18th Brazilian Congress of Thermal Sciences and Engineering
November 16–20, 2020 (Online)

# ENC-2020-0162
# AN INVERSE GEOMETRIC BIOHEAT TRANSFER PROBLEM FOR THE DETECTION OF BREAST TUMOURS

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***Abstract.*** *In recent years, the clinical diagnosis of breast tumours by means of thermal analysis has received increasing attention as a non-invasive and less expensive alternative to classic techniques. In this paper, the inverse problem to estimate the geometric parameters of a circular tumour in a multilayered breast model composed of five tissue layers by taking into account the prior knowledge of the breast skin temperature is analysed. The many forward problems, mathematically modelled by the Pennes' equation, required in the inverse analysis are numerically solved by the Finite Element Method through the software Wolfram Mathematica®. The inverse problem is posed as an optimization problem and solved by a binary Genetic Algorithm (GA) written in C language. Aiming at studying the effectiveness of the GA, a noise (uncertainty) is inserted into the reference data (breast skin temperature) in order to mimic experimental thermal measurement errors inherent in real cases. In both analysed cases (with and without noise) the algorithm was able to find with a considerable accuracy the center and radius of the tumour, revealing to be a promising method in the detection of tumours near the skin surface.*

***Keywords:*** *Geometric Inverse Problem, Genetic Algorithm, Pennes' Bioheat transfer, Finite Element Method*

## 1. INTRODUCTION

Breast cancer was the female cancer with higher incidence in the last years (excluding nonmelanoma skin cancer) with around 2.1 millions of new cases and estimative of 627 thousand deaths in 2018 (Bray *et al.* (2018)). Chakraborty and Rahman (2012) define cancer or malignant neoplasm as a result of genetic alterations or epigenetic in the somatic cells with different risk factors, such as family history of breast cancer, radiation, alcohol consumption among others (Hortobagyi (1998)). According to Weigelt *et al.* (2005), the primary tumour is not the most cause of death, but instead metastases at distant sites, so it is very important to detect breast cancer in early stage, increasing the success rate of treatment. The identification of signs and the tumour itself can be carried out by self-exam and clinical exams.

The most popular clinical exam to detect breast cancer is mammography that creates a set of images of the breast by ionizing radiation. However, this method can impose some risk by radiation, risk of false alarm (Heywang-Köbrunner *et al.* (2011)), high invasion of patient and high costs of operation and maintenance. Hence, researchers have been proposing alternatives techniques for diagnosis and detect breast cancer (e.g., Ekici and Jawzal (2020); Sun *et al.* (2020); Morrow *et al.* (2011); Soltani *et al.* (2019); Menegaz and Guimarães (2019)); and a promising one relies on the use of thermography due to its low cost and non-invasive feature for the patient when compared to the mammography, for example. One way to use thermography to detect breast cancer is measuring the skin temperature and then employing some combination of mathematical modelling and computational resources to find information about the tumour (e.g., Ekici and Jawzal (2020); Figueiredo *et al.* (2019); Gonçalo Filho *et al.* (2017); Mital and Pidaparti (2008); Melo *et al.* (2017)), which consists in a inverse problem. An inverse problem can be described by the determination of unknowns variables, material properties, shape or mathematical model without direct measurement (Iljaž *et al.* (2020)). Experimental results of thermography applied in breast cancer detection can already be seen in some works as in Bezerra *et al.* (2013) and Bahador *et al.* (2018).

In fact, inverse problems have been applied in medicine due to the fast advance of computational resources, often applying evolutionary algorithms like Genetic Algorithms (examples of such applications can be seen in Ghaheri *et al.* (2015)) to solve some type of inverse problem combined with numerical methods like Finite Element, Finite Volume or Finite Difference Method to solve partial differential equations that arise in this kind of problems (e.g., Bousselham *et al.* (2018); Bhowmik and Repaka (2016); Harikumar *et al.* (2004); Reis *et al.* (2014); Valente *et al.* (2018)). Different authors have applied computational modelling to search geometrical parameters and thermophysical properties of breast cancer by thermography using skin temperature as reference; however, many of these models assumed a simplified 2D model composed only of tumour and surrounding tissue.

