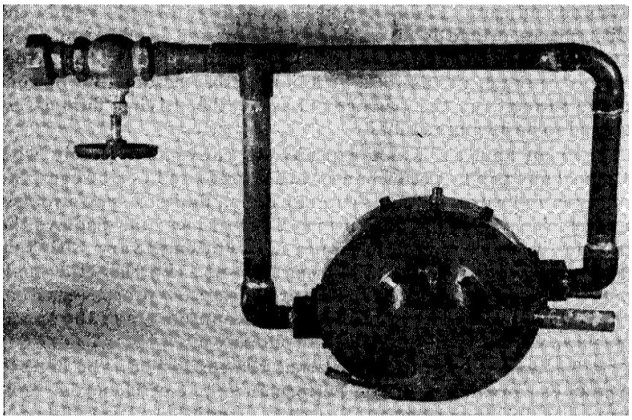

Fig.3. Multiple disc turbine of Rice (1965) on the test bench.

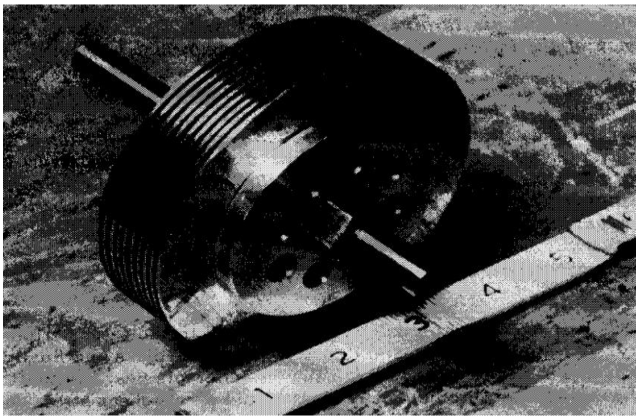

Fig. 4. Rotor of Rice (1965) multiple disc turbine coupled to it's central axis, where the torque and power produced are measured.

It was considered in this study that the working fluid (air) when flowing in the rotor is incompressible, since, as in the turbines classified as an action, the pressure variations in the analysed device occur exclusively inside the injector nozzle. As the flow at the nozzle outlet of a typical MDT can reach supersonic speed, it was assumed that the fluid is compressible inside the nozzle.

Although there are more recent experimental results, study of Rice, 1965, was chosen because it presents MDT's operating conditions and construction characteristics in sufficient detail, thus allowing the selection of the calculation method for efficiency and power and, as a consequence, the realization of the proposed optimization. Thus, based on the data provided, an optimization of the turbine rotor was proposed using bulk-parametric model and the evolutionary algorithm, with the objective of increasing the power extracted by the turbine.

2. MAIN EQUATIONS

2.1 Bulk-parameter method

The friction factor (BP) method is used to determine the efficiency and power obtained by a MDT from the knowledge of the geometric characteristics of the rotor, the operating regime of the equipment and the properties of the working fluid at the outlet of the injector nozzle, being based on considerations related to the interaction between the fluid and the rotor, described through a friction factor f. The equations for calculating the isentropic efficiency and, as a consequence, the turbine power are obtained through the Navier-Stokes equations, considering that in the rotor there is the flow of an inviscid, incompressible fluid and in laminar regime. The resulting equations are solved considering that the viscous forces act as field forces in the flow. The method received several contributions, the most relevant being given by Rice, 1965, and Carey, 2010.

From the analytical methods tested by Maidana, 2015, is the one that best predicts the performance of this type of device. According to the bulk - parameter method, for a fluid in the laminar flow regime, the isentropic efficiency of a MDT can be described by Equation (1):

$$\eta_{iso} = \frac{(\hat{W}_e+1)U_e^2-(\hat{W}_i+\xi_i)\xi_i+U_i^2}{\frac{k}{k-1}RT_e\left[1-\left(\frac{p_i}{p_e}\right)^{(k-1)/k}\right]} \tag{1}$$

where U_e is the peripheral speed of discs turbine, ζ_s the ratio between the outer and inner discs radii and p_s and p_e was inlet and fluid outlet pressure, respectively. In Eq. (1), k and R are the fluid properties and $\hat{W}_e$ and $\hat{W}_s$ are, respectively, the dimensionless relative velocity of the fluid at admission and discharge, being defined by:

$$\hat{W} = \frac{v_\theta - U}{U_e} \tag{2}$$

According to the deductions obtained by Carey, 2010, the dimensionless relative velocity at any point on the surface of the rotor can be calculated by:

$$\hat{W} = \frac{e^{24\zeta^2/\mathrm{Re}_m^*}}{\zeta}\left[\frac{\mathrm{Re}_m^*}{24}e^{-24\zeta^2/\mathrm{Re}_m^*}+\left(\hat{W}_e-\frac{\mathrm{Re}_m^*}{24}\right)e^{-24\zeta^2/\mathrm{Re}_m^*}\right] \tag{3}$$

