

ENC-2020-0593

**APPLICATION OF DIFFUSIVE PARAMETER IDENTIFICATION
TECHNIQUES FOR WOOD DRYING USING OPTIMIZATION METHODS**

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Abstract. *Drying is an essential process before the manufacture of any wood component, part or product. The prevention of defects in drying operations is of fundamental importance due to the potential reduction of the product value caused by the quality reduction of the material. Most of these defects are caused by large tension/deformation gradients which occur by the moisture difference between the surface and the center of the wood. Therefore, careful process control and knowledge of the wood properties are essential for a successful drying operation. In the context of operations using greenhouses, the drying process involves a problem fully coupled with the phenomena of heat and mass transfer (moisture), which demand the knowledge of heat and moisture diffusivity parameters. This paper presents a discussion on the application of optimization methods in the determination of moisture diffusivity parameters. The direct problem, fully coupled, simultaneously solves the energy and mass conservation equations based on the finite volume method. On the other hand, the inverse problem is solved in the time domain using the methods of particle swarm optimization, genetic algorithm and the Nelder-Mead method.*

Keywords: *drying, finite volume method, particle swarm optimization, genetic algorithm, Nelder-Mead;*

1. INTRODUCTION

Drying is one of the processes that has an important value for the wood industry. Reducing the moisture content is essential for most wood use. This reduction influences the material properties, such as dimensional stability, increased mechanical strength, among others. The execution of this process requires large investments and high energy consumption, resulting in large costs. Thus, the intention of the researchers is to improve the drying process without any defects that may interfere with the final use of the product.

So, to improve the drying process, it is essential to have knowledge about the physical processes involved in the heat and mass transfer. Throughout the wood drying process, in general, heat is transferred from the air to the surface and conducted from the surface into the material. The convective coefficient is associated with the heat resistance in the transfer to the surface of the wood, while the resistance to heat conduction inside the wood is influenced by the material thermal conductivity.

The loss of moisture is a consequence of the mass transfer in the material, with the water movement from the interior to the surface through the capillarity mechanisms and transport from the surface to the air by evaporation. Therefore, the resistance to evaporation is related to the convective coefficient of mass and diffusion within the wood. To improve the drying process, the use of mathematical models is an alternative with the aim of helping to reduce costs, time, energy and drying defects.

The first attempt to quantify and also describe the drying of the wood was by the application of the mass diffusion model and, later, by the use of models that considered the coupled heat and mass phenomena. Therefore, the advantages of using these mathematical models come from the ease of use and analysis of the involved parameters sensitivity, to a better analysis of the process

The use of computer programs for the simulation of drying processes presupposes the knowledge of wood properties and the parameters of heat exchange and mass transfer. Due to the coupled nature of the phenomena involved, the determination of heat and mass transfer parameters is not immediate, requiring reliable and robust numerical-experimental techniques. This paper proposes the use of inverse methods to determine these parameters.

Using a common definition, inverse methods are applied when it is desired to determine the causes from the consequences or effects. In the present case, we seek to determine the parameters of heat and mass exchange (cause), knowing the loss of moisture over time (effect). The solution of this class of problems also requires the application of direct methods, that is, the mathematical modeling (and numerical implementation) of the phenomena involved. In the present case, the Finite Volumes Method (FVM) of governing equations discretization (conservation of energy and mass) is applied to the solution of the direct problem. In solving inverse problems, according to Gesualdo (2005), optimization has been one of the most important tools that has been applied in several fields of engineering.

The inverse problem methods have been increasingly used in the study and determination of model parameters, where computer simulations and experimental procedures are not carried out in isolation, but simultaneously in order to obtain the maximum information about the physical problem in the analysis. In this context, some optimization methods can be mentioned, such as: genetic algorithms, particle swarm methods, and Nelder-Mead among others.

Therefore, the purpose of this article is to apply inverse methods and analyze the results of solution time (number of steps) and the possibility of obtaining the global minimum (desired wood and process parameters). This paper objective is to apply these methods to a heat and mass transfer problem to determine the heat exchange coefficient and the diffusive parameters of wood drying.

For this article, the initial drying data form the curve of mass loss over time of a sample and its surface temperature, from which the parameters of heat and mass exchange will be determined. The direct problem is solved by the Finite Volumes Method, while the inverse problem is solved by Particle Swarm Optimization (PSO), generic algorithms (GA), Nelder-Mead (NM) and a global-local hybrid method combining PSO or GA with the Nelder-Mead method.

