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OPTIMIZATION OF COOLING CAVITY WITH THE APPLICATION OF CONSTRUCTAL THEORY

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Abstract. *Based on the constructal principle, the present research focuses on the problem of minimizing the maximum temperature of solid bodies with a rectangular shape and internal heat generation, i. e, maximizing the heat flow that leaves the body. The cavity has a rectangular shape (I), whose area (two-dimensional) is kept fixed while its dimensions, width (L_0) and height (H_0) vary. The cavity optimization occurs through the application of the Constructal Theory associated with the exhaustive search method. This process consists of determining the best relation between the dimensions of the width and height of the cavity for a given fraction of the occupied area so that the heat exchange through the isothermal walls is maximized. The GMSH software was used to generate the geometry and discretization of the computational domain. For the processing step, the laplacianFoam solver was used, which is an integral part of the OpenFOAM software. The transport equations are discretized using the Finite Volume Method (FVM). The studies performed show that for the case of the isothermal cavity, the best geometric shape of the cavity, that minimizes the maximum temperature, occurs in the almost total intrusion of the cavity into the solid.*

Keywords: *Cooling of cavities. Transient regime. Geometry optimization. Heat transfer. Constructal Theory.*

1. INTRODUCTION

The problems of heat conduction and cooling of electronic devices (heat sources), motivate the creation of geometric structures in order to control the temperature, and consequently an optimization in the manufacturing process that result in new miniaturizations (Zhang et al., 2011). Biserni et al., (2004), studied numerically the optimization of 'I' shaped cavities with a heat supplied as a boundary condition and as an internal heat source. Based on results, the authors observed that there was no change in them, proving that for this geometry it does not matter where the heat is generated and the search for the best configuration depends only on the degrees of freedom and a restriction. A T-shaped cavity was investigated by Lorenzini et al., (2011). This shape has more degrees of freedom and consequently more restrictions than the I shaped cavity. Biserni et al., (2007), investigated the H-shaped isothermal cavity. In the optimization of the domain, up to six degrees of freedom are used and it was observed that its thermal performance is three and four times greater than the T and I-shaped cavities, respectively. Lorenzini et al., (2011), analyzed the isothermal cavities in the shape of T and Y, where both reduced their maximum temperature with the increase in the volumetric fraction of their single branches (stem). In the work by Lorenzini et al., (2014b), the X-shaped cavity was optimized. To reduce the maximum temperature, it was necessary to optimize two degrees of freedom. In the work by Rocha et al., (2010), the I shaped convective cavity was evaluated. The authors found that the best performance occurs when there is total insertion of the cavity in the solid for a ratio between height and width less than 2. The investigation by Estrada et al., (2015) consists of finding the best geometric configuration using several I-shaped rectangular convective cavities inserted in a cylindrical body. Thus, the more complex the configuration, the better the system performance will be due to the redistribution of heat points. The study by Lorenzini et al., (2013), consists of investigating the T-shaped cavity. The convective heat flow becomes a boundary condition of the cavity surfaces. The results showed that the best geometry was that in which the stem and the affluent branch had a greater penetration in the domain of the solid. Later, Lorenzini et al., (2014), evaluated the same Y-shaped cavity. The evaluation of the first degrees of freedom was carried out through Exhaustive Search simulations for all geometric shapes and, for more complex problems, the optimization was performed with Genetic Algorithm (GA). In the work of Razera et al., (2019), an E-shaped fin cavity was investigated in order to find the geometry of the system that minimizes the maximum temperature. The geometry was evaluated for different boundary conditions, through the prescribed temperature and the dimensionless parameter of heat transfer by convection. The authors also observed that, with the increase of dimensionless parameter of heat transfer by convection, the heat transfer in the system was favored, with a performance in heat transfer similar to the condition obtained by the prescribed temperature boundary condition.

