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EXPERIMENTAL INVESTIGATION OF AN INVARIANT DESCRIPTOR FOR THE CONJUGATE COOLING OF DISCRETE HEATERS IN A DUCT

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Abstract.
Circuit boards are usually cooled by forced airflow and the heat transfer from the discrete electronic components may be enhanced by conductive boards. An experimental investigation was performed to show that the temperatures of four protruding heaters mounted on the conductive lower wall of a rectangular duct cooled by forced parallel airflow may be conveniently predicted by means of dimensionless conjugate coefficients g^+_{ni}. These coefficients are invariant with the heaters' power dissipation and they may be grouped in a square matrix G^+ of order equal to the number of heaters. The conjugate coefficients were conveniently obtained from tests with a single active heater at a time. They were expressed as functions of the airflow Reynolds number, based on the duct hydraulic diameter, in the range from 2,000 to 6,100. These coefficients constitute the conjugate coefficients matrix G^+ of order four in these experiments. Additional experimental tests where then performed for distinct duct flow rates and arbitrary power dissipation in each heater, with two or three active heaters at a time. The measured heaters' temperatures in these tests were quite well predicted using the previously obtained invariant conjugate coefficients matrix G^+.

Keywords: *Conjugate forced convection-conduction, protruding heaters, dimensionless conjugate coefficients, invariant descriptors.*

1. INTRODUCTION

The semiconductors industry leads the innovations in the field of data processing. This occurs since the development of the MOS transistors, working as logical gates for the binary language. According to Vassighi and Sachdev (2006), the current integrated circuits (ICs) or processors developed in this industry have more than one billion transistors. This was possible only due to a manufacturing scale reduction of these components. Nowadays, the GPU and CPU processing units are produced in industrial scale with up to 16 processing cores, where the transistors are kept 10 nanometers apart from each other, as stated in Stallings (2017). This indicates the high density and compactness of this hardware in the actual integrated circuit boards.The demand for higher performance and operational frequency of these hardware leads, however, to higher energetic cost and, consequently, to larger heat dissipation rates and higher temperatures during their operation.

Therefore, the integration of these devices requires their thermal and energetic management. In most cases, this task is performed employing atmospheric air as the cooling fluid, due to its availability, ease of handling and high dielectric strength. In spite of these advantages, the air presents adverse thermal properties typical of gases, i.e., low thermal conductivity and specific heat. Due to this, much research has been carried out related to the air cooling of electronic components and devices. Several heat transfer enhancement techniques have been developed in order to keep the components' and devices' temperatures within the reliable limits stated by their manufacturers. Due to the compactness tendency of electronic equipment, their internal circuit boards are now usually closely spaced, thus preventing the use of finned surfaces on their components with larger heat dissipation rates. One possible solution to a proper cooling of critical components in these cases, as proposed by Nakayama (1997), was the use of circuit boards with increased thermal conductivity. In this case, the components' cooling would be performed by a conjugate forced convection-conduction mechanism from the components to the cooling air.

As described by Moffat and Anderson (1990), Moffat (1998) and Moffat (2004), when the cooling process of a component on a circuit board occurs essentially by forced convection, it may conveniently be described by an adiabatic heat transfer coefficient. The reference temperature for this coefficient is the cooling fluid mixed mean temperature just upstream of the heated component. Their results showed that this coefficient is invariant with the power dissipation rate in the component - it is then called an invariant descriptor of the discrete convective cooling. When the board contains several power dissipating components, the adiabatic heat transfer coefficient changes from component to component, but it maintains the invariant characteristic with the power dissipation rate for each component.

When conductive circuit boards are employed, however, the adiabatic heat transfer coefficient is no longer an invariant descriptor for the components heat loss due to the addition of conduction loss from the components to the board. In this case, the conjugate mechanism has to be included in the thermal analysis. Davalath and Bayazitoglu (1987) pioneered with a numerical investigation of the forced fluid flow cooling of three identical two-dimensional protruding heaters mounted on the lower wall of a horizontal parallel plates channel. This wall was considered either adiabatic or with a finite thermal conductivity equal to ten times that of the cooling fluid. The numerical algorithm *SIMPLE* was used in the numerical solution of the conservation equations in the domain including the flow between the parallel plates and the solid regions of the heaters and the lower plate thickness. The heaters' height was 25% of the channel height, for a Reynolds number range from 100 to 1500 and the fluid Prandtl number from 0.1 to 2.0. Their results indicated a significant temperature decrease around the heaters when they were mounted on a conductive substrate. In the case of an adiabatic substrate, the local Nusselt number (Nu) was larger on the heaters' upper and front surfaces and the heaters' average Nusselt number was largest for the upstream heater.

In another approach, Kim and Anand (1995) performed a numerical investigation of the cooling of five and a half identical protruding blocks uniformly heated, mounted on the lower conductive wall of a channel. Their results to characterize the conjugate forced convection-conduction cooling of the blocks were expressed in terms of two global thermal resistances. They were based on the temperature difference between the maximum temperature of each heated block and either the cooling fluid temperature at the channel inlet or the fluid mixed mean temperature just upstream of each heated block. Their results were obtained for laminar steady flow and they indicated that the global thermal resistance decreases with both the cooling flow Reynolds number and with the substrate thermal conductivity increase relative to that of the cooling fluid.