2. HEAT AND MASS TRANSFER

The processes of heat and mass transfer can be quantified based on equations of appropriate rates, which are used to calculate the amount of energy and mass transferred per unit time. Fourier's law is the rate equation used for thermal conduction, and for the mass transfer process Fick's law is applied. According to Incropera et. al. (2019), performing a differential control volume dx , dy and dz , and considering the generation and variation of moisture over time, a mass balance is performed in order to obtain the moisture diffusion equation. After using the Taylor series expansion and making components of the control volume output with truncation, and rearranging the equations, the diffusion equation for Cartesian coordinates is obtained as

$$\frac{\partial}{\partial x} \left(D_M \frac{\partial M}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_M \frac{\partial M}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial M}{\partial z} \right) + n_m = \frac{\partial M}{\partial t}. \quad (1)$$

The same procedure is performed to obtain the differential equation of thermal diffusion for Cartesian coordinates, as

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q = \rho c p \frac{\partial T}{\partial t}. \quad (2)$$

According to Incropera et al (2019), for mass and heat transfer phenomena it is necessary to define boundary conditions for the differential equation that describe it. The solution depends on the physical conditions existing at the borders, in the environment, including possible variation with time. The conditions of Dirichlet and Neumann are not addressed in this paper, and the Robin's condition will be used in this article, as described in equations (3) and (4), as

$$-D_M \frac{\partial M}{\partial x} \Big|_{x=0} = h_M [M(x=0, t) - M_\infty], \quad e \quad -D_M \frac{\partial M}{\partial x} \Big|_{x=L} = h_M [M(x=L, t) - M_\infty], \quad (3)$$

$$-H_M \frac{\partial T}{\partial x} \Big|_{x=0} = h_M [T(x=0, t) - T_\infty], \quad e \quad -H_M \frac{\partial T}{\partial x} \Big|_{x=L} = h_M [T(x=L, t) - T_\infty]. \quad (4)$$

The transient heat and mass diffusion processes in a timber layer are modelled by a one-dimensional formulation.

3. HEAT AND MASS TRANSFER COUPLING

The coupling between the heat and mass transfer equations is based on the dependence of mass diffusivity with temperature and the relation between heat and mass exchange coefficients by convection.

The thermal diffusivity equation adopted in this study was used by Musch et. al. (1998) and considers that mass diffusivity depends on temperature according to Arrhenius' law,

$$D_M = D_M(T) = D_\Gamma \exp \left(-\frac{D_\epsilon}{T} \right), \quad (5)$$

where D_M and D_Γ are material parameters characteristic of wood. The relation between the coefficients of heat exchange and mass by convection is established by the analogy between the thermal and mass limit layers, $\delta T/\delta M \approx Le^{1/3}$ (Bergman et al., 2011), so that

$$h_M = h_c \left(\frac{D_{AB} Le^{1/3}}{k_a} \right), \quad (6)$$

where $Le = \alpha/D_{AB}$ is the Lewis number, D_{AB} is the diffusion coefficient of chemical component A (humidity) in chemical component B (air) and k_a is the thermal conductivity of the air.

4. SOLUTION OF DIFFERENTIAL EQUATIONS - DIRECT METHOD

Since the objective of this paper is the study of the different optimization strategies in the solution of the inverse problem, the solution of the direct problem is obtained from a one-dimensional approximation of the physical problem. The discretization uses the Finite Volume Method described in (Maliska, 2004). The thermal diffusion equation can be represented as

$$A_p^T T_p = A_e^T T_e^\theta + A_w^T T_w^\theta - (A_e^T + A_w^T) T_p^\theta + A_p^{T,0} T_p^0, \quad (7)$$

and, similarly, the discretization of the diffusion equation in time and space can be represented by

$$A_p^M M_p = A_e^M M_e^\theta + A_w^M M_w^\theta - (A_e^M + A_w^M) M_p^\theta + A_p^{M,0} M_p^0. \quad (8)$$

The boundary conditions discretization is described in more detail by Maliska (2004) and Patankar (1980).