In this work, a numerical study was carried out with the objective of combine computational techniques with the Constructal Theory associated with the exhaustive search method to build a cooling cavity that has its shape optimized in order to minimize the maximum temperature of the solid, above all, maximizing the heat transfer between the solid and external environment. The main purpose of the study was to analyze the internal generation of heat that varies with time.

2. PROBLEM DESCRIPTION

The necessary procedures for the optimization of the I-shaped cavity are presented for the cooling of a solid body of constant conductivity with internal heat generation in a transient regime.

In this solution, for each geometry studied, the heat conduction equation is solved for a certain finite time. Therefore, the heat generation term, q_t^0, is time dependent and assumes the value

$$\tilde{T}_m = q^0 cos^2(wt + \alpha) \tag{1}$$

where q^0 is a constant heat generation [W], t is time [s], w is the frequency [s^{-1}] and α a dimensionless constants.

Geometry is subject to two restrictions: the total volume and the cavity volume. The solid of constant conductivity material has a fixed volume that is determined by L being the width, H the height and W the thickness. In this body, a cavity with dimensions L_0, H_0 and W is built, as shown in Fig. 1.

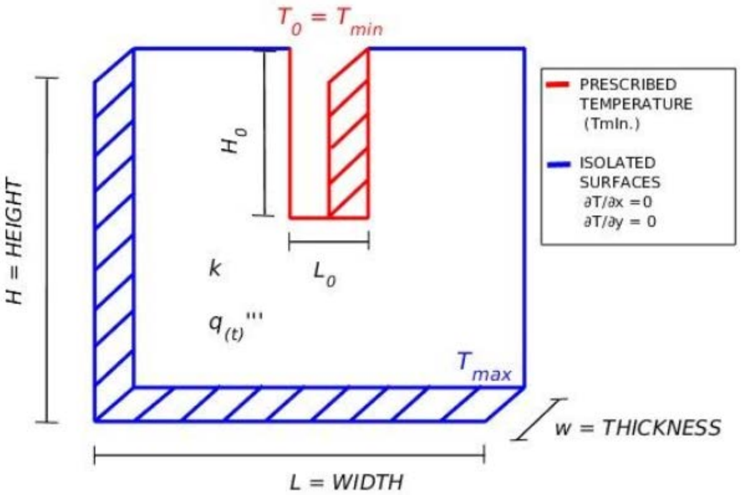

Figure 1 - I-shaped isothermal cavity.

For the present work, which will have a solid with internal heat generation in a transient regime, the energy equation that describes the physics of the problem, can be written in the form (Çengel, 2009).

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + q_t''' = \rho c\frac{\partial T}{\partial t} \tag{2}$$

where k is the thermal conductivity (W/mK), T the temperature (K), x and y are the Cartesian coordinates (m), q_t''' the volumetric internal heat generation (W/m^3), ρ the density (kg/m^3), c the specific heat (J/kg K) and t the time (s).

Based on the work of Almogbel and Bejan (1999), Ledezma, Bejan and Errera (1997) and Souza and Ordonez, (2011), the volumetric heat generation, q_t''' (W/m^3) is defined by:

$$q_t''' = \frac{q_t^0}{A_s W} = \frac{q_t^0}{(A - A_c)W} \tag{3}$$

where q_t^0 is the absolute heat generation as a function of time [W], A_s solid area [m^2], W the solid thickness [m] (W = 1 for 2D model), A for total area [m^2] and A_c the cavity area [m^2].

Combining the Eqs. (2 and 3) energy equation can be rewritten as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{q_t^0}{(A - A_c)W} = \rho c\frac{\partial T}{\partial t} \tag{4}$$

The boundary conditions for the proposed problem are determined as follows: the outer surface of the solid is isolated

$$\frac{\partial T}{\partial n} = 0 \tag{5}$$

where *n* is the normal direction to the surface.