Numerical simulations of the conjugate forced convection-conduction air cooling of discrete two-dimensional protruding heaters mounted on the conductive lower wall of a horizontal parallel plates' channel were performed by Alves and Altemani (2012). Their results indicated that for an arrangement of N discrete heaters mounted on the conductive plate, the average temperature increase of the nth heater of the array $(T_{h,n})$, above the fluid inlet temperature in the channel (T_0), is related to the conjugate cooling rate (q_i) of all heaters of the array. They expressed this relationship in terms of conjugate coefficients g^+_{ni} as in Eq. (1):

$$T_{h,n} - T_0 = \frac{1}{\dot{m}c_p} \sum_{i=1}^{N} g^+_{ni} q_i \tag{1}$$

In this equation, $\dot{m}$ represents the channel mass flow rate and c_p, the specific heat of the cooling fluid. It is noteworthy that the conjugate coefficients are invariant with the conjugate cooling rate from each heater of the array. This is a very important feature for the thermal design of circuit boards with discrete power dissipation components, The coefficients are valid for a specific heaters' distribution on the conductive plate and they depend also on the cooling fluid properties and on its mass flow rate in the channel. For an arrangement of N discrete heaters on a circuit board, the (N^2) conjugate coefficients can be arranged in a square matrix (G^+) of conjugate coefficients and the temperature prediction of each heater may be obtained as in Eq. (2).

$$\begin{bmatrix} T_{h,1} - T_0 \\ T_{h,2} - T_0 \\ \vdots \\ T_{h,n} - T_0 \end{bmatrix} = \frac{1}{\dot{m}c_p} \begin{bmatrix} g^+_{11} & g^+_{12} & \cdots & g^+_{1N} \\ g^+_{21} & g^+_{22} & \cdots & g^+_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ g^+_{N1} & g^+_{N2} & \cdots & g^+_{NN} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix} \tag{2}$$

The conjugate coefficients (g^+_{ij}) are dimensionless and they are equivalent to the superposition kernel function (g^*) developed by Anderson and Moffat (1992) for the case when the cooling occurs only by the convection mechanism. The diagonal terms g^+_{ii} of the conjugate coefficients matrix (G^+) indicate the self heating effect of a heater on its own temperature increase. The off-diagonal terms indicate the effect of the conjugate cooling of any heater on the temperature increase of the remaining heaters of the array.

The most convenient way to obtain the conjugate coefficients is to perform tests with a single active heater of the array at a time and to measure the temperature increase of each heater relative to the cooling fluid inlet temperature $(T_{h,n} - T_0)$ in the channel. Then, each conjugate coefficient can be obtained from Eq. (1) or from Eq. (2). This procedure must be repeated for each heater of the array in order to complete the matrix of conjugate coefficients. These tests may be performed either by experiments or by numerical simulations, as indicated by Loiola and Altemani (2015), where either one or two protruding heaters were mounted on the lower conductive wall of a rectangular duct cooled by a parallel forced air flow.

For the case of either one or two rows of discrete protruding heaters mounted on the lower wall of a rectangular duct cooled by two impinging flows exiting from two square openings at the duct upper wall, the conjugate coefficients were obtained from laboratory experiments, as reported by (de Marchi Neto and Altemani, 2017). Some results of this work will be compared to those of the present investigation.

The purpose of the present experimental investigation was to evaluate the conjugate coefficients matrix for the cooling of four protruding heaters mounted in two rows of two heaters on the lower conductive wall of a rectangular duct cooled by a parallel forced air flow. The ratio of the heaters to the duct height was equal to 0.3. The conjugate coefficients were obtained by laboratory experiments, under steady state conditions, for a range of the air flow mass flow rate in the duct. The coefficients were expressed as functions of the duct flow Reynolds number (Re_D) in the duct, ranging from 2000 to 6100. These coefficients were the terms of the square matrix (G^+) of order equal to four - the number of discrete heaters in the duct. In addition to the tests performed to obtain all the conjugate coefficients, with a single active heater at a time, additional tests were also performed afterwards with either 2 or 3 active heaters at a time, with distinct air flow rates in the duct. These tests were performed with arbitrary power dissipation in each heater of the array, so that the heaters' temperatures could be compared with the predictions made by Eq. (2), making use of the previously determined conjugate coefficients matrix (G^+).

2. EXPERIMENTAL INVESTIGATION

2.1 Experimental Assembly and procedure

The experimental apparatus is shown schematically in Fig. 1. The experimental procedure to evaluate the duct flow Reynolds number (Re_D), the conjugate coefficients (g_{ni}) and the discrete heaters' temperatures will be described next.