This work carries out a study on an inverse bioheat conduction problem for the detection of a circular breast tumour (center and radius) from the prior knowledge of breast skin temperature in a multilayered breast model. To this end, a computer code based on the binary Genetic Algorithm (GA) and Finite Element Method (FEM) has been developed. Genetic Algorithm is an stochastic optimization algorithm proposed by Holland (Holland (1992)) based on the theory of evolution where fittest individuals tend to survive and maintain some of their characteristics over the next generations, whereas the Finite Element Method is a numerical technique for solving partial differential equations that subdivides a system into elements, discretizing the space (Reddy (2006)).

## 2. STATEMENT OF THE INVERSE PROBLEM

The problem to be analysed herein consists of estimating the geometric parameters of a breast tumour through a thermal analysis. More precisely, based on prior knowledge of the temperature on the breast skin surface obtained by other means (named herein as reference temperature), an inverse analysis is carried out to detect the tumour by minimizing the following error norm (also called objective function):

$$J(\mathbf{y}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{T}_i - T_i(\mathbf{y})\right)^2} \tag{1}$$

where $\mathbf{y}$ stands for a vector containing the design variables, $\hat{T}_i$ is the reference breast skin temperature measured at $N$ points and $T_i(\mathbf{y})$ is the estimated (computed) one for a given design variable vector.

In the present work a 2D model is considered by assuming the breast to be a semi-circle and including distinct tissue layers (namely, epidermis, dermis, fat, gland and muscle) as depicted in Fig. 1. In addition, to assess the proposed methodology, the tumour to be detected is already known and assumed to be a circle given by the domain $\Omega_T = \{x, y \in \mathbb{R} : (x-27)^2 + (y-45)^2 \leq 4^2\}$. In this way, the design variable vector containing the geometric parameters to be estimated by the inverse analysis can be defined as $\mathbf{y} = \begin{bmatrix} x_r & y_r & r \end{bmatrix}^T$.

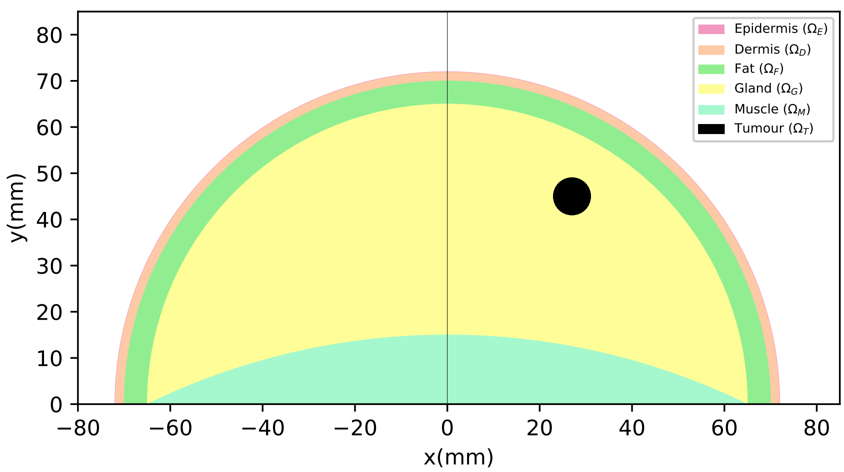


Figure 1: Geometry of the breast model with a tumour.

## 3. SOLUTION PROCEDURE

### 3.1 Genetic algorithm

Once the inverse problem can be recast into a minimization problem given by Eq. (2) (recalling that it contains 3 design variables, namely, circle center position ($x_r$, $y_r$) and radius ($r$) of the tumour), a deterministic or a stochastic optimization algorithm must be employed.

$$\begin{aligned} &\min J(\mathbf{y}) \\ &s.t.\ \ \Omega_T(\mathbf{y}) \subset \Omega_G \end{aligned} \tag{2}$$

Here, a binary GA has been employed to estimate the design variable vector $\mathbf{y}$. The GA is characterized by some distinct features, namely: i) the optimal variables are found from a population of possible solutions rather than a single solution; ii) no requirement for continuity in the derivatives of the fitness (or objective function); in fact, no information regarding gradients of the objective function is required during the search process, being necessary only the evaluation of objective function values; iii) due to the randomness nature of the GA, it tends to find the global minimum, avoiding stagnation in a local minimum; and iv) the GA is quite suited for parallel implementations. The algorithm can be briefly described as: starting with a population with $I$ individuals randomly selected in the search space, each of them with $m = 3$ design variables where each variable is a $string$ composed by $n_{bits}$ whose binary value is represented in an interval $[0, 2^{n_{bits}})$, a generation procedure is then initiated until find the optimal design variables, which must satisfy the constraints. The conversion real/discrete is defined by Eq. (3).

$$y_i = y_i^{\min} + \frac{y_i^{\max} - y_i^{\min}}{2^{n_{bits}} - 1} \sum_{k=0}^{n_{bits}-1} 2^k string_k \tag{3}$$

where $y_i^{\max}$ and $y_i^{\min}$ are, respectively, the upper and lower values in the search space for each design variable, such that the estimated (computed) tumour domains $\Omega_T(\mathbf{y})$ are always inside the gland layer according to Eq. (2).