Re_m^* is called, in the bulk-parameter method, the modified Reynolds number, which is defined by:

$$Re_m^* = \frac{D_h}{r_i}Re_{BP} \tag{4}$$

where D_h is the hydraulic diameter of the rotor, being equal to twice the spacing between the rotor discs. By the bulk-parameter-method, the Reynolds number, Re_{BP}, is given by:

$$Re_{BP} = \frac{\dot{m}}{\pi r_i n \mu_i} \tag{5}$$

where $\dot{m}$ and μ_i correspond, respectively, to the mass flow rate and the viscosity of the working fluid at the outlet of the injector nozzle, where n is the number of disks that make up the turbine rotor.

The Eq. (3) was obtained by the author considering only laminar flow and rotor with smooth discs. In order to consider the roughness effect and the consequent variation in the flow regime, the following equation was obtained for this study, deduced from the bulk-parameter method concepts:

$$\hat{W} = \frac{e^{Po\zeta^2/\mathrm{Re}_m^*}}{\zeta}\left[\frac{\mathrm{Re}_m^*}{Po}e^{-Po\zeta^2/\mathrm{Re}_m^*} + \left(\hat{W}_e - \frac{\mathrm{Re}_m^*}{Po}\right)e^{-Po\zeta^2/\mathrm{Re}_m^*}\right] \tag{6}$$

In Eq. (6), *Po* is called the Poiseuille factor which, according to Romain, 2012, is a parameter is a parameter that is directly related to the friction factor and, consequently, the flow regime and surface roughness. Thus, the Poiseuille factor can be obtained from the knowledge of the friction factor through the following equation:

$$f = \frac{Po}{24} \tag{7}$$

The efficiency of the rough disk turbine can be determined using Eq. (7) in conjunction with Equations (2), (3) and (6). Thus, the power generated by the turbine can be determined from the Equation (8):

$$\dot{W}_{util} = \eta_{iso}\dot{W}_{iso} \tag{8}$$

where $\dot{W}_{iso}$ is the isentropic power generated by the working fluid when flowing through the rotor turbine (ideal condition) e $\dot{W}_{util}$ é a potência útil obtida no eixo da turbina e $\dot{W}_{iso}$ a potência isentrópica que pode ser determinada pela Eq. (9):

$$\dot{W}_{iso} = \dot{m}\frac{k}{k-1}RT_i\left[1 - \left(\frac{p_i}{p_o}\right)^{k/(k-1)}\right] \tag{9}$$

The constant *k* is the ratio between the specific heats of the working fluid, *R* is the difference, in module, between these two properties.

2.2 Injector nozzle modelling

As suggested by Carey, 2010, the behaviour of the fluid in an injector nozzle can be modelled considering the one-dimensional and compressible flow. Thus, knowing that the nozzle of MDT of Rice, 1965, is of the convergent type, the maximum speed that the working fluid could reach in this device is that of sound, which can be determined through Eq. (10):

$$v_{\theta,ideal} = \sqrt{kRT_i} \tag{10}$$

The actual speed at the outlet of the injector nozzle, $v_{\theta,i}$, which corresponds to the injection speed at the periphery of the rotor discs, can be determined from the nozzle efficiency definition:

$$\eta_{bocal} = v_{\theta,i}^2/v_{\theta,ideal}^2 \tag{11}$$

Carey, 2010, determined from the results obtained from experimental measurements performed on the turbine of Rice, 1965, that the nozzle used in the analysed MDT had an efficiency between 0.44 and 0.52. Thus, it was considered for this study that the nozzle efficiency is equal to 0.48, which corresponds to the average of the determined efficiency values. The fluid velocity, $v_{\theta,i}$, calculated through this procedure is used as a parameter for determining $\widehat{W}$ in the intake section of the rotor in Equation 2 and 6.