Cavity walls are isothermal, i. e., constant temperature T_0 is set to them

$$T = T_0 \tag{6}$$

To solve the problem in a dimensionless form, the following variables must be defined:

$$(\tilde{x}, \tilde{y}) = \frac{(x,\ y)}{A^{1/2}} \tag{7}$$

$$\tilde{T} = \frac{T - T_c}{q^0 / kW} \tag{8}$$

$$\tau = \frac{kt}{\rho c A} \tag{9}$$

$$\Phi = \frac{A_c}{A} \tag{10}$$

$$\Omega = w \frac{\rho c A}{k} \tag{11}$$

The dimensionless shape of the total domain area can be expressed as

$$\tilde{H}\tilde{L} = \frac{HL}{A} = 1 \tag{12}$$

$$(\tilde{H}, \tilde{L}, \tilde{D}) = \frac{(H,\ L,\ D)}{A^{1/2}} \tag{13}$$

Energy equation (Eq. (4)) can now be rewritten as

$$\frac{\partial}{\partial \tilde{x}}\left(\frac{\partial \tilde{T}}{\partial \tilde{x}}\right) + \frac{\partial}{\partial \tilde{y}}\left(\frac{\partial \tilde{T}}{\partial \tilde{y}}\right) + \frac{1}{1 - \Phi} cos^2(\Omega\tau + \alpha) = \frac{\partial \tilde{T}}{\partial \tau} \tag{14}$$

subjected to the following boundary conditions

$$\frac{\partial \tilde{T}}{\partial \tilde{n}} = 0 \tag{15}$$

on external surfaces, and

$$\tilde{T} = 0 \tag{16}$$

on the cavity faces.

The thermal optimization of the problem has the objective of minimizing the resistance to heat flow, which in the dimensionless formulation is the dimensionless temperature given by Eq. (8).

2.1 Computational model

The transient heat conduction problem defined by Eq. (14) and the control conditions Eqs. (15-16) will be solved with the OpenFOAM software that uses the finite volume discretization in converting the continuous problem to the discrete problem. More specifically, the *laplacianFoam* solver will be used. The exhaustive search technique will be used to obtain the optimal geometry of the cavity.

Table 1 shows the main configurations used in the solution of the isothermal cavity problem.

Table 1 – OpenFOAM Control parameters for solution of the isothermal cavity.

Variable	**Parameter**
Software version	7.0
Algorithm	SIMPLE
Solvers Temperature	 PCG
Interpolation Schemes Transient Gradient Laplacians	 Euler Gauss linear Gauss linear corrected

2.2 Verification of the computational model

In the work of Biserni et al., (2004), the thermal resistance ($\tilde{T}$) was minimized in order to achieve the lowest maximum temperature for different values of Φ as a function of the geometry of the cavity, represented by the ratio between its height and its width, H_0/L_0.

The case used as reference was the geometry of the optimal I-shaped cavity, with a value of Φ = 0.9 for the $(H_0/L_0)_o$ = 0.94 relation where $\tilde{T}$ = 0.00011234 was obtained. The optimized geometry obtained by Biserni et al., (2004) is shown in Fig. 2.