5. PARAMETER IDENTIFICATION - INVERSE METHODS

The inverse problems of heat and mass transfer make use of temperature and / or mass measurements, in order to estimate unknown parameters in the analysis of physical problems. Therefore, this study adopts a research model where computer simulations and experimental measurements are not carried out isolated, but in a coupled way, in order to obtain the most realistic information about the material and process parameters.

The present inverse problem consists in determining the coefficients of the mass diffusivity equation and the heat exchange coefficient from the measurement over time of the total sample mass and the surface temperature. Due to the different diffusion rates, the temperature measurement time is shorter than that of the mass (the sample comes into thermal equilibrium with the environment more quickly).

The objective function associated with the mass diffusion problem, $g^M(\mathbf{p}^M)$, evaluates the relative difference between the mass of the wood sample measured experimentally and the corresponding mass calculated numerically during the total drying time,

$$g^M(\mathbf{p}^M) = \sqrt{\frac{1}{N_M} \sum_{i=1}^{N_M} \left(\frac{m_{num}^T - m_{exp}^T}{m_{exp}^T} \right)^2}. \quad (9)$$

The objective function for the thermal problem, $g^T(\mathbf{p}^T)$, evaluates the difference between the surface temperature of the sample measured experimentally and the corresponding temperature calculated numerically during the first hours of drying,

$$g^T(\mathbf{p}^T) = \sqrt{\frac{1}{N_T} \sum_{i=1}^{N_T} \left(\frac{T_{num}^S - T_{exp}^S}{T_{exp}^S} \right)^2}. \quad (10)$$

The identification of the mass diffusion and heat transfer parameters is done jointly through the use of a combined objective function, defined in this paper by

$$g(\mathbf{p}) = \lambda^T g^T(\mathbf{p}^T) + \lambda^M g^M(\mathbf{p}^M). \quad (11)$$

In this way, the methods that will be used to solve the inverse problem will be presented briefly throughout the next sections.

5.1 Genetic algorithm method

The genetic algorithm (GA) method is a heuristic computational technique used to find approximate solutions in search and optimization problems. This method was developed thinking according to how living beings survive and pass their genetic material on to future generations, using the principles of natural selection proposed by Charles Darwin (Correa, 2000; Mitchell, 1996).

The basic structure of the method consists of two fundamental steps: selection and reproduction, that is, application of genetic operators (Stahlschmidt, 2010).

In the GA method, the population is formed by a set of individuals (potential solutions) represented by a chromosome (vector of design variables, \mathbf{p}_m) and genes (design variables p_i) successively redefined until a convergence criterion is reached (Vaz Jr. et al, 2013).

The first step is the random definition of the initial population delimited by the search space (lower and upper limits of each parameter),

$$\mathbf{p}^{(0)} = \{p_1^{(0)}, p_2^{(0)}, \dots, p_m^{(0)}, \dots, p_{n_p}^{(0)}\}. \quad (12)$$

Reaching a set of potential solutions n_p , so that each variable of an individual project \mathbf{p}_m corresponds to a single material parameter (Vaz Jr. et al, 2013) using binary coding,

$$\mathbf{p}_m^{(0)} = \mathbf{p}_m^{D,(0)} \cup \mathbf{p}_m^{T,(0)} = \{p_1^{(0)}, p_2^{(0)}, \dots, p_i^{(0)}, \dots, p_{n_s}^{(0)}\}, \quad (13)$$

where p_i is a design variable and superscripts D and T indicate the parameters of the mass diffusion and heat conduction problems, respectively.

In general, in the method of genetic algorithms some steps are followed sequentially. Thus, before selecting the individuals to be used in the reproduction phase, according to Goldberg and Sastry (2011), a method called scaling is applied, which has the objective of avoiding super individuals from dominating the reproduction process.