3. MDT OPTMIZATION

For the proposed optimization, each MDT rotor disk analysed was divided into 10 parts of section Δr_j, in order to determine the ideal roughness as a function of the radius for each rotor section, as shown in the Fig 2, between the radius of the fluid intake (r_e) and the radius where the fluid leaves the rotor (rI):

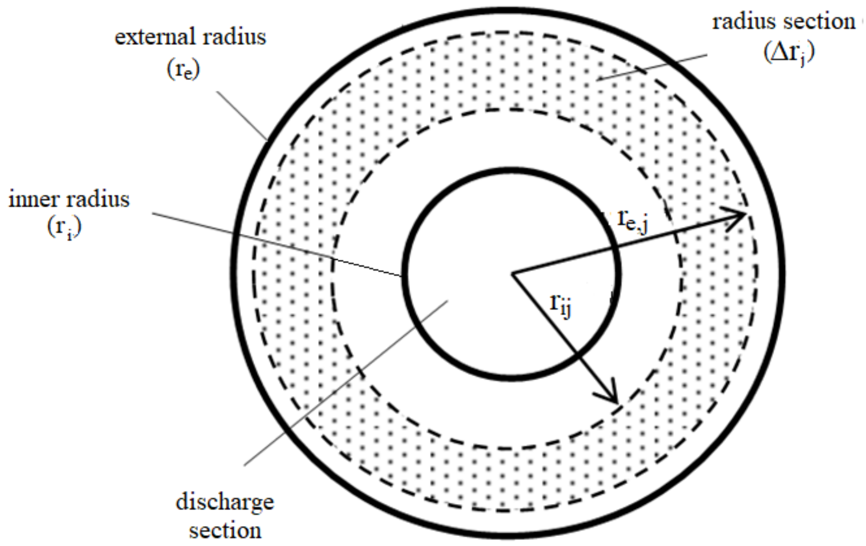

Figure 2. Schematic drawing showing the proposed discretization of the turbine rotor discs used for the optimization study.

The suggested proposal for the optimization of MDT consists of increasing the power generated by each rotor section by increasing the surface roughness of the disks, taking into account that the sections close to the axis produce less power due to the lower torque, in order to reduce the variation in power produced between the different sections of the rotor. Thus, the proposed optimization is equivalent to minimizing the objective function given by Eq. (12):

$$S_{\dot{W}_j} = \sqrt{\frac{1}{(n_j-1)}\sum\left|\Delta\dot{W}_j - \Delta\bar{\dot{W}}_j\right|} \tag{12}$$

where $S_{\dot{W}_j}$ is the standard deviation related to the power generated in each set of rotor sections with size Δr_j, n_j is the number of hypothetical rotor divisions (10) and $\Delta\bar{\dot{W}}_j$ is the average of the power obtained considering all sections. The power value $\Delta\dot{W}_j$ in each section Δr_j is determined by applying the bulk-parameter method.

For the optimization process, a routine was used based on the evolutionary optimization method of PIKAIA, version 1.2 (Charbonneau, 2002). The operating conditions (constant values) established for the optimization process were obtained from the average values turbine shaft rotation and thermodynamic conditions and fluid flow at the inlet nozzle, which were established by Rice, 1965, in his experimental work. These average values are shown in Table 1:

Table 1. Operating conditions considered in the MDT optimization process.

Rotation (rpm)	Inlet Pressure (kPa)	Inlet Temperature (K)	Flow Rate (kg/s)
6300	965	347	0.04536

The roughness of the surface of the rotor discs was used as the main restriction in the optimization algorithm. Thus, it was established that the roughness could assume values between 0 and 300 μm. According to Momber (2008), this range of roughness values can be obtained through typical processes of modification of surface finish.

The operating conditions tested experimentally by Rice (1965) for the turbine analysed in this work and the respective performance values obtained are presented in Table 2. These values were used to compare the results, that is, the increase in power and efficiency, as well as for calculating the constants in Table 1:

Table 2. Operating conditions and results obtained experimentally by Rice, 1965, for the performance of the analysed turbine.

Rotação (rpm)	Pressão de Entrada (kPa)	Temperatura de Entrada (K)	Vazão (kg/s)	η_{exp} (%)
6300	377	368	0.0213	21.7
8500	377	368	0.0213	25.4
9200	377	368	0.0213	25.8
8000	515	352	0.0306	21.2
10000	515	352	0.0306	23.8
11000	515	352	0.0306	24.4
8000	552	356	0.0263	21.7
10000	552	356	0.0263	23.8
11000	552	356	0.0263	23.8
9000	690	353	0.0331	21.6
11000	690	353	0.0331	24.1
12000	690	353	0.0331	19.0
9300	827	347	0.0401	21.2
11000	827	347	0.0401	22.9
12200	827	347	0.0401	13.5
11800	965	347	0.0454	23.2
12500	965	347	0.0454	11.9

where η_{exp} is the efficiency of the MDT determined experimentally by:

$$\eta_{exp} = \dot{W}_{axis}/\dot{W}_{iso} \tag{13}$$

In Eq. (13), $\dot{W}_{axis}$ is the useful power measured experimentally on the turbine axis for each of the operating conditions in which the turbine was tested, being equal the $\dot{W}_{util}$ if minor losses are not considered. $\dot{W}_{iso}$ is determined by Equation (9).

