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AUTOPARAMETRIC NON-IDEAL SYSTEM AND VIBRATION ENERGY HARVESTING

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Abstract: There has been much recent interest on the study of autoparametric vibration absorbers for mechanical structural systems. Small-scale laboratory model were tested to gain understanding of the numerous requirements for optimum performance of these vibration suppression devices. They have many practical implementations in civil and mechanical engineering. In the other hand, the study of non-ideal vibrations has drawn special attention of engineering researchers in recent years. Let us consider a unbalanced motor on an elastic foundation. If we consider the region before resonance in the Frequency-Response curve of this system, we note that as the power supplied to the motor increases, its speed of rotation increases accordingly. However, this behavior doesn't continue indefinitely. The closer the motor speed moves toward the resonant frequency, the more power is required to increase the motor speed, as part of the energy is consumed moving the supporting structure. A large change in the power supplied to the motor results in a small change in its frequency and a large increase in the amplitude of the resulting elastic support oscillations. Thus, near resonance, it appears that additional power supplied to the motor only increases the amplitude of the response of the supporting structure while having little effect on the RPM of the motor. Another area of recent interest is Energy Harvesting. In this process, electrical energy is obtained through conversion of mechanical energy from an ambient vibration. In our application, a non-ideal vibration source and a vibration absorber system using piezoelectric material of linear of a nonlinear type is used. The employed methodology to carry out the analysis of this work is as follows. We present the derivation of coupled nonlinear differential equations of the three subsystems. The motor-structure subsystem is defined as a nonideal vibrating system that consists of a nonlinear cantilever beam excited by a small dc motor with eccentric mass, with limited power supply and located in the opposite extremity to the fixed side. The second subsystem is defined as an autoparametric vibration absorber that consists of a flexible beam with tip mass perpendicularly coupled to cantilever beam in its free extremity. The third subsystem consists of piezoelectric devices installed in the base of the absorber beam and an electric circuit is connected to the piezoelectric material in order to produce usable voltage out-put. Next, several numerical simulations were carried out focused on passage through resonance, when the motor rotational frequency is near the cantilever beam natural frequency and when the non-ideal subsystem frequency is approximately twice the absorber beam frequency (two-to-one internal resonance).

Keywords: Autoparametric vibration absorber, non-ideal system, energy harvesting

INTRODUCTION

In practical situations, the dynamics of a mechanical system with a forcing source cannot be considered as given a priori, as the excitation can be influenced by the dynamics of the whole system. This is particularly important when the source has a limited-power supply, as occurs with a DC electric motor, for example. Thus, the dynamics of the power supply is influenced by the response of the system being forced. The study of non-ideal vibrating systems when the excitation is influenced by the response of the system has been considered a major challenge in theoretical and practical engineering research. There are more details on non-ideal systems theory in Kononenko (1969), Balthazar *et al.* (2003), Felix *et al.* (2009), Warminski *et al.* (2001), Nayfeh and Mook (1979), among others. Other previous researchers have detected the Sommerfeld effect (Samarantay *et al.*, 2010; Felix *et al.*, 2009) in structural models excited by non-ideal power supply: as the motor accelerates to reach near resonant conditions, a considerable part of its output energy is consumed to generate large amplitude motions of the structure and not to increase its own angular speed. For certain parameters of the system, the motor can get stuck when in resonance not having enough power to reach at higher rotation regimes. If there is any more power, jump phenomena may occur from near resonance to considerably higher motor speed regimes and no stable motions is possible between these two.

The resulting responses could include energy transfer from one mode of the system to another or saturation phenomenon was studied in non-ideal problem depending of the physical and geometric properties of the portal frame to tune the natural frequencies of the two main modes into a 1:2 internal resonance and the non-ideal excitation frequency is near of the second natural frequency (Palacios *et. al.*, 2002; Brasil *et. al.*, 2001). The implementation of the active saturation control to suppress steady-state vibrations of a non-ideal simple portal frame structure was proposed by Felix *et. al.* (2005) connecting it to a second-order controller using quadratic position coupling terms. Other example of these systems that saturation phenomenon was applied are the L-shape beam structure by Haddow *et. al.* (1984), Bux and Roberts (1986) and the nonlinear saturation controller was applied for nonlinear beam (Mitura and Kecik, 2016; Hamed and Amer, 2014; Xu *et.al*., 2015). The nonlinear energy harvesting material in non-ideal problems with saturation phenomenon was introduced as in Iliuk *et al.* (2013a,b).