5.2 Particle swarm optimization

To implement the PSO method were used inertial, social and cognitive components. It was used populations with particles number n_p with “ n ” parameters (Beppler Jr., 2014), where \mathbf{p}_m represents an individual particle, so that

$$\begin{aligned} \mathbf{p}^{(k)} &= \{p_1^{(k)}, p_2^{(k)}, \dots, p_m^{(k)}, \dots, p_{n_p}^{(k)}\} \\ \mathbf{p}_m^{(k)} &= \mathbf{p}_m^{D,(k)} \cup \mathbf{p}_m^{T,(k)} = [p_1^{(k)}, p_2^{(k)}, \dots, p_i^{(k)}, \dots, p_{n_s}^{(k)}]^T \end{aligned} \quad (14)$$

is a vector formed by the coordinates p_i of the particles in the space of the design variables, that is, it is composed of the parameters to be identified. The population particle velocity set is given by equation (15) and with each element of the set represented by the velocity of an individual particle,

$$\begin{aligned} \mathbf{v}^{(k)} &= \{v_1^{(k)}, v_2^{(k)}, \dots, v_m^{(k)}, \dots, v_{n_p}^{(k)}\} \\ \mathbf{v}_m^{(k)} &= [v_1^{(k)}, v_2^{(k)}, \dots, v_i^{(k)}, \dots, v_{n_s}^{(k)}]^T \end{aligned} \quad (15)$$

Thus, the speed of each particle has the social and cognitive inertial components and new position of the population particles are evaluated by

$$\begin{aligned} \mathbf{v}^{(k)} &= w\mathbf{v}^{(k)} + C_1 R_1 \otimes (\mathbf{p}_{lbest}^{(k)} - \mathbf{p}^{(k)}) + C_2 R_2 \otimes (\mathbf{p}_{gbest}^{(k)} - \mathbf{p}^{(k)}) \\ \mathbf{p}^{(k+1)} &= \mathbf{p}^{(k)} + \mathbf{v}^{(k)} \end{aligned} \quad (16)$$

It must be limited the use of search before joining the method. Because if the updated position is outside the established limits, the component must receive the boundary value (Beppler Jr., 2014).

5.3 Nelder-Mead method

Another optimization method that does not use gradient concepts is the Nelder-Mead (NM) method, proposed in 1965 by Nelder and Mead (Vaz Jr. et al., 2015).

According to Vaz Jr. et al (2015), the Nelder-Mead method defines a simplex with $n + 1$ vertices, (in a n dimension project space) that moves towards the optimum by replacing the worst vertex with its reflection around the centroid of the hyperplane, composed of the remaining vertices. Other operations are made with the simplex in order to obtain an even better vertex than the previous reflection and, thus, increase the convergence rate.

The Nelder-Mead algorithm has three important elements:

- a. The creation of an initial simplex based on an initial estimate $\mathbf{p}^{(0)}$;
- b. Search for a new vertex along a certain direction and formation of a new simplex by replacing the worst vertex, $\mathbf{p}_{(n+1)}^{(k)}$, with the new one chosen from the following possible operations: reflection, $\mathbf{p}_r^{(k)}$, expansion, $\mathbf{p}_e^{(k)}$, internal contraction, $\mathbf{p}_{ic}^{(k)}$, or external contraction, $\mathbf{p}_{ec}^{(k)}$, as defined respectively by (Vaz Jr. et al., 2015),

$$\begin{aligned} \mathbf{p}_r^{(k)} &= \mathbf{p}_0^{(k)} + \rho(\mathbf{p}_0^{(k)} - \mathbf{p}_{n+1}^{(k)}) \\ \mathbf{p}_e^{(k)} &= \mathbf{p}_0^{(k)} + \gamma(\mathbf{p}_r^{(k)} - \mathbf{p}_0^{(k)}) \\ \mathbf{p}_{ic}^{(k)} &= \mathbf{p}_0^{(k)} + \beta(\mathbf{p}_{n+1}^{(k)} - \mathbf{p}_0^{(k)}) \\ \mathbf{p}_{ec}^{(k)} &= \mathbf{p}_0^{(k)} + \beta(\mathbf{p}_r^{(k)} - \mathbf{p}_0^{(k)}) \end{aligned} \quad (17)$$

where, $\mathbf{p}_0^{(k)}$ is the centroid of the hyperplane formed by the vertices $\mathbf{p}_1^{(k)} \dots \mathbf{p}_n^{(k)}$ and ρ, γ, β are reflection, expansion and contraction control coefficients (Vaz Jr. et al., 2015).

- c. If the new vertex is worse than the worst vertex, $\mathbf{p}_{(n+1)}^{(k)}$, the simplex is shrunk towards the best vertex.