After creating an initial population randomly, the stopping criterion for the generations is usually assumed as either a pre-specified number of generations or a pre-specified objective function value. For each new generation three basic genetic operators are applied, namely, selection, crossover, and mutation in addition to the elitism strategy.

The operators employed here for the binary GA are described as: tournament for selection that consists in selecting the two bests within four chosen randomly in the population; single-point crossover that pairs the genes of two parents and then exchange the information by not choosing the extreme positions, therefore, generating two children. This crossover is only applied if a random number is less than or equal to a crossover probability, otherwise the parents are hold for the next generation. Mutation bit by bit that inverts a bit given a probability and, finally, simple elitism that preserves the best individual on a population for the next generation replacing the worst individual of the current generation.

### 3.2 Forward Problem via FEM

Once the geometric parameters are estimated by the GA, the forward bioheat transfer problem modelled by the Pennes' equation given by Eq. (4) ( Murthy *et al.* (2000)) with the current estimated tumour domain $\Omega_T(\mathbf{y})$ can be solved.

$$\nabla \cdot k\nabla T + \rho_b c_b \omega_b (T_b - T) + Q_m = 0, \qquad \mathbf{x} \in \Omega = \bigcup_{i=T,M,G,F,D,E} \Omega_i \tag{4}$$

where $\rho_b$, $c_b$, $T_b$ and $\omega_b$ represent, respectively, the density, specific heat, arterial temperature and perfusion rate with subscript $b$ denoting blood. For tissue properties we have $T$, $k$ and $Q_m$ that correspond, respectively, to temperature, thermal conductivity and metabolic heat generation.

The boundary condition at $y_{min} = 0$ (inner portion of the muscle) is assumed to be at the core temperature (Dirichlet boundary condition):

$$T = 37^\circ C \tag{5}$$

In order to simulate the heat transfer with the environment, the outer portion of the epidermis, named $\Gamma_E$ (i.e., breast skin surface), is subjected to the following convective boundary condition (Robin boundary condition):

$$k\nabla T \cdot \mathbf{n} = h(T_\infty - T) \tag{6}$$

where $h = 10$ W/(m$^2$°C) and $T_\infty = 25$°C stand, respectively, for the heat transfer coefficient and environmental temperature. Such a value of $h$ stems from the natural convection flow on the skin surface.

The starting point of the FEM formulation consists in deriving the weak form of the Pennes' equation as:

$$\int_\Omega \left(\nabla v \cdot k\nabla T - v\rho_b c_b \omega_b (T_b - T) - vQ_m\right) d\Omega - \int_{\Gamma_E} v\,(T_\infty - T)\, d\Gamma = 0 \tag{7}$$

where $v(\mathbf{x})$ is the test function.

Next the domain $\Omega$ is here partitioned into $nel$ triangular nonoverlapping elements, i.e., $\Omega \approx \Omega^h = \bigcup_{e=1}^{nel} \Omega^e$. Hence, the conforming finite element space for the approximate temperature field is defined as:

$$S^h = \left\{T^h \in C^0(\Omega) : \left. T^h \right|_{\Omega^e} \in P_2(\Omega^e), \forall \Omega^e \in \Omega^h \right\} \tag{8}$$

where $P_2(\Omega^e)$ is the finite element interpolating space (notice that quadratic triangular elements are being employed). Thus, the finite element solution for the temperature field can be defined as:

$$T(\mathbf{x}) \approx T^h(\mathbf{x}) = \sum_{i=1}^{n} N_i(\mathbf{x})\, T_i \tag{9}$$

where $n$ is the total number of nodes in the mesh, $N_i(\mathbf{x})$ is the global shape function defined according to $S^h$ and $T_i$ is the nodal value of the temperature. Assuming that the test function $v(\mathbf{x})$ is also defined in $S^h$ with the same approximation given by Eq.(9) but with $v(\mathbf{x}) = 0$ on the Dirichlet boundary, the approximate counterpart of the weak form leads to a system of equations of the form $\mathbf{KT} = \mathbf{F}$. The conductance matrix $\mathbf{K}$ and the heat load vector $\mathbf{F}$ are formed by assembling their element counterparts which are evaluated by numerical integration.