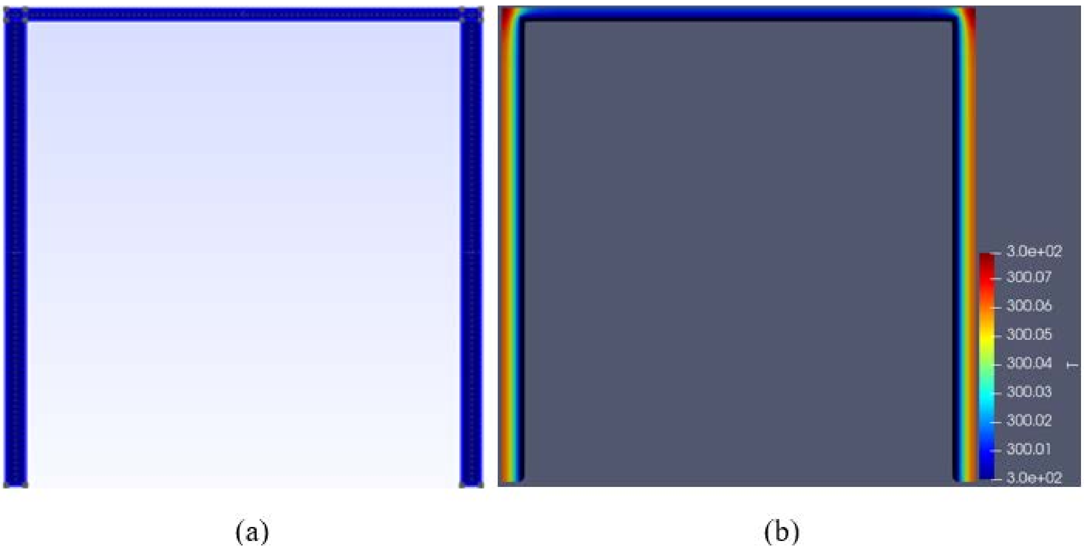

Figure 2 - Optimal cavity - Format for the lowest maximum temperature of the solid: (a) construction in the GMSH software; (b) visualization in the ParaView software.

The computational domain was discretized with the GMSH software. A mesh independence test was carried out to determine the maximum size of the mesh element that ensure a solution independent of discretization. It was assumed that a percentage difference of less than 1% was sufficient to ensure the quality of the results. The percentage difference was calculated by

$$D_{max} = \left| \tilde{T}^{j}_{max} - \tilde{T}^{j+1}_{max} \right| / \tilde{T}^{j}_{max} < 0.01 \quad (17)$$

Table 2 - Mesh independence test: maximum temperatures at each refinement for Φ = 0.9 and $(H_0/L_0)_o$ = 0.94.

MESH	ELEMENT SIZE	NUMBER OF ELEMENTS	$\tilde{T}_{max}$ [K]	D_{max}
1	0.007	3036	0.00011352	---------
2	0.005	7000	0.00011234	0.0104
3	0.003	20206	0.00011139	0.0084

According to Tab. 2, the precision criterion was reached in the first refinement, with a mesh of 7000 elements and an element size of 0.005. It is important to notice that the number of elements for each geometry will vary, however the size of the elements will be constant.

The solucion verification was performed through a direct comparison with the results presented by Biserni et al., (2004), for a cavity with H/L = 1 e Φ = 0.3. This comparison is shown in Tab. 3, where the results obtained with other software are considered in the comparison. It is important to highlight that, in this verification, the solution is permanent, that is, the transient term of Eq. (13) is zero and source term is 1/(1- Φ) only.

Table 3 - Verification of the numerical solution.

H_0/L_0	$\tilde{T}_{max}$ (FIDAP*)	$\tilde{T}_{max}$ (MATLAB*)	$\tilde{T}_{max}$ (OPENFOAM)
1.875	0.1873	0.1873	0.1864
1.2	0.1436	0.1435	0.1432
0.8334	0.10865	0.1086	0.1086
0.4686	0.06574	0.0657	0.0658
* Biserni et al., 2004.			

For the results of Biserni et al., (2004), shown in Tab. 3, both softwares uses the Finite Element Method (FEM) for the discretization of the transport equations.

3. RESULTS

The thermal optimization for the cavity shown in Fig. 1 consists of determining the temperature field for a number of configurations. The investigation is based on assigning some values of the relation A_c/A, which was called Φ, and for each value of this several values for the H_0/L_0 ratio will be tested.

From the results analysis, it can be seen that as the Φ value increases, the maximum temperature decreases, which is as a result of minimizing the overall thermal resistance. They are inferior to those obtained in the permanent solution because the total amount of energy supplied to the system is also small for a defined period of time. Figure 3 compares results for different values of Φ, both in permanent and transient regime. It is important to highligth that optimal geometry, the H_0/L_0 value for the lowest maximum temperature, is obtained with almost the total insertion of the isothermal cavity in the solid body.