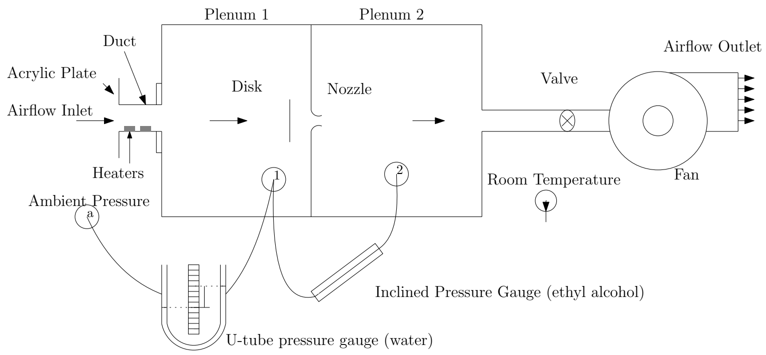


Figure 1. Schematic view of the experimental apparatus.

The experimental apparatus was assembled in the Energy Department Laboratory. It consisted of a rectangular duct where four protruding heaters were mounted on its lower conductive horizontal wall. Air from the laboratory was forced through the rectangular duct in suction mode by a fan, as indicated in Fig. 1. Downstream the rectangular duct, the air flow crossed a large plenum box with a partition wall before being channeled to a tube with a flow control valve and to the fan downstream, to be discarded to the atmosphere outside the laboratory. The partition wall in the plenum box contained a calibrated nozzle with 0.017 m diameter, to measure the air mass flow rate $\dot{m}$ through the duct. The air flow pressure loss through the calibrated nozzle was measured by means of an inclined manometer (Merian, model 40HE35, USA), with a resolution scale of 0.254x10^{-3} m, using ethyl alcohol (relative specific weight 0.7876) as the manometric fluid.

The rectangular duct cross section was (0.16 x 0.02) m and it had a length equal to 0.25 m. The duct lateral and top walls were made of Plexiglas. The bottom wall, where the protruding heaters were mounted, called the substrate plate, had a thickness of 0.002 m and it was made of Aluminum. The four heaters were distributed on the Aluminum plate as indicated in Fig. 2.

The heaters consisted of square Aluminum blocks with an edge equal to 0.050 m and a height of 0.006 m. They were made from two square Aluminum plates with sides equal to 0.05 m: a base with a height of 0.0045 m and a top with a height of 0.0015 m. A 0.254x10^{-3} m diameter Teflon coated Chromel wire (Omega Eng., USA) was inserted into 12 equally spaced square grooves with a side equal to 0.5x10^{-3} m machined in one face of the heater's base. Thus, the electrical resistance of each heater, at 24,1 °C, varied from 8,766 Ω to 8,945Ω. The interface between the base and top of each heater, and the interface between each heater and the Aluminum substrate plate, were filled with a thin thermal grease layer in order to improve their thermal contact. Each heater was fixed to the Aluminum substrate plate by means of two small screws, in order to retain their position and to get a better thermal contact.

The final assembly of the rectangular duct with the Aluminum substrate is shown in Fig. 3. View (a) presents the frontal end, where the air flows into the duct. The region below the Aluminum substrate plate was filled with a 0.020 m thick thermal insulation layer of polyurethane foam. This insulation layer was sealed on the back side of the duct by a small Plexiglas plate in order to avoid any air leakage through the polyurethane foam. Figure 3(a) also shows that all the wiring of the instrumented test duct and heaters entered the duct through the frontal cross section under the Aluminum

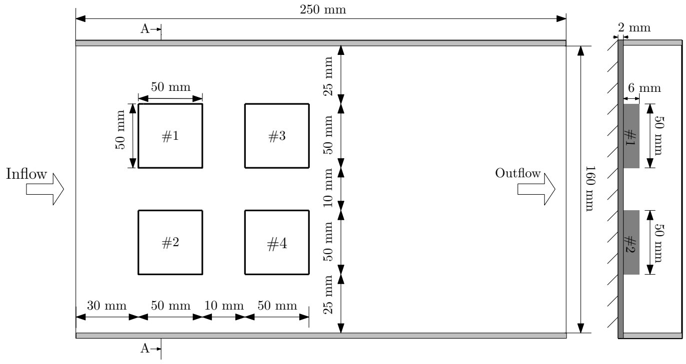
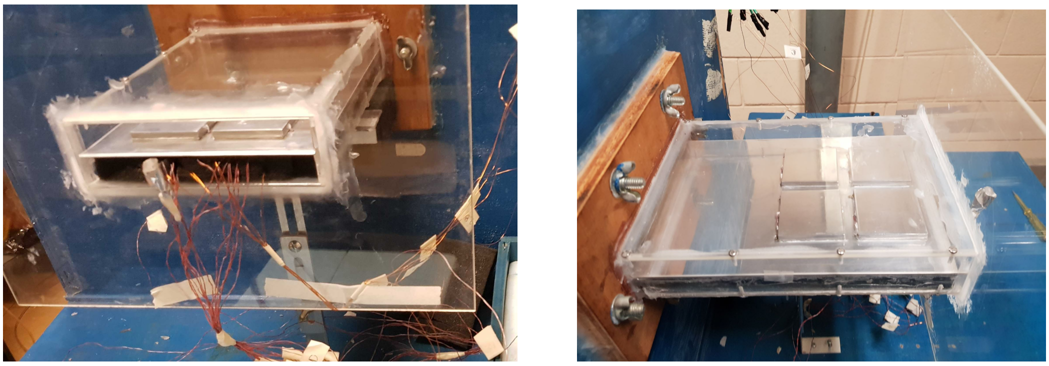


Figure 2. Rectangular duct view and cross section with the four protruding heaters.

plate. Figure 3 (b) shows the lateral view of the duct and its connection to the plenum box. Both Figures 3 (a) and (b) also show the frontal plate fixed at the duct inlet cross section to minimize the air flow head loss at the duct entrance.