Autoparametric vibration absorber has been used to reduce vibrations in single-degree-of-freedom structures (Lee and Cho, 2000) and to reduce vibrations in non-ideal systems (Sado and Kot, 2007), while attaching an energy harvester to an autoparametric vibration response (Erturk and Inman, 2008; Kecik and Borowiec, 2013, Jia and Seshia, 2014) would be of interest for the purposes of enhancing its control effects and use of the harvested power. In this paper, we described results from ambient vibration harvesters that utilize the two beam coupling of type L-shaped beam where the horizontal beam will be non-ideal system while the vertical beam will be the piezoelectric energy harvester. The two beams are tuned to a natural frequency ratio of 2:1. The aim of this work is to tune the passive autoparametric vibration absorber to cancel the non-ideal beam vibration and produce maximum vibration of the absorber vertical beam to act as an energy harvester.

MODELING OF THE AUTOPARAMETRIC NON-IDEAL SYSTEM AND ENERGY HARVESTER

The energy harvesting/autoparametric non-ideal problem consists of a horizontal cantilever beam of supporting an unbalanced motor with limited power at its free end of ($x_1, m_1, \varphi, m_0, r$) displacement, mass, angular displacement, unbalanced mass, eccentricity. Furthermore, a vertical cantilever beam is coupled with tip mass based on piezoelectric material of (x_2, m_2) displacement and mass, as illustrated in Fig. 1.

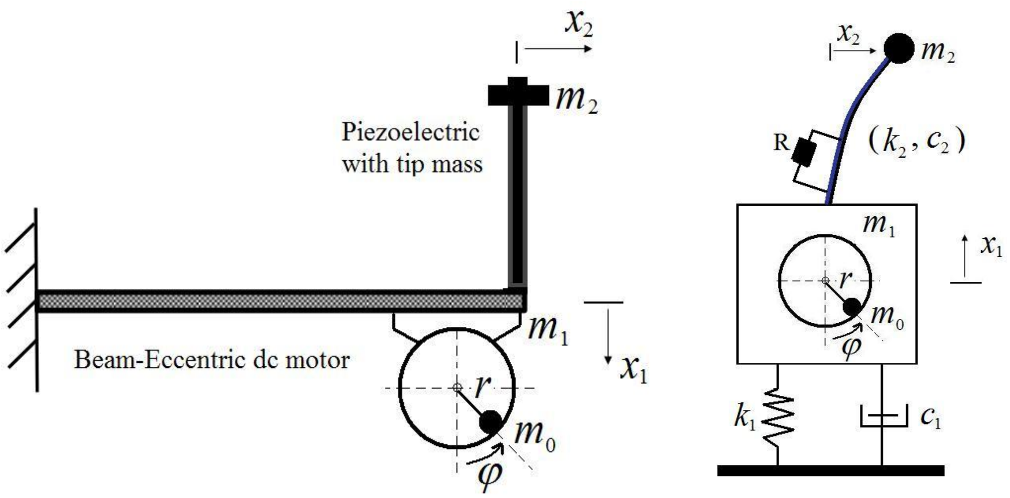


Figure 1 - Schematic diagram of autoparametric non-ideal system and energy harvester.