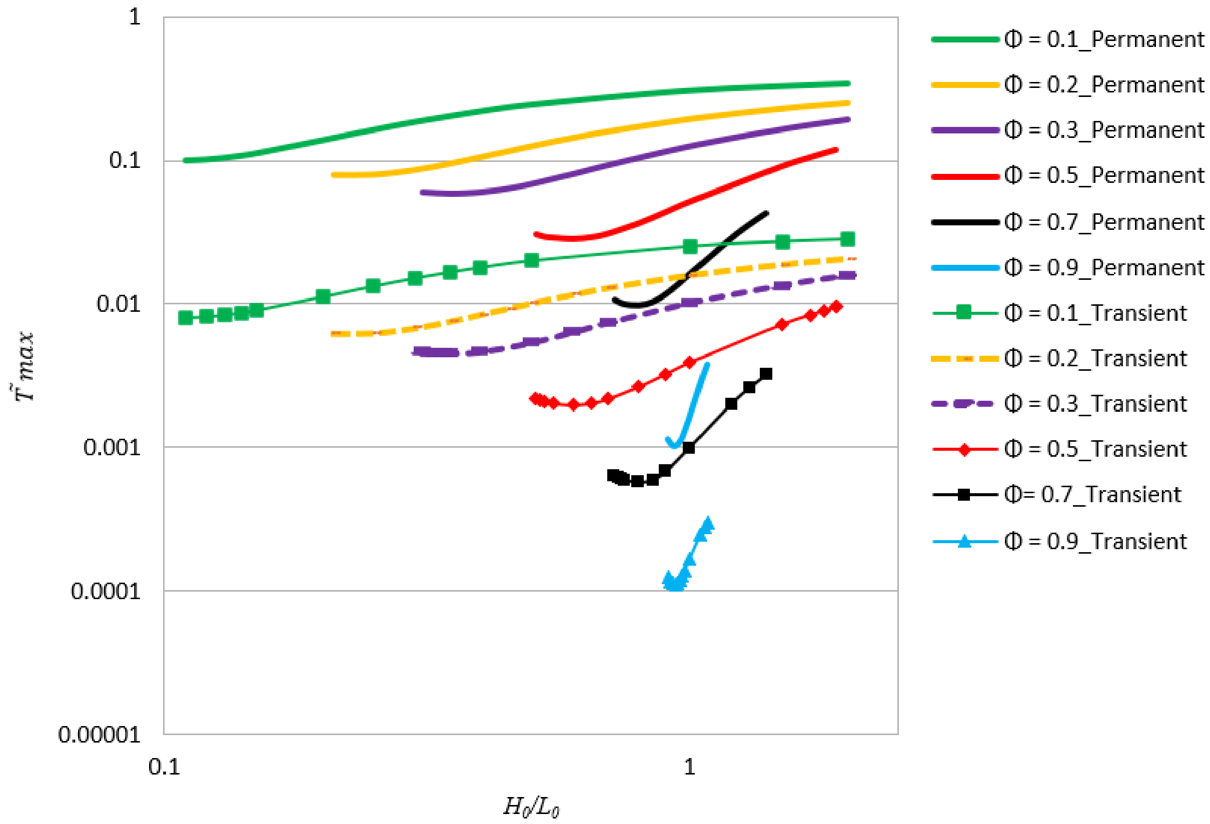

Figure 3 - Comparison of results for permanent and transient regimes.

Table 4 presents the values of Φ related to the optimal values of the H_0/L_0 ratio of the I-shaped cavity, as well as the minimum maximum temperatures obtained. Mesh 2 was used in all simulations, as shown in Tab. 2.

Table 4 - Optimization of the I-shaped cavity. Optimal values of $\tilde{T}_{max}$ for different values of Φ.