(a) View of the air flow inlet. (b) Lateral view of the duct with four heaters.

Figure 3. Rectangular duct with four heaters on the Aluminum substrate.

2.2 Temperature Measurement and Power Supply

The temperatures of the heaters and the duct in the experimental tests were measured by means of $0.254 \text{x} 10^{-3}$ m diameter Teflon coated Type E thermocouple wires (Chromel-Constantan, Omega Eng., USA). The thermocouple wires were connected to a selector switch (OSW5-20, Omega Eng., USA) and each temperature was read from a digital transducer (OSW5-20, Omega Eng., USA) with a resolution of 0.1 °C. A total of 22 thermocouples were used: three in each heater and the remaining on the substrate plate, on the upper and lower duct walls, near the duct air flow inlet, on the thermal insulating layer covering the duct and in the plenum box near the flow nozzle. The outer surfaces of the duct, prior to the experimental tests, were covered by an insulating thermal layer, as indicated in Fig. 4. It consisted of a polyurethane foam (k = 0.035 W/m.K) with distinct thicknesses, equal to 0.060 m at the upper wall and 0.035 m at the other three walls.

Figure 4. Rectangular duct surrounded by insulation.

The electrical resistance in each heater was connected by copper wiring to a DC power supply (Instrutherm, model FA-3005, BRA) and to a digital multimeter (HP, model 34401A, USA). The power dissipation in each heater was obtained from the product of the DC electrical current and the corresponding voltage drop across the heater resistance.

The experimental data from the tests were obtained under steady state conditions, three to four hours after the beginning of each test. This condition was assumed when all the thermocouples' readings were within 0.1 °C of the previous readings during a time interval equal to 30 minutes. In addition to the temperature readings, the power dissipation in the heaters, the duct mass flow rate, and the atmospheric pressure were also recorded every 30 minutes.

2.3 Conjugate Coefficients

As mentioned, the conjugate coefficients were obtained from experiments with a single active heater at a time. In all tests, the three thermocouples inserted in each Aluminum block heater always indicated a uniform temperature. In each test, a specific air flow rate was set by the flow control valve. Then, the power dissipation in the active heater was adjusted so that its temperature remained close to either 40°C or 50°C. This procedure to obtain data with two distinct heater temperatures was adopted to verify whether the conjugate coefficients are invariant with the power dissipation in the heaters. Due to the geometric symmetry of the heaters' position on the substrate plate, these experimental tests were performed assuming a thermal symmetry between the heaters $\#1$ and $\#2$ and also between the heaters $\#3$ and $\#4$, indicated in Fig. 2.

The conjugate coefficients (g_{ni}^{+}), for each heater mounted on the substrate plate were obtained from the evaluation of the conjugate heat transfer rate (q_i) from the active heater, as indicated in Eq. (3).

$$q_i = p_w - (q_{rd} + q_f + q_{iso}) \tag{3}$$

In Eq. (3), p_w indicates the electric power dissipation in the heater and the remaining three terms on the right side represent the thermal losses, where q_{rd} indicates the radiation losses to the environment surfaces by the heater and the substrate plate surface, q_f represents the conductive losses through the thermocouple and power wires connected to the heater, and q_{iso}, the thermal losses through the insulation layer covering the duct. All the heater and the Aluminum substrate surfaces were finely polished and their global hemispherical emissivity was assumed equal to 0.1.

The results obtained from Eq. (3) were used as inputs to Eq. (4) in order to obtain the conjugate coefficients (g_{ni}^{+}) from the experiments with a single active heater at a time. For the case when n = i, the corresponding coefficients (g_{nn}^{+}) are described as the self-heating coefficients and they occupy the diagonal terms of the square matrix (G^{+}). The off-diagonal terms (g_{ni}^{+}), corresponding to the cases when the subscripts n $\neq$ i, represent the effect of the i^{th} heater heating on the temperature increase of the n^{th} heater of the array.

$$g_{ni}^{+} = \frac{\dot{m}c_p}{q_i}(T_{h,n} - T_0) \tag{4}$$

All the (g_{ni}^{+}) were obtained as functions of the air flow Reynolds number based on the rectangular duct hydraulic diameter, as indicated in Eq. (5). In this equation, (P_m) represents the wetted perimeter of the duct and (μ), the air viscosity. The experimental tests were performed in the range of (Re_D) from 2,000 to 6,100.

$$Re_D = \frac{4\dot{m}}{\mu P_m} \tag{5}$$

2.4 Uncertainty analysis

The uncertainties of the experimental results were estimated following the uncertainties propagation method described by Kline (1953). They were evaluated by the method of sequential perturbations, described by Moffat (1982), as cited in Coleman and Steele (2018). The *Software EES (F-Chart Software)* was employed for these evaluations. The uncertainty ΔR of an experimental result $R = f(x_1, x_2, \ldots, x_n)$, obtained from a set of measurements $x_1, x_2, \ldots, x_n$, was evaluated by means of Eq. (6).