### 3.3 Computer Implementation

The developed code is written in C language for the implementation of the GA due to its speed (besides being an imperative and well-established language), whereas the finite element solution of the Pennes' equation is readily performed by the built-in function **NDSolveValue** provided by the software *Mathematica*® (Wolfram language) that since version 10 provides the FEM analysis. The user-friendly of this function with many capabilities accessed through a list of options allows a straightforward generation of the computational geometry and mesh, as well as set up of the parameters of the model.

The link between the two languages is carried out through the C code that calls the Mathematica Kernel which evaluates the *Mathematica* file, with the estimated design variables being read from an input file created by the GA code. Subsequently, the breast skin temperature calculated by the *Mathematica* is written to an output file and then read by the C code that computes the objective function. Figure 2 summarizes the main steps for the developed code, whereas Fig. 3 depicts the remeshing procedure required due to the randomness of the estimated tumour domains $\Omega_T(\mathbf{y})$ from the GA. In fact, the regeneration of the whole unstructured mesh by setting up a local refinement into the tumour domain according to its radius is a quite important step of the code. Finally, it is worth mentioning that a mesh sensitivity analysis has been performed, leading to the conclusion that mesh sizes shown in Fig. 3 are enough to yield accurate results.

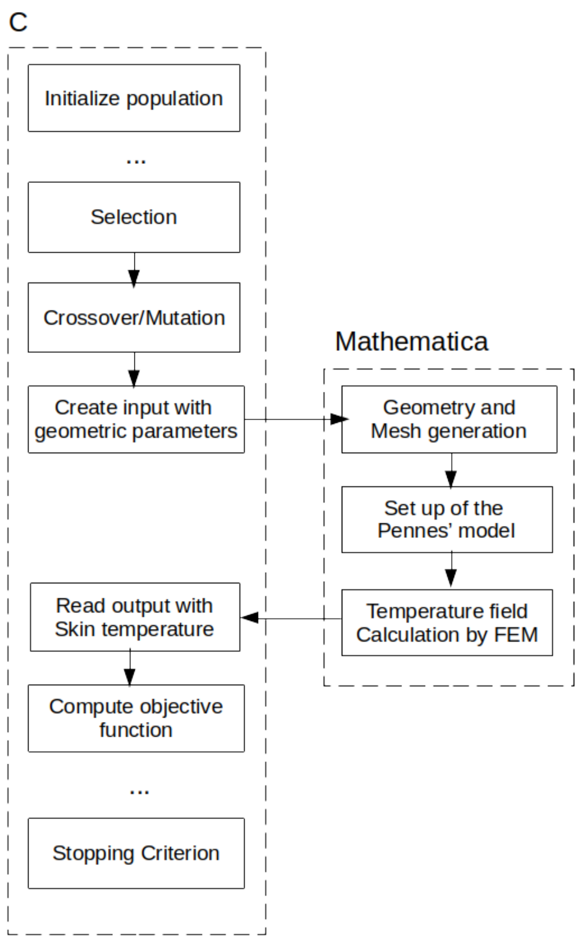
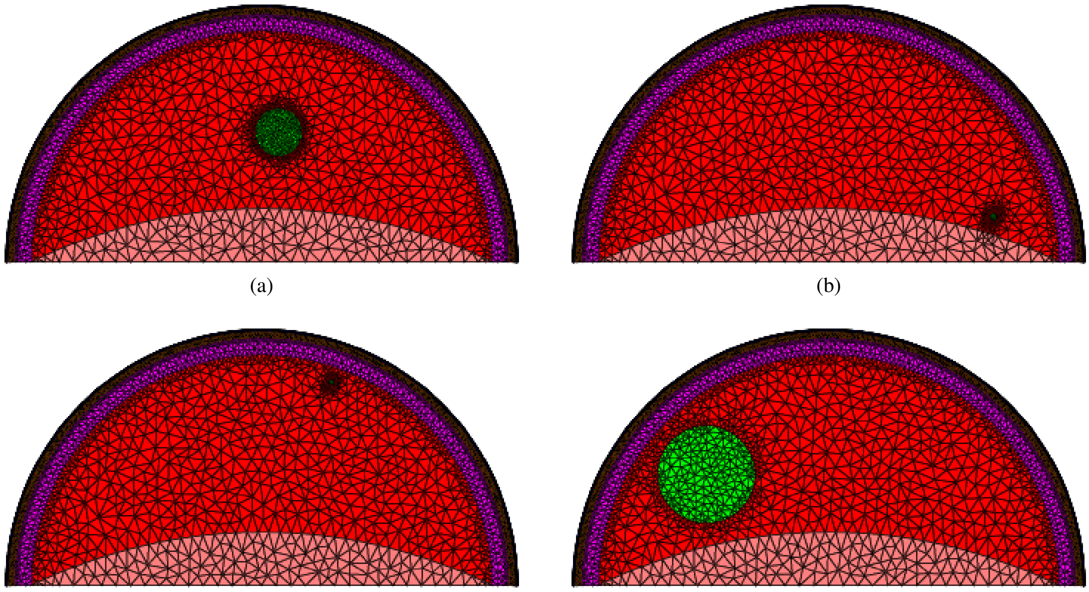



