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GENETIC ALGORITHM OPTIMIZATION APPLIED TO POLLUTION SOURCE ESTIMATION IN THE ATMOSPHERE

Roseane A. S. Albani
Rio de Janeiro State University, Nova Friburgo, RJ, Brazil
roseanealves75@gmail.com
Vinicius V.L. Albani
Federal University of Santa Catarina, SC, Brazil
v.albani@ufsc.br

Antonio J. Silva Neto
Rio de Janeiro State University, Nova Friburgo, RJ, Brazil
ajsneto@iprj.uerj.br

Abstract. *This work proposes a new methodology to estimate atmospheric emissions using a combination of composite misfit functions in the Tikhonov-type regularization, and the Genetic Algorithm, to minimize this functional. A Morozov-like discrepancy principle is used to choose the regularization parameter according to the available data. The joint use of composite misfit functions, Tikhonov-type regularization, and the Genetic Algorithm defines the inverse modeling for source identification. Besides, the dispersion model (or forward problem) is solved using the Galerkin/Least-Squares formulation combined with parametric profiles to account for the relevant process in the atmospheric dispersion considered in this work. The proposed methodology is evaluated with a single source dispersion field experiment, presenting accurate values for the source position and strength.*

Keywords: *Genetic Algorithm, Atmospheric Dispersion, Source Estimation, Tikhonov Regularization, Composite Misfit*

1. INTRODUCTION

Furtive emissions of hazardous gases to the atmosphere are usually difficult to detect. They may originate from human activities, such as industrial production, natural sources, and others. Accidental releases of gases may be harmful to people's health and the environment. Thus, the fast identification of the leakage origin, the time of release, and the mass of emitted gases are of great importance for the decision-makers to reduce damages. Such source identification can be addressed using dispersion modeling combined with inversion techniques.

Source estimation of atmospheric emissions using mathematical modeling has been an active research field in the last decades. In general, it combines several elements, such as defining the input datasets, setting up the forward problem, as well as its solution method, and defining the inverse modeling technique. Given its practical and academic importance, extensive literature is devoted to source identification. See, for example, Addepalli *et al.* (2011); Albani and Albani (2019, 2020); Albani *et al.* (2020); Ma *et al.* (2013, 2014); Wade and Senocak (2013); Yee *et al.* (2008), and references therein.

Different techniques have been used to solve atmospheric source identification problems and they differ mainly on the forward problem modeling, that can be, for example, numerical (Albani and Albani, 2019, 2020; Yee *et al.*, 2008) or analytic (Addepalli *et al.*, 2011; Ma *et al.*, 2013; Wade and Senocak, 2013), and the inversion technique, that can be based, for example, on Bayesian inference tools (Wade and Senocak, 2013; Ma *et al.*, 2013; Yee *et al.*, 2008) or on optimization methods (Albani and Albani, 2019, 2020; Albani *et al.*, 2020; Ma *et al.*, 2013).

In the present work, we propose a source estimation tool that combines a Galerkin/Least-Squares (GLS) finite elements formulation (Hughes *et al.*, 1989) to solve the forward problem with Tikhonov-type regularization to address the inverse problem. The GLS formulation allows the introduction of general coefficients for the wind components, turbulent diffusion, and others, as functions of space and time, thereby, allowing the incorporation of realistic physical conditions. Moreover, the use of adaptive mesh refinements saves computational time without compromising solution accuracy. The proposed Tikhonov-type regularization technique makes use of composite data misfit functions, handling noise, and model uncertainties more appropriately. The regularization parameter, that balances the importance of data and *a priori* information, is set according to the available data by a discrepancy-based rule (Morozov, 1966; Albani and De Cezaro, 2019; Albani *et al.*, 2017). The penalty term is the negative form of the Boltzmann-Shannon entropy, which is a well-known regularizing term. The minimization of the resulting Tikhonov-type functionals is undertaken by a *Genetic Algorithm*

(Holland, 1975; Camps Echevarría *et al.*, 2019), which is an accurate global minimizer. It is particularly useful in the present context, since the objective functional is nonlinear, and may have different local minimum points.

The resulting estimation technique is sufficiently general to handle real source identification problems, as the numerical results using the Copenhagen Tracer Experiments datasets show.