The governing equations of motion of the non-ideal system will be considered as the simplified model of Felix *et al.* (2005) and the modeling of the autoparametric vibration absorber proposed by Lee and Cho (2000). The modeling of the piezoelectric energy harvester is considered as in Erturk *et al.* (2009). The governing equations of motion of the coupled system are given in dimensionless form in Eq. (1).

$$
\begin{aligned}
&\ddot{u}_1 + \mu_1\dot{u}_1 + u_1 - \delta(u_2\ddot{u}_2 + \dot{u}_2^2) = \gamma_1(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) \\
&\ddot{u}_2 + \mu_2\dot{u}_2 + \omega^2 u_2 + \rho\delta^2(u_2\ddot{u}_2 + \dot{u}_2^2)u_2 - \chi_1 u_3 = \delta u_2\ddot{u}_1 \\
&\ddot{\varphi} = \Gamma(\dot{\varphi}) + \gamma_2\ddot{u}_1\sin\varphi \\
&\dot{u}_3 + \lambda u_3 + \chi_2\dot{u}_2 = 0
\end{aligned}
\tag{1}
$$

where the dot denote differentiation related to the dimensionless time $\tau = \omega_1 t$. $\Gamma(\dot{\phi}) = a - b\dot{\varphi}$, a is the dimensionless motor torque that can be changed according to the voltage of the DC motor, b is a dimensionless constant for each model of DC motor considered and $\dot{\varphi}$ is the angular velocity (speed of rotation) of the motor. The static displacement is given as $x_0 = \frac{(m_0 + m_1 + m_2)g}{k_1}$ and the harvested voltage across the load resistance is denoted by $u_3 = V$. The dimensionless displacements of the beams are defined as $u_1 = \frac{x_1 + x_0}{x_0}$, $u_2 = \frac{x_2}{x_0\sqrt{m_0 + m_1 + m_2}}$. The dimensionless parameters are defined as $\delta = x_0\eta_1$, $\rho = \frac{\eta_2(m_0 + m_1 + m_2)}{\eta_1^2}$ (where η_1 and η_2 are coupling constants), $\gamma_1 = \frac{m_0 r}{(m_0 + m_1 + m_2)x_0}$, $\mu_1 = \frac{c_1}{(m_0 + m_1 + m_2)\omega_1}$, $\mu_2 = \frac{c_2}{m_2\omega_1}$, $\omega = \frac{\omega_2}{\omega_1}$ (where $\omega_1 = \sqrt{\frac{k_1}{m_0 + m_1 + m_2}}$ and $\omega_2 = \sqrt{\frac{k_2}{m_2}}$), $\gamma_2 = \frac{m_0 r x_0}{I + m_0 r^2}$. $\chi_1 = \frac{\theta}{x_0\omega_1^2\sqrt{m_0 + m_1 + m_2}}$, $\chi_2 = \frac{\Theta x_0\sqrt{m_0 + m_1 + m_2}}{C_p}$, $\lambda = \frac{1}{RC_p\omega_1}$ (where R is the load resistance; C_p is the capacitance of the piezoelectric layer; θ and Θ are the electromechanical coupling coefficients that depending of material constant and design of energy harvester (Sodano *et al.*, 2004).

The harvested power (P) is given by Eq. (2).

$$P = \frac{V^2}{R} \tag{2}$$

NUMERICAL SIMULATIONS AND DISCUSSIONS

In this section, the numerical results used to illustrate the proposal of this work. A numerical model of the presented autoparametric non-ideal system, using the system parameter values shown in Table 1, was constructed in MATLAB with ODE23 solver in the steady variables $X_1 = u_1$, $X_3 = u_2$, $X_6 = \dot{\varphi}$ and $X_7 = u_3$.

Table 1 – Properties of the energy harvester auroparametric non-ideal system.

Parameters	Values
μ_1; μ_2	0.1; 0.007
δ; ρ	3.1; 6.2
γ_1; γ_2; b	0.001; 13.33; 0.5
χ_1; χ_2; λ	0.03; 0.05;0.01

Therefore, the next subsection will show the numerical results provided by the governing equations of motion of the non-ideal system in Eq. (1).

Autoparametric non-ideal system numerical results

In this section, the simulations were without considering the energy harvesting material, will be presented some discussions about numerical simulations of the considered autoparametric non-ideal system. Figure 2 presents an example for case a = 0.545 of resonance capture (Sommerfeld effect) where the angular velocity (speed of rotation) is in resonance with natural frequency of the non-ideal beam ($X_6 = \dot{\varphi} \approx 1$).