Figure 2: Flowchart of the main steps required by the inverse analysis.



(a) (b)

(c) (d)

Figure 3: Remeshing during the GA evolutionary process.

## 4. RESULTS AND DISCUSSION

Preliminary tests showed that a population with 120 individuals and a stopping criterion of 100 generations were sufficient to obtain accurate results; but, on the other hand, a sensitivity analysis is still necessary to obtain a better effectiveness of the GA, seeking the best combination of parameters. The mutation probability ($p_m$) cannot have large values, which could transform the search into a purely random search. Thus, some mutation is required to prevent the premature convergence to suboptimal solutions (Srinivas and Patnaik (1994)); and values of 0.01 and 0.03 were chosen to test the mutation probability $p_m$. In addition, we have the crossover probability ($p_c$) that controls the rate that new solutions are introduced in the population, causing exchange of genetic material between solutions (Srinivas and Patnaik (1994)). Values of 0.80, 0.90 and 1.0 were chosen to test $p_c$. The $n_{bits}$ was fixed to a value of 32.

With the goal of assessing the effectiveness of the GA when real data are employed, a noise is inserted into the known reference breast skin temperature in order to mimic experimental measurement errors inherent in real cases. The expression is given by:

$$\hat{T}_i^* = \hat{T}_i + \xi \hat{T}_u \tag{10}$$

where $\xi$ represents a random number $\xi \in [-1, 1]$, $\hat{T}_u$ represents the level of uncertainty and $\hat{T}_i$ the reference temperature. Based on Iljaž *et al.* (2020) we have used uncertainty with values 0mK representing exact measurement as well as 25mK representing measurement with noise as depicted in Fig. 4.

The thermophysical properties of the tissue layers of the present model (see Fig. 1) are presented in Tab. 1 (Jiang *et al.* (2010); Çetingül and Herman (2011); Ng and Sudharsan (2001)). In addition, $\rho_b = 1060kg/m^3$ and $c_b = 3770J/(kgK)$.

Table 1: Thermophysical properties of the tissue layers

| | | Thickness [mm] | Thermal conductivity $k$ [W/(mK)] | Blood perfusion rate $\omega_b$ $10^{-4}$ [m$^3$/s/m$^3$] | Metabolic heat generation $Q_m$ [W/m$^3$] |
|---|---|---|---|---|---|
| 1 | Epidermis | 0.1 | 0.235 | 0 | 0 |
| 2 | Dermis | 1.9 | 0.445 | 2 | 368.1 |
| 3 | Fat | 5 | 0.21 | 1.8 | 400 |
| 4 | Gland | 65 | 0.48 | 5.4 | 700 |
| 5 | Muscle | 15 (at $x = 0$) | 0.48 | 8.1 | 700 |
| 6 | Tumour | - | 0.48 | 108 | 50000 |

Given the nature of stochastic algorithms, a statistical analysis is performed taking into account the GA parameters previously defined. The results are shown in Tabs. 2-5, recalling that the reference geometric parameters are defined as $x_r = 27mm$, $y_r = 45mm$ and $r = 4mm$ (see Fig. 1). It is worth mentioning that 15 simulation runs for each combination were carried out.