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial R}{\partial x_2}\delta x_2\right)^2 + \ldots + \left(\frac{\partial R}{\partial x_n}\delta x_n\right)^2} \tag{6}$$

The partial derivatives in this equation were discretized in order to be evaluated numerically and the uncertainty of each measured variable (δx_n) was estimated from the measuring instruments and from previous experience in the laboratory. Tab. 1 presents a list of the absolute and relative estimated uncertainties of the measured variables in the experiments. The largest source of uncertainty on the results was that of the temperature measurements, which was estimated equal to 0.1°C. As a result of this procedure, the relative uncertainty for the air flow Reynolds number Re_D in the investigated range was 5.1% and the relative uncertainty for the conjugate coefficients (g_{ni}^{+}) was 5.3%.

Table 1. Values adopted for uncertainties.

Variable	Uncertainties
Thermocouple temperature [°C]	0.1
Inclined manometer hight [in alcohol]	0.005
Coefficient k of the nozzle [-]	0.02
Barometric pressure [hPa]	3.0
Aluminum properties [%]	2.5
Air properties [%]	1.0

3. RESULTS AND DISCUSSION

The experimental tests to evaluate the conjugate coefficients g^+_{ni} were performed with a single active heater at a time. The average air flow velocity in the duct during the tests varied from 0.97 m/s to 2.90 m/s, corresponding to a range of the Reynolds number (Re_D) from 2,000 to 6,100. The dimensionless conjugate coefficients were correlated to the Reynolds number by a power as indicated by Eq. (7), where the constants C and b were obtained by curve fitting to the discrete experimental results for each heater.

$$g^+_{ni} = CRe^b_D \tag{7}$$

After the conjugate coefficients had been obtained as described, additional experimental tests were performed with more than one active heater at a time. The purpose was to compare the measured heaters' temperatures in these tests with predictions obtained from Eq. (2), using the previously evaluated matrix (G^+). The agreement of the measured and the predicted temperatures would indicate that the conjugate coefficients are invariant to the conjugate forced convection-conduction heat transfer rate from the heaters.

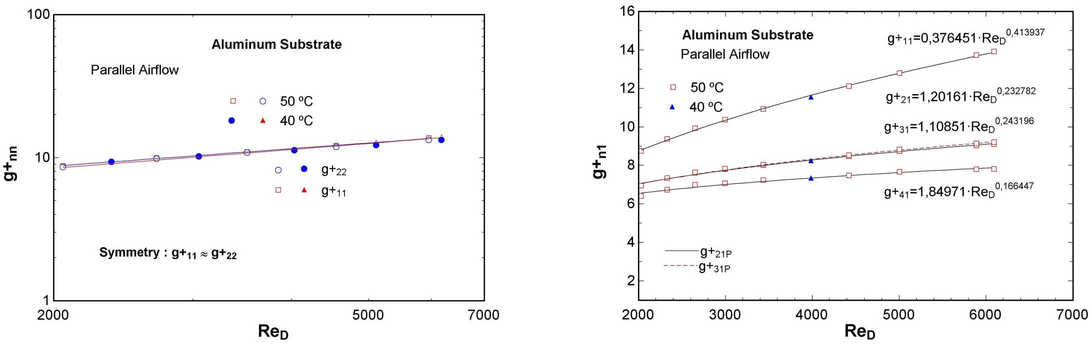
3.1 Results for the Aluminum Substrate Plate

For the tests with a single active heater at a time, due to the mentioned geometric symmetry of the four heaters on the substrate plate of the rectangular duct, only eight conjugate coefficients had to be determined. They were g^+_{n1} and g^+_{n3}, where n may vary for the N heaters on the substrate plate - from 1 to 4 in the present investigation. Thus, the experiments were performed either with heater $\#1$ active or with heater $\#3$ active. Some additional tests were also performed with heater $\#2$ active in order to verify the assumed symmetry.

Due to the Aluminum substrate high thermal conductivity and its thermal contact with the heaters, the conjugate forced convection-conduction heat transfer rate from the heaters was estimated in the range from 87% to 94% of their electric power dissipation. The balance of the remaining power dissipation in the heaters was removed mainly by radiation and by conduction through the insulation layer around the duct.

The experimental results for the self-heating conjugate coefficients g^+_{11} are presented in Fig. 5 as a function of the air flow Reynolds number (Re_D) in the duct. The results for g^+_{22} are also presented in the same figure, indicating that both are virtually the same, satisfying the assumed symmetry for the present heaters' configuration in the duct. In addition, the figure shows the results for tests performed with distinct operating temperatures (either 40 °C or 50 °C) of both heaters. They show that the conjugate coefficients are invariant to the distinct power dissipation required to attain the distinct temperatures in both heaters.