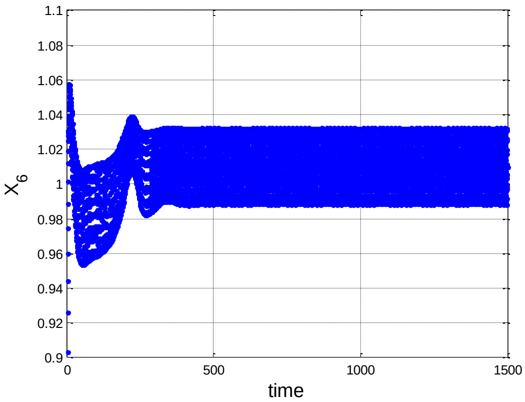
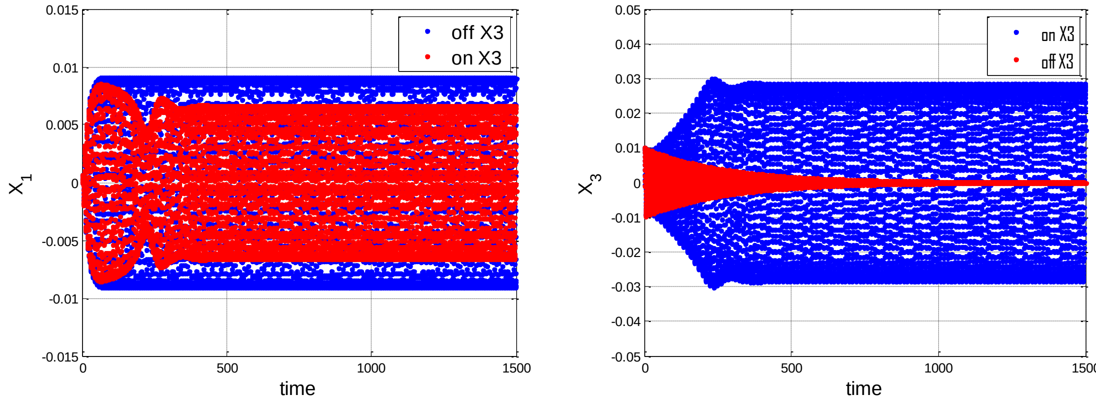


Figure 2 – Angular velocity for the case with resonance capture.

Figure 3 presents the numerically simulated time domain response of both non-ideal horizontal beam (X_1) and the autoparametric vibration absorber (X_3 controller). The control technique based on the saturation phenomenon suppressed the high amplitude of the first-mode vibration of a reduced non-ideal horizontal beam. Because of the two-to-one internal resonance condition kept between the non-ideal beam and the vertical beam controller (ω = 0.5 ↔ $\omega_1 = 2\omega_2$), the amplitude of the non-ideal beam becomes saturated and its vibrational energy in excess was partially-transferred to the vertical beam in the time range [50, 1500].



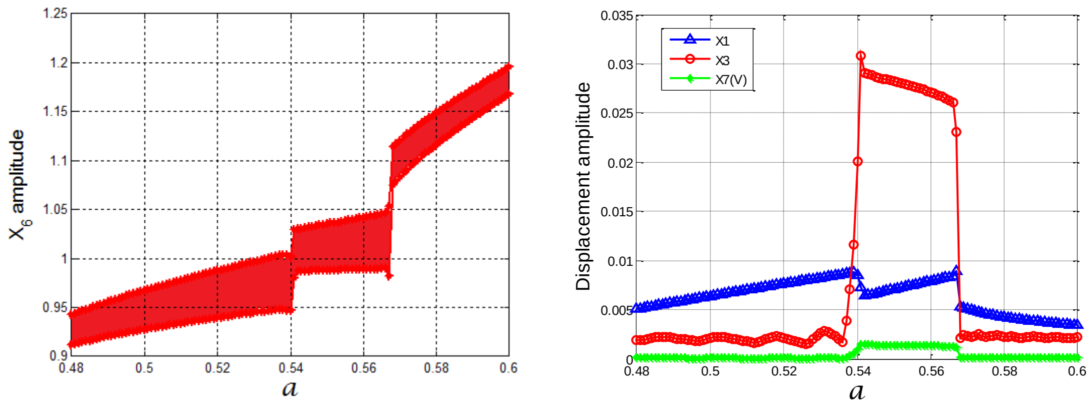
(a) (b)

Figure 3 – Displacement of the system with saturation phenomenon.