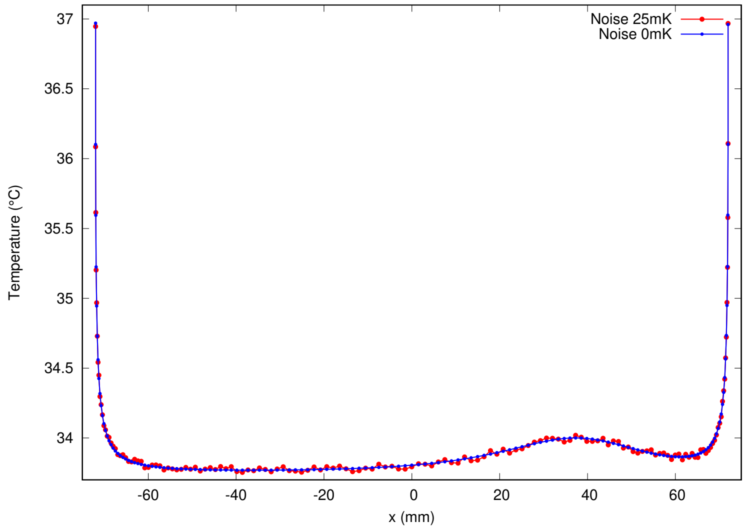


Figure 4: Reference breast skin temperatures with 0mK and 25mK of noise.

Table 2: Statistical results for 0mK noise.

| $p_m$ | $p_c$ | Median | Mean | St. dev. | $p_m$ | $p_c$ | Median | Mean | St. dev. |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.80 | 3.53E-4 | 8.53E-4 | 9.33E-4 | 0.03 | 0.80 | 3.38E-4 | 9.07E-4 | 1.99E-3 |
| 0.01 | 0.90 | 3.77E-4 | 9.94E-4 | 9.81E-4 | 0.03 | 0.90 | 3.38E-4 | 9.80E-4 | 2.02E-3 |
| 0.01 | 1.00 | 2.09E-3 | 2.08E-3 | 2.54E-3 | **0.03** | **1.00** | **2.22E-4** | **4.78E-4** | **6.56E-4** |

Table 3: Statistical results for 25mK noise.

| $p_m$ | $p_c$ | Median | Mean | St. dev. | $p_m$ | $p_c$ | Median | Mean | St. dev. |
|---|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.80 | 0.0153092 | 0.0154619 | 5.097E-4 | 0.03 | 0.80 | 0.0152140 | 0.0152595 | 1.214E-4 |
| 0.01 | 0.90 | 0.0152234 | 0.0154378 | 6.393E-4 | 0.03 | 0.90 | 0.0152126 | 0.0153254 | 4.185E-4 |
| 0.01 | 1.00 | 0.0152159 | 0.0153412 | 4.157E-4 | **0.03** | **1.00** | **0.0152125** | **0.0152141** | **2.938E-6** |

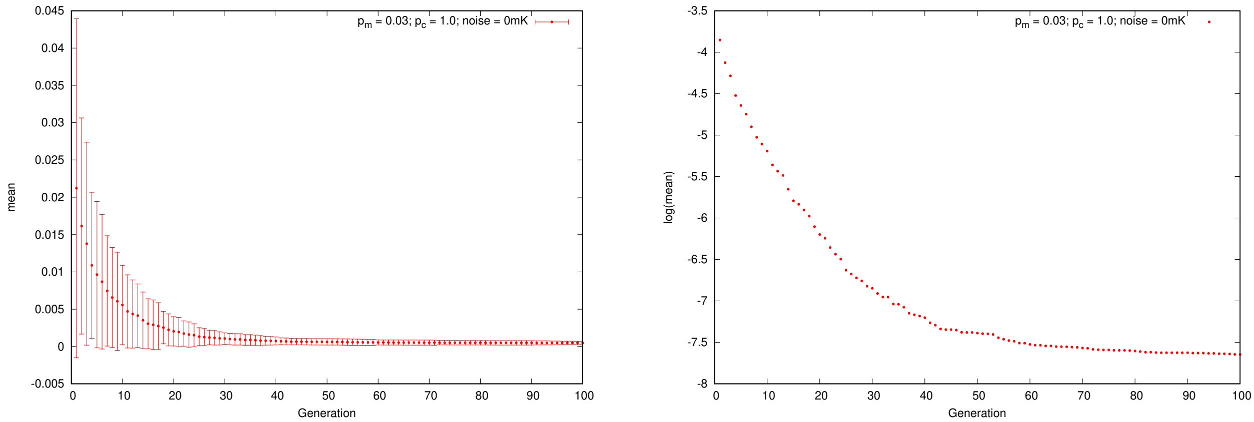
Table 4: Best results for 0mK with respective relative errors.