The main contributions of the present work to atmospheric source estimation are summarized as follows:

1. The combination of accurate solutions of the forward and inverse problems, by using the GLS formulation to solve the dispersion model and Tikhonov-type regularization to estimate the source parameters.

2. The use of composite data misfit functions to handle noise and uncertainties.

3. The application of Genetic Algorithms to minimize the Tikhonov-type functional.

4. Setting the regularization parameter according to a discrepancy-based rule.

5. Testing the methodology with experimental data.

The article is divided as follows: Section 2 presents the mathematical description of the forward problem. Section 3 describes the inverse modeling techniques. The numerical results for the forward and inverse problems are presented in Section 4. Concluding remarks can be found in Section 5.

2. FORWARD PROBLEM

A study case considering the single source Copenhagen Tracer Experiments (Gryning *et al.*, 1987; Gryning and Lyck, 2002), is presented here. During this experiment, the tracer gas SF_6 was released without buoyancy from a TV tower at 115 meters and then collected at sampling units at 2-3 meters above the surface level and arranged in concentric radial arcs distant 2,4 and 6 km from the source. Figure 1 shows the arrangement of sensors and source emission during the tracer experiments.

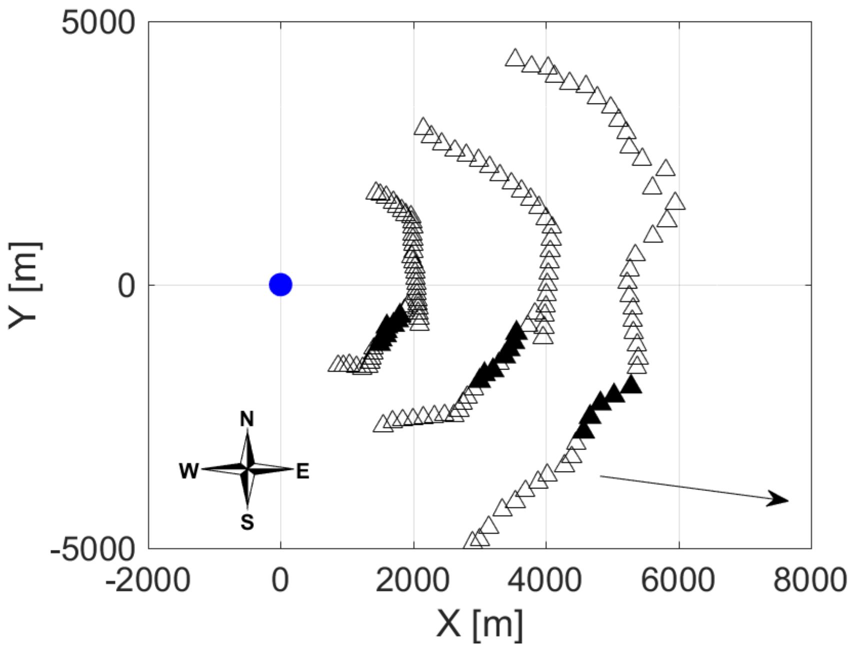

Figure 1. Position of the source and sampling units during the Copenhagen Tracer experiment. The filled circle represents the emission source, the empty triangles the not used sensors and the filled ones, the sensors used in the source estimation. The arrow indicates the wind direction.

The Copenhagen experiment was performed during unstable atmospheric conditions. Meteorological data were measured at a TV tower at several heights. Concentration data were sampled every 20 minutes, along with one hour of the experiment. The simulations were performed using one hour of averaged concentration data. For a better explanation concerning this experiment, see Gryning *et al.* (1987); Gryning and Lyck (2002). The Copenhagen Experiment provided meteorological and concentration data necessary to validate the proposed methodology to estimate the source parameters. Considering the characteristics of this experiment, for modeling purposes, the following hypotheses are considered:

1. The steady-state regime was established.

2. No chemical reaction, radioactive decay, or deposition process occur.
3. The wind flows over flat and homogeneous terrain.
4. The wind did not present a considerable change over the average period used here.