Convergence of the method is defined from a normalized index, calculated using the objective function of the worst, $\mathbf{p}_{(n+1)}^{(k)}$, of the best, $\mathbf{p}_1^{(k)}$, vertices of the simplex,

$$\phi_{NM}^{(k)} = \frac{g(\mathbf{p}_{n+1}^{(k)}) - g(\mathbf{p}_1^{(k)})}{g(\mathbf{p}_{n+1}^{(0)}) - g(\mathbf{p}_1^{(0)})}. \quad (18)$$

5.4 Hybrid method

According to Stahlschmidt (2010), hybrid methods consist of a combination of two or more optimization methods. This combination of methods is mainly used to improve the quality of the results and to increase the convergence rate of the optimization process. In this paper will be used the method known as global-local, which combines population-based methods (global step), such as genetic algorithms or the particle swarm method, followed by the application of the Nelder-Mead method (local step) using the best determined individual/particle when a transition criterion is found.

In the first step, the application of population-based methods makes it possible to reduce the problem domain to a region close to the global minimum of the problem. The second step aims to reach the minimum of the problem more quickly.

6. RESULTS AND DISCUSSION

The idealized problem consists of determining parameters of heat and mass exchange knowing the moisture loss and temperature evolution over time in a wood sample.

Solution of this class of problems also requires application of direct methods, that is, the mathematical modeling (and numerical implementation) of the phenomena involved. For this paper, was used the finite volumes method which required some data for implementation, shown in Table 1.

6.1 Nelder-Mead method application to the problem

The analysis of the Nelder-Mead method consists in the evaluation of success rate in obtaining the reference parameters for 20 random initial estimates. The initial estimates of D_r , D_e and h_c were generated within a search range, which was used for all applications.

For the NM method was studied the application of the initial point at the vertex and centroid of the simplex. It was noticed that for the initial estimate at the simplex vertex the results achieved were better than the application at the centroid. Thus, it was found that the Nelder-Mead method has a high sensitivity to the initial estimate point.

Finally, as much as the Nelder-Mead method presents itself as a fast method, for this study it did not recover the reference values nor did it reach convergence, thus affirming the importance of hybrid method application.

Table 1. Data for the finite volume program

Parameters	Value
Room moisture and temperature	0 (kg/kg) / 353.15 K
Initial wood temperature and concentration	0.12 (kg/kg) / 298.15 K
Thermal conductivity of wood	0.18 [W/mK]
Specific heat of wood	1883.0 [J/KgK]
Dry wood specific mass	825.0 [Kg/m ³]
Air specific mass	0.94577 (kg/m ³)
Total drying time	100 h
Wood thickness (x), width (y) and length (z)	0.04 × 0.2 × 0.2 m

6.2 Particle Swarm Optimization (PSO) application to the problem

For the present study, PSO simulations were performed for three different populations in order to observe the success or not of convergence for each case.

- i. case a): 25 particles;
- ii. case b): 50 particles;
- iii. case c): 500 particles.

All simulations performed used the global best of each step as the global best of the method. In other words, an evaluation of all the particles positions was performed and the one with the lowest objective function was taken by reference of the swarm global best position in that step. All cases were performed up to a maximum number of steps, N_{max} .

Figure 1 shows the evolution values of the global best objective function for the three cases studied. It is possible to notice that the number of objective function calculations or the number of times in which the direct program of finite volumes was called was high. It is still notable that convergence has not been achieved even with 10,000 interactions for a population of 500 particles.

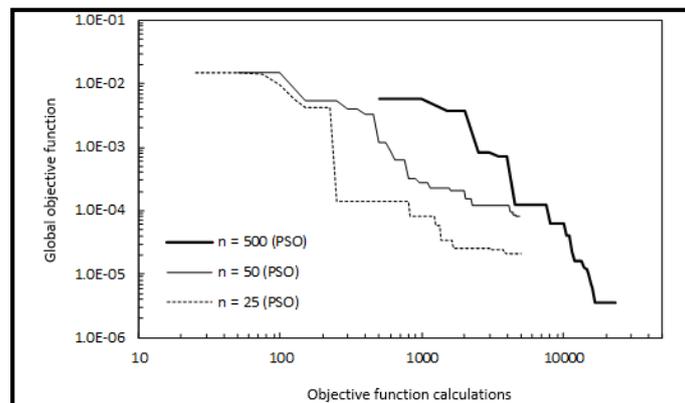


Figure 1. Evolution of the objective function for number of particles $n_p = 25, 50 e 500$

Therefore, even with the increase in population, the convergence rate has not increased, on the contrary, it has become slower. This same fact was observed for other applications as in the work of Vaz Jr. et al. (2013) where the authors applied the PSO method to identify parameters of inelastic materials and concluded that smaller populations may compromise the research capacity of the method causing uncertain convergence values. They also observed that larger populations do not improve convergence and may indicate dubious convergence as well as values getting stuck in non-convex areas of the problem studied.