| $p_m$ | $p_c$ | Objective function | $x_r(mm)$ | $y_r(mm)$ | $r(mm)$ | $e_x(\%)$ | $e_y(\%)$ | $e_r(mm)$ |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.80 | 2.11E-5 | 27.004 | 44.997 | 4.000 | 0.0145 | 6.78E-3 | 4.95E-3 |
| 0.01 | 0.90 | 4.17E-6 | 27.001 | 45.001 | 3.999 | 3.06E-3 | 3.18E-3 | 0.014 |
| 0.01 | 1.00 | 1.14-6 | 27.000 | 45.000 | 4.000 | 1.27E-4 | 2.34E-4 | 3.75E-4 |
| 0.03 | 0.80 | 1.77E-5 | 26.995 | 44.996 | 4.002 | 0.0187 | 8.64E-3 | 0.0378 |
| 0.03 | 0.90 | 1.76E-6 | 27.000 | 44.999 | 4.000 | 1.67E-3 | 1.44E-3 | 4.35E-3 |
| 0.03 | 1.00 | 2.27E-5 | 26.995 | 45.001 | 4.001 | 0.01677 | 2.53E-3 | 0.02089 |

Table 5: Best results for 25mK with respective relative errors.

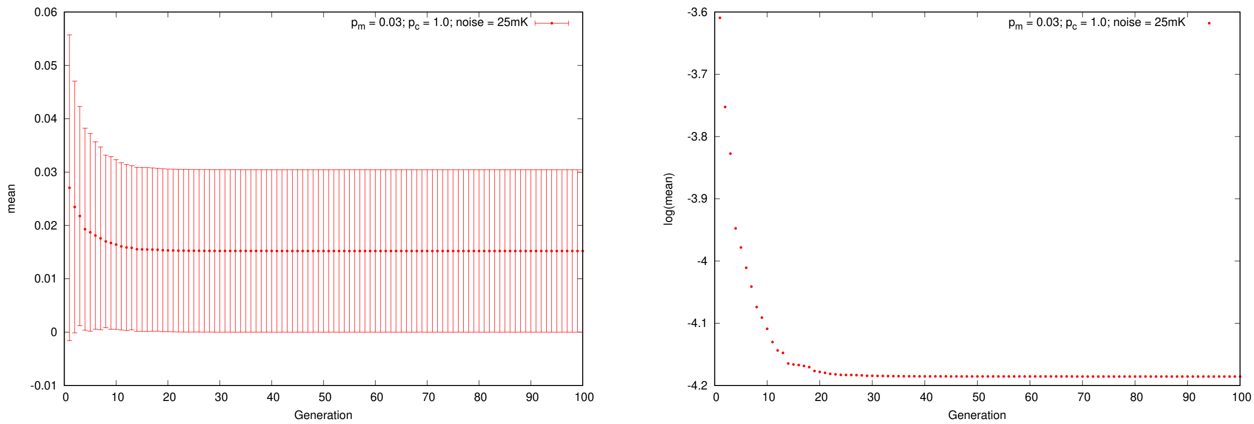
| $p_m$ | $p_c$ | Objective function | $x_r(mm)$ | $y_r(mm)$ | $r(mm)$ | $e_x(\%)$ | $e_y(\%)$ | $e_r(mm)$ |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.80 | 0.0152125 | 27.415 | 45.392 | 3.802 | 1.538 | 0.871 | 4.955 |
| 0.01 | 0.90 | 0.0152125 | 27.410 | 45.386 | 3.804 | 1.521 | 0.858 | 4.886 |
| 0.01 | 1.00 | 0.0152124 | 27.401 | 45.363 | 3.810 | 1.485 | 0.806 | 4.741 |
| 0.03 | 0.80 | 0.0152124 | 27.398 | 45.356 | 3.813 | 1.475 | 0.791 | 4.682 |
| 0.03 | 0.90 | 0.0152124 | 27.397 | 45.357 | 3.813 | 1.472 | 0.793 | 4.698 |
| 0.03 | 1.00 | 0.0152124 | 27.400 | 45.363 | 3.811 | 1.483 | 0.806 | 4.728 |

Analysing these results it is possible to conclude the following main points: i) The GA parameters set to $p_m = 0.03$ and $p_c = 1.0$ yield the best results; ii) as expected, a lower objective function value was achieved for the case with 0mK, indicating that the geometric parameters are accurately estimated; iii) results with noise indicate a faster stagnation when compared to those without noise (Fig. 5 and Fig. 6); and iv) results for the case with 25mK are slightly less sensitivity




(a) (b)

Figure 5: Statistical results during generations for $p_m = 0.03$, $p_c = 1.0$ and 0mK: (a) Mean of the objective function with standard deviation; (b) log of mean of the objective function.