The off-diagonal coefficients of the matrix (G^+) indicate the thermal influence of the active heaters on their neighbors. Thus, the conjugate coefficient g^+_{n1} indicates the influence of the active heater $\#1$ on the temperature rise of any of its n neighbors, from $\#2$ to $\#4$ in the present investigation. The results for the diagonal and off-diagonal conjugate coefficients for heater $\#1$ are presented in Fig. 5.b. They indicate, first, that the largest temperature increase caused by the active heater $\#1$ is in itself, in other words, the self-heating. Second, the almost coincidence of the coefficients g^+_{21} and g^+_{31}, in the investigated range of the Reynolds number, indicate that conduction is the dominant conjugate heat transfer mechanism from the heaters due to the highly conductive Aluminum substrate.



(a) The diagonal terms of the conjugate matrix G^{+} for the Aluminum substrate plate.

(b) The conjugate coefficients g^{+}_{n1} for the Aluminum substrate plate.

Figure 5. Conjugate Coefficients g^{+}_{nn} for the Aluminum Substrate.

The experimental results for the self-heating conjugate coefficient g^{+}_{33} were obtained in tests where heater #3 was the single active heater. These results are presented in Fig. 6 and they were correlated to the Reynolds number in the form of a power law, as indicated in the figure. There is a close agreement with the results of g^{+}_{11}, already presented in Fig. 5.b. These results indicate that, due to the considered proximity of the heaters' distribution on the highly conductive Aluminum substrate plate, the self-heating conjugate coefficients for the first and the second rows of heaters are almost the same. There is, in this case, a large contribution of conduction to the conjugate heat transfer from the heaters.

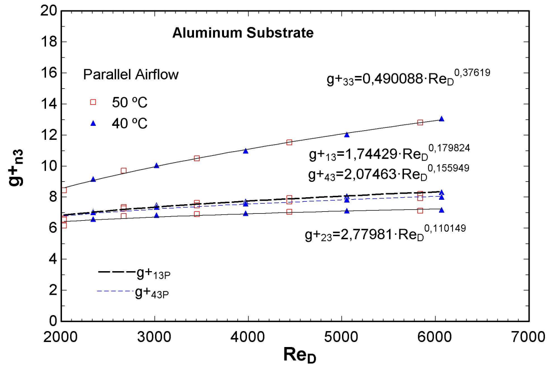
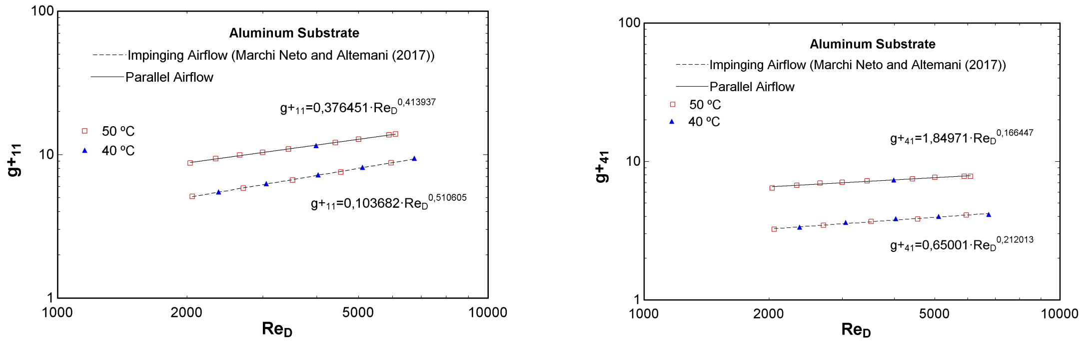


Figure 6. The conjugate coefficients g^{+}_{n3} for the Aluminum substrate plate.

3.2 Comparison between Parallel Airflow and Impinging Airflow

A previous investigation, performed by de Marchi Neto and Altemani (2017), reported results for the conjugate coefficients considering a similar heaters' distribution and duct geometry. They employed, however, impinging flow on the first row of heaters, instead of the parallel flow configuration of the present investigation. Considering both configurations, the experimental results for the conjugate coefficient g^{+}_{11} of both investigations are compared in Figure 7.a. They show that this coefficient is smaller for the impinging flow configuration in the entire Reynolds number range and the difference between both results varies from 41.5% to 32,4% as the Reynolds number increases. Considering both flow configurations, these results indicate that for the same heat transfer rate from heater #1, its temperature will be lower using the impinging flow configuration. From another view point, if this heater is kept at the same temperature in both configurations, the conjugate heat transfer rate will be distinctively higher for the incident flow configuration.

As another example, the comparison of the experimental results for the effect that the conjugate heat transfer rate from heater #1 had on the temperature increase of heater #4 for both flow configurations is presented in 7.b. The conjugate coefficient g^{+}_{41} for the parallel flow configuration is always above the results for the impinging flow, the difference varying from 49.4% to 46.5% as the Reynolds number increases in the investigated range. These results indicate that the temperature increase of heater #4, due to the same conjugate heat transfer rate from heater #1 in both configurations, will be smaller for the impinging flow configuration than for the parallel flow configuration.