In summary, for this work, the PSO method was not effective because it did not recover the reference values nor did it reach convergence.

6.3 Genetic algorithm (GA) method application to the problem

Simulations were performed with the genetic algorithms method for three different populations in order to observe the success or not of convergence for each case.

- i. case a): 25 particles;
- ii. case b): 50 particles;
- iii. case c) : 500 particles.

All simulations performed used the global best for each particle. All cases were performed up to a maximum number of steps, N_{max} .

Figure 2 shows that the increase in the population did not influence the results of convergence nor promoted a reduction in the calculations of the objective function. The direct finite volume program had a high number of executions with more than 10,000 calls and the reference values were not recovered.

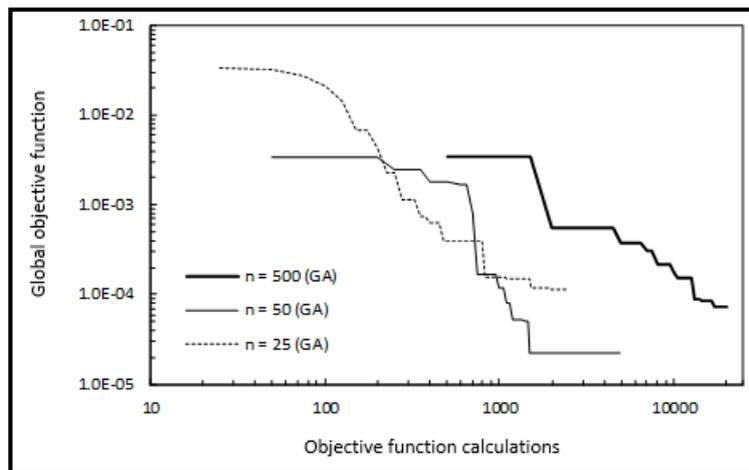


Figure 2. Evolution of the objective function for number of particles $n_p = 25, 50 e 500$

Therefore, for this study the method of genetic algorithms was inefficient because it presented a low rate of convergences and, for the populations and maximum number of generations studied, it did not recover the reference values, requiring a large number of solutions to the direct problem (called to the finite volume program). Despite the inefficiency for the present physical problem, GA is able to reduce the search space and allow the use of another and more efficient method as a second step.

6.4 Hybrid method PSO-NM

As described in the previous chapter, in order to increase the efficiency of the parameter identification process, the global-local hybrid method PSO-NM was used. Thus, the NM method was applied to the results obtained in the PSO method simulation.

In this way the PSO method was applied until a convergence index with $\phi_g \leq 0.1$ was obtained, and then the best particle was used for the NM method initial estimate. Then, applying the Nelder-Mead to the best particle found in the PSO method, the convergence of the NM step was admitted when the convergence index reached $\phi_g^{NM} \leq 10^{-10}$.

The calculations of the PSO method objective function become high when compared to the calculations performed by the PSO-NM method, so, it is noticed a reduction in calls from the direct program and which directly influences the simulation time. The analysis of objective function $k^{PSO} + k^{NM}$ total calculations shows that they passed the amount a little when compared only in numbers between the two methods, but, taking into account the convergence index and the computational simulation time, the method hybrid remains advantageous. Thus, Figure 3 shows the evolution of the convergence index, where it can be seen that for the three cases studied from the transition point, it happened more quickly.