The estimation of the source parameters from atmospheric emissions is established in terms of an advection-diffusion Partial Differential Equation (PDE), or dispersion model, and an inverse problem technique. The atmospheric dispersion problem consists of finding the averaged concentration $c(x, y, z)$, that is the solution of the following PDE problem:

$$\mathbf{u} \cdot \nabla c - \nabla \cdot (\mathbf{K} \nabla c) = Q\delta(x - x_s)\delta(y - y_s)\delta(z - z_s) \text{ in } \Omega, \tag{1}$$

where Ω represents the computational domain, $\mathbf{u} = (u, v, 0)$ stands for the wind field, with components $u = |\mathbf{u}| \cos(\theta)$, $v = |\mathbf{u}| \sin(\theta)$, where θ represents the wind direction angle, ∇ is the gradient operator and $\mathbf{K}$ is a diagonal matrix, which non-zero entries K_x, K_y and K_z denote the turbulent diffusion in the x, y and z directions respectively. The wind field and turbulent diffusion coefficients are highly dependent on the meteorological conditions. Further, Q is the source strength, δ is the Dirac delta distribution and $(x_s,\ y_s,\ z_s)$ are the source coordinates. Equation (1) is completed with the following boundary conditions:

$$\nabla c \cdot \mathbf{n} = 0, \tag{2}$$

with $\mathbf{n}$ denoting the outward normal at $z = z_0$ as well as $z =$ H, and

$$c = 0 \quad \text{elsewhere} \tag{3}$$

where z_0 and H denote the lower and the upper limits of Ω. Considering the linearity of Eq. (1), it is possible to write it as the so-called adjoint state PDE. This new version of Eq. (1) establishes a direct relationship between the receptors and the sources, wherein each experimental concentration $C^{obs}(x_k, y_k, z_k)$ at the kth sensor can be written as

$$C^{obs}(x_k, y_k, z_k) = \int_\Omega c_k^*(x, y, z) S(x, y, z)dxdydz = \langle c_k^*, S \rangle, \tag{4}$$

where

$$S(x, y, z) = Q\delta(x - x_s)\delta(y - y_s)\delta(z - z_s)$$

represents the point source and c_k^* is the solution of the adjoint-state PDE

$$-\mathbf{u} \cdot \nabla c^* - \nabla \cdot (\mathbf{K} \nabla c^*) = L_k, \tag{5}$$

where

$$L_k(x, y, z) = \delta(x - x_{\text{sensor},k})\delta(y - y_{\text{sensor},k})\delta(z - z_{\text{sensor},k}),$$

and $(x_{\text{sensor},k}, y_{\text{sensor}}, z_{\text{sensor},k})$ is the coordinate location of the kth sensor. The boundary conditions associated to Eq. (5) are given by

$$\nabla c^* \cdot \mathbf{n} = 0, \tag{6}$$

with $\mathbf{n}$ the outward normal at $z = z_0$ as well as $z = H$, and

$$c^* = 0 \quad \text{elsewhere.} \tag{7}$$

Equation (5) is solved using the Galerkin/Least-Squares (GLS) formulation proposed by Hughes *et al.* (1989).

3. THE SOURCE ESTIMATION TECHNIQUE

Assume, for instance, that the set of observed concentrations at the sensors is given by the expression in Eq. (4). Hence, we want to find a set of parameters $(x^\dagger, y^\dagger, z^\dagger, Q^\dagger)$ solving the equation

$$C(x^\dagger, y^\dagger, z^\dagger, Q^\dagger) = C^{\text{obs}}, \tag{8}$$

where C^{obs} stands for the set of observed concentrations. Unfortunately, the model in Eq. (1)-(3) is not perfectly accurate and the concentration data is, in general, corrupted by noise. Then, the inverse problem in (8) cannot be solved directly in a stable way. To address this issue, following Albani and Albani (2019, 2020); Albani *et al.* (2020), we apply Tikhonov-type regularization with composite misfit functions. Tikhonov-type regularization is a class of regularization techniques where the original problem (8) is replaced by the minimization of the so-called Tikhonov-type functional. The Tikhonov-type functional is

$$\mathcal{F}(x, y, z, Q) = \phi(x, y, z, Q) + \alpha f(x, y, z, Q), \tag{9}$$

where ϕ denotes the composite data misfit function, $\alpha > 0$ is the regularization parameter and f is the penalty term.