(a) (b)

Figure 6: Statistical results during generations for $p_m = 0.03$, $p_c = 1.0$ and 25mK: (a) Mean of the objective function with standard deviation; (b) log of mean of the objective function.

to the change of GA parameters when compared to the those without noise. These points are completely connected with the accuracy furnished by the collected reference data (skin temperature). Clearly, reference data with noise impose some difficulty in the inverse problem. However, the GA was able to estimate the geometric parameters with considerable effectiveness in the sense that estimated values were not affected by the different choice of the GA parameters as can be observed in Tab. 5. Finally, Fig. 7 shows the temperature results for the best individual considering the case with noise.

## 5. CONCLUSIONS

The detection of breast cancer by means of thermography has received attention due to low cost, besides being a non-invasive technique, when compared to conventional clinical exams. This work has conducted a numerical study on an inverse bioheat transfer problem to estimate the geometric parameters (center and radius) of a circular breast tumour from the prior knowledge of the breast skin temperature, taking into consideration a 2D multilayered breast model modelled by Pennes' equation. For this purpose, a computer code based on the FEM for the solution of forward problems and the GA for solving the inverse problem has been developed. The built-in function **NDSolveValue** of the *Mathematica*® for the FEM analysis reveals to be an easy and powerful tool; besides, the coupling procedure with the C code concerning the GA is very straightforward.

To assess the effectiveness of the GA, two cases with and without noise inserted in the reference breast skin temperature were presented. In addition, a sensitivity analysis was performed on both cases to study the influence of GA parameters in obtaining good estimates. It was possible to observe that even with a poor accuracy of the reference skin temperature with noise, the GA provides good results in estimating the geometric parameters, independently of the employed parameters. Finally, the present study indicates that thermography can aid in the detection of breast tumours near the skin surface.

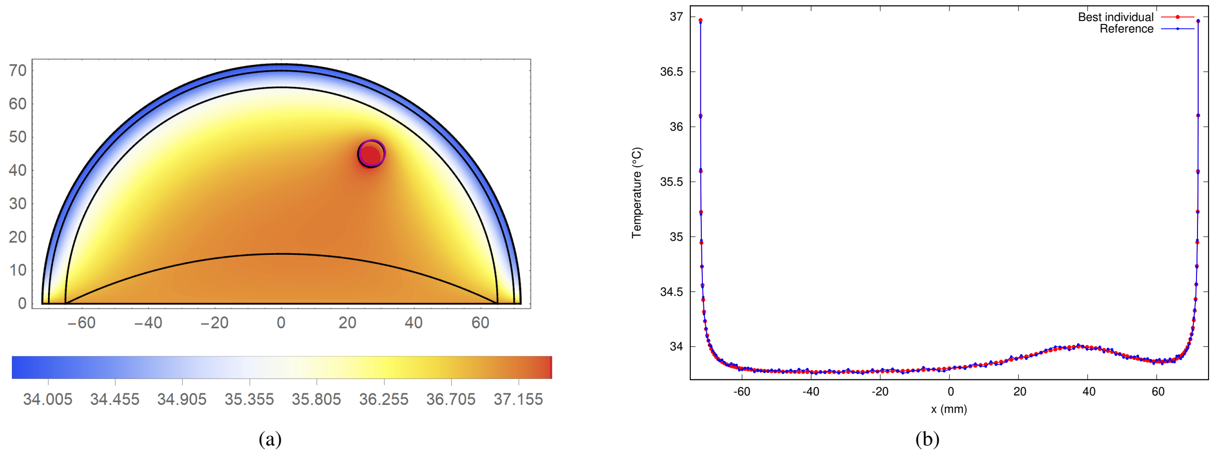



Figure 7: Results for the best individual with $p_m = 0.03$, $p_c = 1.0$ and $25mK$. (a) Temperature field for the breast model with the reference tumour and the estimated one (magenta); (b) Comparison between the reference skin temperature and the one computed taking into account the estimated parameters.

## 6. ACKNOWLEDGEMENTS

The financial support provided by CNPQ, FAPEMIG and UFSJ is greatly acknowledged.

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