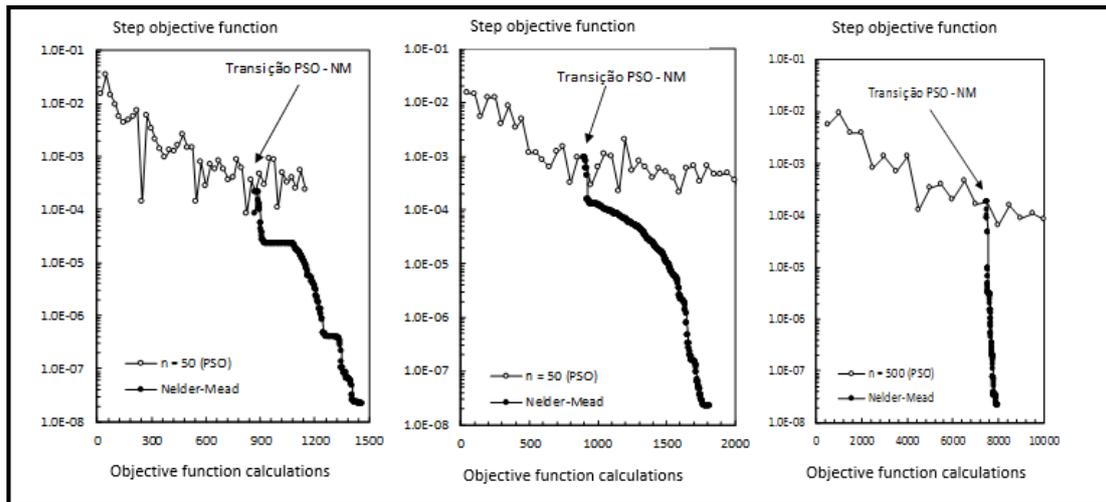


Figure 3. Convergence index and objective function evolution of the step for number of particles $n_p = 25, 50$ e 500

In Figure 3 it is possible to notice oscillations after the hybrid method is applied, which confirms the sensitivity of the Nelder-Mead method to local minimums. It can be seen in the figures that the step with Nelder-Mead method showed a high convergence rate, that is, the reduction of convergence index and the objective function with a number of objective function calculations much faster than the step only with PSO.

After applying the hybrid method, all simulations recovered the reference parameters, so the optimization problem obtained the global minimum.

6.5 Hybrid method GA-NM

In order to increase the parameter identification process efficiency, the global-local hybrid method GA-NM was applied. The Nelder-Mead method was applied to the results found in the simulation of the GA method described in the previous chapter.

Therefore, the GA method was used until a convergence index with $\phi_g \leq 0.1$ was reached, so the best individual was chosen for the initial estimate of NM method. The convergence of the hybrid method step was admitted when the convergence index reached $\phi_g \leq 10^{-10}$.

It can be seen that the isolated application of the GA method was not sufficient to recover the reference values for the established convergence index. Thus, with the hybrid method GA-NM application based on the GA transition value, the results obtained at the end of the simulation were satisfactory, as they managed to reach the reference values.

The objective function calculations of the GA method become high when compared to the calculations performed by the GA-NM method. In this way, there is a reduction in calls from the direct program, which influences directly the simulation time. Evaluating the total calculations of the objective function $k^{GA} + k^{NM}$ they passed the quantity a little when compared only in numbers between the two methods, but, taking into account the convergence index and the computer simulation time the hybrid method it remains advantageous.

Thus, Figure 4 shows the convergence index evolution, where it can be seen that for the three cases studied from the transition point, it presented more quickly.

6.6 Hybrid method GA-NM and PSO-NM comparison

Finally, the performance of the two methods applied together with the hybrid method was evaluated to conclude which among them had the best result for this study.

Figure 5 shows that the GA-NM method achieved convergence before PSO-NM. It can still be noticed that with the application of the GA and PSO method alone, the convergence would be slow and the amount of calculations of the objective function would be high.