Carey, 2010, showed that there is a good agreement between the friction factor method and the experimental data when the objective is to determine the useful power produced by multiple disc turbines. Thus, the η_{exp} values in Tab. 2 are those determined by the application of the friction factor method for a turbine with the same geometric and operating characteristics as that tested by Rice, 1965.

4. MAIN RESULTS

The rotor surface roughness values obtained from the optimization process are presented, for each Δr_j section, in Table 2:

Table 2. Surface roughness values (ε) obtained for each of the rotor disc discretization sections (Δr_j), where j = 10 corresponds to the section adjacent to the inner radius (r_i) and j = 1 refers to the section adjacent to the outer radius (r_e).

j	ε_j (μm)
1	4.55
2	6.16
3	9.48
4	23.6
5	60.4
6	171
7	268
8	280
9	290
10	295

In Tab. 2, j = 10 refers to the Δr_j discretization section closest to the rotor axis and j = 1 the disk section closest to the outer periphery of the disks. The mutation rate chosen in the evolutionary algorithm was 0.1, this value being obtained after some simulations with mutation rates between 0.1 and 0.7, where it was found that the lowest mutation rates cause less variation in the results of the objective function used. In total, 161,000 evaluations of the objective function were carried out, through 248 generations of individuals, that is, the sequence of bits obtained from the free variables of the problem.

The evaluating the Rice turbine, 1965, with the roughness given in Tab. 2, the power increase in relation to the conventional rotor formed only by smooth discs, is shown in Tab. 3 for each operating condition in which the turbine was experimentally tested:

Table 3. Power increase obtained with the rotor optimization for each operating condition of the analysed turbine.

Inlet Pressure (kPa)	Inlet Temperature (K)	Mass Flow Rate (kg/s)	Turbine Shaft Rotation (rpm)	Δη (%)
377	368	0.02134	6300	11.2
377	368	0.02134	8500	11.0
377	368	0.02134	9200	10.9
515	352	0.03069	8000	6.25
515	352	0.03069	10000	6.40
515	352	0.03069	11000	6.75
552	356	0.02629	8000	5.34
552	356	0.02629	10000	5.66
552	356	0.02629	11000	5.84
690	353	0.03311	9000	6.83
690	353	0.03311	11000	7.16
690	353	0.03311	12000	7.34
827	347	0.04007	9300	7.97
827	347	0.04007	11000	8.25
827	347	0.04007	12200	8.46
965	347	0.04536	11800	9.01
965	347	0.04536	12500	9.16

For the calculations of efficiency increments presented in Table 3, the values of this physical quantity obtained with the application of the friction factor method in each presented operating condition were used as a reference, where the turbine was tested in it's original configuration (rotor with smooth discs). According to Eq. (8), the same increase in efficiency in the turbine is also obtained in the power.

The efficiency gain, Δη, predicted for the turbine with the optimized rotor that were used to determine the respective values calculated in Table 2 were determined from Eq. (14) described below:

$$\Delta\eta = \frac{\eta_{opt} - \eta_{exp}}{\eta_{exp}} \tag{14}$$

where η_{opt} is the value of the MDT isentropic efficiency analyzed by applying the friction factor method considering the roughness configuration of the rotor discs shown in Table 2.

5. CONCLUSIONS

The change in surface roughness obtained through the application of the friction factor method in an evolutionary optimization algorithm, shows that it would be possible to obtain an increase in power of up to 11.2% in a turbine with the same geometry as that tested by Rice, 1965, and which operates under the same conditions.

It's possible to verify that the highest possible values of efficiency in the determined optimization occur at low rotor speeds and flow rates, which correspond to the ranges of the lowest Reynolds numbers of the flow inside the rotor. These values of increment of generating power were determined comparing with the value obtained by the friction factor method for the standard turbine, since, according to Carey, 2010, and Maidana, 2015, this method is able to predict with good approximation the values performance of multi-disc turbines. This increase was possible due to the increase in the roughness of the discs in the sections close to the outlet holes of the rotor fluid, thus providing a greater contribution of these sections in the generation of power.

More expressive increases in power could be obtained through other methods that allow the increase of the friction factor on the rotor surface. In addition, as described by other authors, such as Truman *et al.*, 1978, Lemma *et al.*, 2008, and Krishnam *et al*. 2011, there are other possible ways to increase the power of multi-disc turbines, since several components, in addition to the rotor, are subject to optimization, such as the use of supersonic nozzles.

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