In the present work the misfit assumes one of the following three forms:

$$\phi_{\ell_2,\ell_{1.001}}(x, y, z, Q) = 0.5\frac{\|C(x, y, z, Q) - C^{\text{obs}}\|^2_{\ell_2}}{\|C^{\text{obs}}\|^2_{\ell_2}} + 0.5\frac{\|C(x, y, z, Q) - C^{\text{obs}}\|^{1.001}_{\ell_{1.001}}}{\|C^{\text{obs}}\|^{1.001}_{\ell_{1.001}}}, \tag{10}$$

$$\phi_{\ell_2,\log}(x, y, z, Q) = 0.5\frac{\|C(x, y, z, Q) - C^{\text{obs}}\|^2_{\ell_2}}{\|C^{\text{obs}}\|^2_{\ell_2}} + 0.5\frac{\|\log C(x, y, z, Q) - \log C^{\text{obs}}\|^2_{\ell_2}}{\|C^{\text{obs}}\|^2_{\ell_2}}, \tag{11}$$

$$\phi_{\log,\ell_{1.001}}(x, y, z, Q) = 0.5\frac{\|\log C(x, y, z, Q) - \log C^{\text{obs}}\|^2_{\ell_2}}{\|\log C^{\text{obs}}\|^2_{\ell_2}} + 0.5\frac{\|C(x, y, z, Q) - C^{\text{obs}}\|^{1.001}_{\ell_{1.001}}}{\|C^{\text{obs}}\|^{1.001}_{\ell_{1.001}}}, \tag{12}$$

where the ℓ_2 term is related to Gaussian-distributed noise, the $\ell_{1.001}$ term to sparse noise, i.e., when only for a small number of observations noise is larger, and the logarithmic term is related to log-normal noise. Composite misfit functions are useful to handle different uncertainty sources in estimation. The regularization parameter defines the balance between the misfit and the penalty term. In general, it must be chosen according to the data. Here, it is considered that the penalty term is given by

$$f(x, y, z, Q) = (x + 2000)\log(x + 2000) + (y + 5000)\log(y + 5000) + z\log(z) + Q\log(Q), \tag{13}$$

which is the negative of the Boltzmann-Shannon entropy (Cover and Thomas, 2006).

Since the Tikhonov-type functional in Eq. (9) may have a large number of local-minimum points, its minimization may be solved by *Genetic Algorithms* (GA) (Holland, 1975), which is a powerful global minimizer.

The regularization parameters are chosen according to a discrepancy rule based on the Morozov principle (Morozov, 1966). More precisely, while the value of the misfit is larger than a given threshold, a decreasing sequence of values for α is tested. It is well-known that the misfit function evaluated at the minimizer is monotone non-decreasing for α (Albani and De Cezaro, 2019; Albani *et al.*, 2017).

The threshold is chosen according to the model for the forward problem and the concentration data. The minimization of the Tikhonov-type functional in Eq. (9) with a small value for α is repeated, say 10^{-15}, using the misfit functions in Eqs. (10)-(11). Since the true source parameters are known, the threshold is chosen as the larger misfit value, such that the corresponding minimizers are close to the inverse problem solution. In practice, any available information, like concentration distribution in the sensors array and meteorological data, must be used to decide if the estimated parameters make sense. This helps to set the misfit threshold.

4. RESULTS

This section is devoted to the numerical solution of the forward and the inverse problems using the concentration and meteorological data from the Copenhagen Tracer Experiments.

4.1 Forward Problem

Meteorological conditions strongly influence atmospheric dispersion. Hence, to obtain a reliable source parameter estimation, it is necessary to use realistic coefficients for the wind speed and the turbulent diffusion. Considering the homogeneity hypothesis made in Section 2, it is possible to employ parametric profiles for the wind intensity and the turbulent diffusion rather than to solve the Navier-Stokes equations. Thus, in this work, the parametric profiles proposed by Ulke (2000) will be employed to describe the vertical turbulent diffusion coefficients and wind intensity. For the unstable condition, ($H/L < 0$), they are given respectively by

$$|\mathbf{u}(z)| = \frac{u_{*0}}{\kappa}\left\{\ln\left(\frac{z}{z_0}\right) + \ln\left(\frac{1+\mu_0^2}{1+\mu^2}\right) + 2\ln\left(\frac{1+\mu_0}{1+\mu}\right) + 2[\arctan(\mu) - \arctan(\mu_0)] + \frac{2L}{33H}(\mu^3 - \mu_0^3)\right\} \quad (14)$$