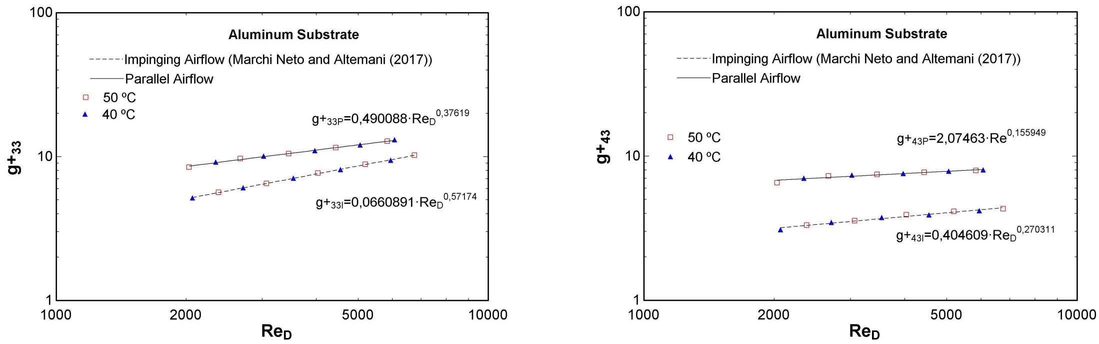


(a) Conjugate Coefficient g^+_{11} for the Upstream Heaters. (b) Conjugate Coefficient g^+_{41} for the Aluminum Substrate.

Figure 7. Conjugate Coefficients g^+_{n1} for the Aluminum Substrate.

The experimental results for the conjugate coefficients g^+_{33} and g^+_{43} are compared in Fig. 8 for both configurations. Both coefficients show the same trend that they are smaller for the impinging flow configuration, with similar meanings as those presented in Fig. 7. The results presented in Fig. 7.a and in Fig. 8.a allow a comparison of the g^+_{33} and the g^+_{11} self-heating conjugate coefficients obtained with the aluminum substrate. For the parallel air flow configuration, the g^+_{33} coefficients are from 2.2% to 6.7% below the g^+_{11} coefficients, while for the incident flow configuration, the g^+_{33} are 0.78% to 8.72% above the values obtained for g^+_{11}. Considering the same duct air flow rate and the same heaters' temperatures, these results indicate that, for the parallel flow configuration, the heaters on the second row are able to transfer a higher conjugate heat transfer rate than those in the first row, due mainly to the flow vortexes generated downstream the first pair of heaters. On the other hand, for the configuration with the impinging flow on the first pair of heaters, there will be a larger conjugate heat transfer rate from the heaters in the first row, mainly due to higher convective heat transfer coefficients at the heaters' top surface and also at the substrate plate area surrounding the upstream row heaters.

The conjugate coefficient g^+_{43}, from one heater on the second row to its neighbor in the same row is presented in Fig. 8.b. The results for the parallel flow configuration are above those for the impinging flow configuration in the range from 46,19% to 52,91%. This may be credited to the fact that for the parallel flow in the duct, conduction through the substrate plate will be the dominant mechanism for the conjugate heat transfer between these two heaters. On the other hand, the impinging flow will not be aligned along the duct near the second row of heaters, and thus there will be an additional convective contribution to the conjugate heat transfer.



(a) Conjugate Coefficient g^+_{33} for the Downstream Heater. (b) Conjugate Coefficient g^+_{43} for the Aluminum Substrate.

Figure 8. Conjugate Coefficients g^+_{n3} for the Aluminum Substrate.

3.3 Superposition tests

The previous tests with a single active heater at a time were performed to obtain the conjugate coefficients matrix (G^+). Additional tests were then performed with more than one active heater at a time and distinct power dissipation in each heater, for the same range of the duct flow Reynolds number. The purpose of these tests was to verify the mentioned invariant property of the conjugate coefficients to the power dissipation and the superposition indicated by Eq. (2), comparing the predictions of this equation with the measured heaters' temperatures in each test. The results are presented in Table 2. It presents, for each test, the duct flow Reynolds number, the electrical power dissipation in each heater and the corresponding conjugate heat transfer rate q_i from each heater.

Table 2. Superposition tests for the substrate plate.

Reynolds number Re_D and T_e				Measured power dissipation			Estimated conjugate heat loss rate		
	#	Re_D [-]	T_e [°C]	P_{w1}[W]	P_{w2} [W]	P_{w3} [W]	q_1 [W]	q_2 [W]	q_3 [W]
Aluminum	1	2032	27.3	6.43	-	1.69	5.42	-	0.88
	2	2031	25.2	1.64	-	6.73	0.80	-	5.67
	3	3006	24.6	3.72	-	3.84	3.08	-	3.19
	4	3002	25.3	10.0	-	10.3	8.15	-	8.44
	5	3480	26.1	10.1	-	3.80	8.86	-	2.80
	6	3486	25.7	3.68	-	10.4	2.66	-	9.16
	7	3440	23.6	2.11	2.21	2.18	1.66	1.76	1.72
	8	5050	25.6	-	15.30	8.39	-	13.80	7.00
	9	5036	24.4	2.11	-	2.17	1.87	-	1.93
	10	5050	25.3	10.1	-	10.3	8.81	-	9.09
	11	5031	24.2	2.11	2.21	2.18	1.77	1.87	1.84
	12	5897	25.4	-	10.80	2.62	-	10.0	2.01
	13	5870	25.6	-	2.50	10.4	-	1.90	9.68
	14	6081	24.1	6.59	6.74	-	5.93	6.08	-
	15	6088	24.3	6.60	-	6.75	5.93	-	6.08
	16	6098	24.5	6.59	2.62	6.74	5.81	1.91	5.95