Since the control technique suppressed the high amplitudes of the non-ideal beam and the vertical beam becomes higher than the non-ideal one, a piezoelectric material was coupled to harvest energy from the vibration energy of the vertical beam, which will be detailed in the next subsection.

Energy harvesting strategy numerical results

Figure 4 shows the steady state responses of system, represented by Eq. (1), during passage through fundamental resonance region ($\dot{\varphi} \approx 1$) corresponding to output of X_6 may be analyzed through a control parameter a with an amplitude diagram. Each amplitude consists of the maximum absolute value of the amplitude oscillation of X_1, X_3 and X_7 that are the output of the horizontal beam, vertical beam and the harvested voltage, respectively. The control parameter a, depending of voltage applied to motor, is applied in the range $0.48 \le a \le 0.6$, considering an increment $\Delta a = 0.001$. The numerical simulations were carried out in the dimensionless time domain $0 \le \tau \le 1500$.



(a) (b)

Figure 4 – Sommerfeld effect: a) X_6 angular velocity and saturation phenomenon: b) X_1 horizontal beam, X_3 vertical beam, X_7 voltage (V)

Figure 4a shows the Sommerfeld effect. It is noted that as the power supplied to the motor increases, its speed of rotation (angular velocity $\dot{\varphi}$) increases indefinitely on $0.48 \le a < 0.54$. The closer the motor speed moves toward the resonant frequency ($\dot{\varphi} \approx 1$), the more power is required to increase the motor speed, as part of the energy is consumed moving the supporting horizontal beam. A large change in the power supplied to the motor results in a small change in its frequency and a large increase in the amplitude of the resulting elastic support oscillations. Thus, passage through resonance is observed in $0.54 \le a \le 0.566$. With the additional power supplied to the motor, it is observed that the angular velocity continues to be captured in the resonance range while the amplitude of the response of the supporting horizontal beam is saturated and then transferred to the vertical beam due to saturation phenomenon, as it is possible to see in Fig. 4b.

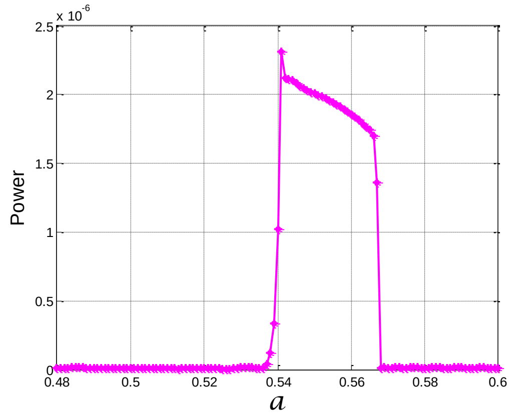


Figure 5 – Electric power from vertical beam and non-ideal source.

As in the resonance range the saturation phenomenon occurs, the energy harvesting is possible as shown in Fig. 5. In this range, the harvested power is improved from approximately zero to an interval of $1.2 < P < 2.5$ (Erturk *et al*., 2009).

As the energy harvesting was possible, some conclusions can be taken out.

CONCLUSIONS

In summary, we have investigated the dynamical behaviour of two beam coupling of type L-shaped beam where the horizontal beam is a non-ideal system while the vertical beam is the piezoelectric energy harvester. The two beams are tuned to a natural frequency ratio of 2:1. The aim is to tune the absorber to cancel the non-ideal beam vibration and produce maximum vibration of the absorber vertical beam to act as an energy harvester.

With the natural frequency ratio 2:1, the system was set to a two-to-one internal resonance which is an important condition to saturation phenomenon occurs. When the unbalanced motor was in resonance with the non-ideal beam, the saturation occurred and transferred part of its vibrating energy to the vertical beam, making possible to harvest energy from its vibration.

The linear torque of the motor was set as a control parameter to a general analysis of the resonance region. Moreover, Sommerfeld effect was detected. With the maintain of resonance from the Sommerfeld effect, the electric power output had wide bandwidth.

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