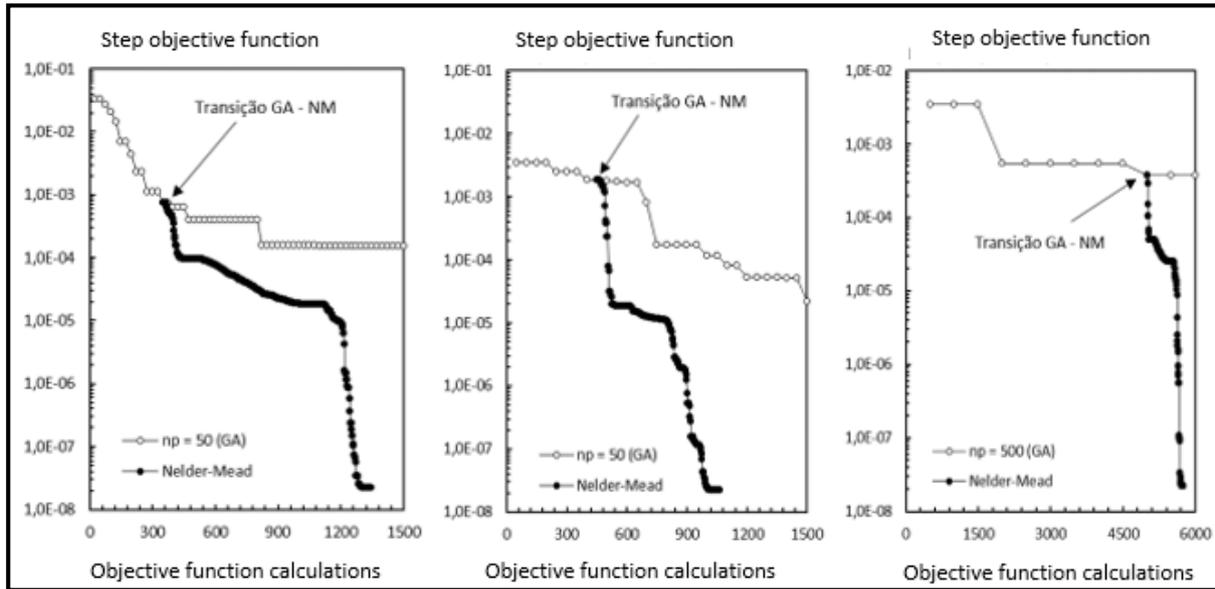


Figure 4. Convergence index and objective function evolution of the step for number of particles $n_p = 25, 50$ e 500

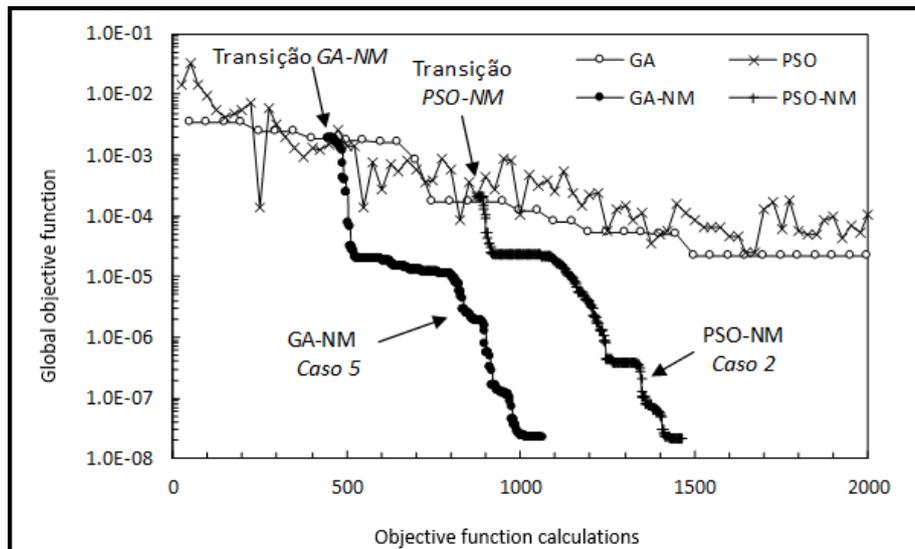


Figure 5. Hybrid method GA-NM and PSO-NM comparison

So, it can be concluded that for this study the hybrid GA-NM method obtained the best result, since it reached the convergence, recovered the original parameters and demonstrated less amount of calculations of the objective function, resulting in a time gain in the simulation.

7 CONCLUSION

Analyzing the results presented for the Nelder-Mead method application, it was noticed that it presented sensitivity to the initial estimate. Thus, it can be verified that with the isolated application of the PSO and GA methods, the values found at the end of the simulation were not satisfactory, since none of them recovered the reference values and neither reached the convergence of the problem. However, it is noteworthy that the application of these was important to reduce the search space for the study second stage. With the application of the hybrid method, the results achieved were satisfactory, recovering the reference values and reaching convergence. Even though for the GA-NM method the

calculations of the objective function were high when compared to GA alone, the computational simulation time was reduced, this highlights the advantage of the method.

Finally, it can be concluded that the association of heuristic methods with the hybrid method were effective for the completion of this work.

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