As can be seen in Table 2, the conjugate cooling rate q_i from each heater represents a large fraction of its electric power dissipation P_{wi}. The overall average was equal to 89,9% in these tests. In these additional tests, the average value T_{hi} of the three measured temperatures in each heater under steady state conditions and the values Tp_{hi} predicted by Eq. (2) are presented in Tab. 3. The largest relative deviation of the predictions from the measured values was close to 2%, while the mean quadratic deviation was less than 0.3 °C for any heater in all these tests. These results lend confidence that the conjugate coefficients matrix can be employed to predict the heaters' temperature under conditions of conjugate forced convection-conduction heat transfer.

Table 3. Comparisons of measured and predicted heaters' temperatures.

		Measured T_{hi} [°C]				Predictions Tp_{hi} [°C] from q_i			
#	Re_D	T_{h1}[°C]	T_{h2}[°C]	T_{h3}[°C]	T_{h4}[°C]	Tp_{h1}[°C]	Tp_{h2}[°C]	Tp_{h3}[°C]	Tp_{h4} [°C]
1	2032	43.73	40.70	41.60	40.00	44.05	40.92	41.71	40.16
2	2031	39.70	38.23	42.00	38.60	39.37	38.00	41.84	38.44
3	3006	35.97	33.97	36.00	33.80	36.14	34.07	36.15	33.95
4	3002	56.20	50.63	56.30	50.30	56.17	50.62	56.18	50.29
5	3480	47.07	42.27	44.23	41.30	47.06	42.19	44.10	41.26
6	3486	43.57	40.90	46.40	41.20	43.29	40.68	46.31	41.15
7	3440	32.27	32.10	32.00	31.00	32.20	32.08	31.94	31.00
8	5050	47.20	53.07	49.10	46.90	47.69	53.94	49.19	47.64
9	5036	29.03	27.97	29.00	27.80	29.14	28.00	29.14	27.92
10	5050	47.90	42.40	47.90	42.07	47.70	42.33	47.70	41.95
11	5031	30.83	30.70	30.60	29.70	30.96	30.85	30.71	29.80
12	5897	36.30	40.67	36.23	36.20	36.82	41.57	36.52	36.87
13	5870	35.40	35.47	39.60	35.20	35.56	35.57	39.70	35.30
14	6081	37.60	37.70	34.20	34.10	38.13	38.35	34.53	34.60
15	6088	37.33	33.87	37.40	33.60	37.44	33.93	37.51	33.67
16	6098	39.70	37.37	39.43	36.00	39.52	37.14	39.30	35.82

4. CONCLUSIONS

The conjugate forced convection-conduction cooling rate of four protruding heaters mounted on a conductive substrate plate in a rectangular duct with forced flow was related to their temperature increase above the flow inlet temperature in the duct. This relation was expressed by means of dimensionless conjugate coefficients g^+_{ni} grouped in a square matrix G^+ of order equal to the number of heaters on the plate. The conjugate coefficients were obtained from experimental tests with a single active heater at a time on an Aluminum substrate plate, with parallel forced air flow in the duct. The resulting conjugate coefficients were related by means of a power law to the duct air flow Reynolds number, in the range from 2,000 to 6,100. Additional experimental tests were then performed with more than one active heater at a time, each heater with distinct power dissipation. These tests indicated that the measured heaters' temperature increase were very close (mean quadratic deviation of less than 0.3°C) to the predictions made with the conjugate coefficients matrix. This agreement indicated that the conjugate coefficients are invariant to the power dissipation in the heaters, and that the simultaneous heating of more than one heater can be described by the conjugate coefficients matrix by linear superposition. Comparisons were made between the conjugate coefficients obtained in this work with the values previously obtained for a similar configuration with impinging flow on the first row of heaters. It was shown that when the conjugate coefficients for both configurations are plotted as functions of a common Reynolds number, the coefficients for the parallel flow configuration are always above those for the impinging flow. This comparison indicates that for the same conjugate heat transfer rate from the heaters in both configurations and the same flow rate in the duct, the heaters' temperatures will be lower for the impinging flow configuration.

The cooling of protruding heaters mounted on a conductive substrate in a duct with parallel forced flow occurs by the conjugate forced convection-conduction mechanism. The results presented in this work indicate the convenience to employ a conjugate coefficients matrix to predict the heaters' temperature increase, due to two main reasons. First, the invariant property of these coefficients to the power dissipation in the heaters and second, the superposition property to evaluate the effect of more than one active heater at a time with distinct conjugate cooling rates.

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