Φ	$(H_0/L_0)_o$	$(\tilde{T}_{max})_o$
0.1	0.11	0.00806819
0.2	0.21	0.00624677
0.3	0.35	0.00455751
0.5	0.6	0.00199175
0.7	0.8	0.00057212
0.9	0.94	0.00011234

Table 5 shows the maximum temperature values for each H_0/L_0 referring to Φ = 0.9 (optimized form) for both cases (permanent and variable heat source).

Table 5 - $\tilde{T}_{max}$ for Φ = 0.9 with permanent and variable heat sources.

H_0/L_0	$\tilde{T}_{max}$ (Permanent source)	$\tilde{T}_{max}$ (Variable source)
0.91	1.14992x10^{-3}	1.2474x10^{-4}
0.92	1.08321x10^{-3}	1.1775x10^{-4}
0.93	1.03523x10^{-3}	1.1337x10^{-4}
0.94	1.02898x10^{-3}	1.1234x10^{-4}
0.95	1.05107x10^{-3}	1.1455x10^{-4}
0.96	1.10715x10^{-3}	1.1976x10^{-4}
0.97	1.18340x10^{-3}	1.2806x10^{-4}
0.98	1.31378x10^{-3}	1.3865x10^{-4}
1.0	1.62455x10^{-3}	1.6590x10^{-4}
1.05	2.85289x10^{-3}	2.4964x10^{-4}
1.07	3.48140x10^{-3}	2.8219x10^{-4}
1.08	3.82489x10^{-3}	2.9846x10^{-4}

4. CONCLUSIONS

Based on the Constructal Theory, current research addresses a study of an isothermal cavity in the form of an I inserted into a solid body with internal heat generation in a transient regime, whose external surface is isolated from thermal exchange. The purpose is to minimize the maximum temperature body by decreasing the thermal resistance of the system so that heat transfer occurs from the volume of constant conductivity material to the external environment through the optimization of the geometry shape. With the application of the Constructal Theory, associated with an exhaustive search process, it was sought the best geometry that maximizes the performance of the system in terms of heat transfer. The system is cooled by direct contact between the cavity, which is kept at a minimum temperature and the solid material. The geometric configurations developed and investigated have the shape of a type I cavity capable of minimizing the temperature of the domain. For each constructed geometry, some fixed values of Φ related to different values of the H_0/L_0 ratio are assigned. For each value of Φ there is an optimal value of H_0/L_0 which corresponds to the minimum maximum temperature dissipated from the solid, which is minimized as the value of Φ increases. The best geometric cavity configuration that maximizes the heat transfer of the solid, by decreasing the overall thermal resistance, occurs when there is an increase in Φ related to the optimal H_0/L_0 ratio for this case. It is also evident that there is no total insertion of the optimal cavity in the solid wall when the lowest maximum temperature is found. From the transient analysis it was observed that the optimal shape of the cavity (Φ = 0.9 e H_0/L_0 = 0.94) is the same as that found with the permanent solution. However, the $\tilde{T}_{\max}$ in transient regime is lower than in the permanent regime because, for a given period of time, less energy is supplied to the solid. This behavior is explained by the fact that the cavity with the optimal geometry is one in which the resistance to heat flow is minimum and the results of $\tilde{T}_{\max}$ depend on the source of heat.

Therefore, it is noteworthy that the decrease in maximum temperatures is of great importance in all areas of heat generation, where the proper functioning of components and equipment that are increasingly compacted and of smaller dimensions depend on the maximization of thermal dissipation. The comparison of the results in a transient regime compared to the literature (permanent regime), means that the solution with the variable source in time influences the value of the maximum temperatures for this form of cavity. In view of the results, the transient regime has the best solution in cooling solid bodies with internal heat generation, where greater demands in terms of electrical current and operating frequency cause a high flow of heat in the components so that the thermal management becomes a fundamental part.

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