with

$$\mu = \sqrt[4]{1 - 22\frac{H}{L}\frac{z}{H}},$$

$\mu_0 = \mu(z_0)$ and

$$K_z(z) = \kappa u_* H \mu \frac{z}{H}\left(1 - \frac{z}{H}\right) \quad (15)$$

where $z_0 = 0.6$m is the surface roughness length and H is the Atmospheric Boundary Layer (ABL) height. Datasets from October 19, 1978 are adopted for the simulations. Hence, $H = 1120$ $[m]$, $u_* = 0.39$ $[m/s]$ and $L = -108$ $[m]$. The wind direction average during the sampling interval was $\theta = 290^o$. Furthermore, $K_x = K_y = 50$ $[m^2/s]$, as suggested by (Arya, 2001, p. 272), for unstable conditions.

A precise source estimation strongly depends on the adjoint problem solution, which is applied to the misfit functional in the Tikhonov regularization. In this work, the stabilized finite element method GLS is applied jointly with adaptive mesh refinements to obtain the numerical solution for the dispersion model. Thus, the mesh is refined according to an error norm, where the highest solution gradients occur, that is, following the tracer plume propagation. Considering the adjoint state PDE solved here, each sensor is treated as a source. Hence, the Eq. (5) is solved for 20 sensors, which provided non-zero concentrations. The computational domain Ω with dimensions $[-2, 8] \times [-5, 5] \times [z_0, H]$ $[km]$ was meshed into about 3.500.000 linear tetrahedral finite elements. The number of elements depends on the sensor position. However, it doesn't change much from one simulation to another.

The assessment of the numerical solution for the dispersion model (5) can be performed using the statistical indices proposed by Hanna (1989). These indices evaluate the agreement between observed (o) and predicted (p) concentrations, given by:

Normalized Mean Square Error: NMSE $= \dfrac{\overline{(C_o - C_p)^2}}{\overline{C_o C_p}}$

Correlation Coefficient: R $= \dfrac{\overline{(C_o - \overline{C_o})(C_p - \overline{C_p})}}{\sigma_o \sigma_p}$

Fractional Bias: FB $= \dfrac{\overline{C_o} - \overline{C_p}}{0.5(\overline{C_o} + \overline{C_p})}$

Fractional Standard Deviations: FS $= \dfrac{(\sigma_o - \sigma_p)}{0.5(\sigma_o + \sigma_p)}$

Factor of two : FAC2 $= 0.5 \leq \dfrac{C_p}{C_o} \leq 2$

where the bars represent the mean values of each quantity.

Table 1. Statistical indices considering the GLS solution of Eq. (5) and the Copenhagen Tracer Experiments datasets.

Method	NMSE	R	FB	FS	FAC2
Ideal values	0	1	0	0	1
Numerical (GLS)	0.03	0.92	0.02	0.12	1

The small and positive FB indicates that the proposed numerical solution for the dispersion problem overestimates the experimental concentration. The Correlation Coefficient near their ideal value, that is 1, shows that there is a high correlation between experimental and numerical concentrations. Besides, FAC2 demonstrates that 100% of the predicted concentrations between the half and the double of the experimental concentrations. The close agreement with experimental data is verified, since the statistical indices are very close to their ideal values. This shows that the proposed methodology to solve the forward problem was successful.

4.2 Source Estimation

The Tikhonov-type functional in Eq. (9) is minimized using the GA function from the global optimization toolbox of Matlab R2017a, with *Population size* $= 400$ chromosomes, *Crossover offspring function* $=$ *Intermediate*, *crossover* *rate* $= 0.8$ and *Mutation function = Adaptive Feasible*. The tolerance limit is set as 10^{-8} and the maximum number of iterations as 400.

The threshold for the misfit value is set as 0.15 and is chosen as described in Section 3. The regularization parameter α in the Tikhonov-type functional (9) is selected according to the discrepancy rule presented in the same section. More precisely, the largest α values is selected such that the corresponding data misfit is below the threshold. This helps to avoid overfitting the data, which may lead solutions to reproduce noise and to introduce bias in reconstructions. The tested values of α for each data misfit function in Eqs. (10)-(12) are:

$$\alpha = 10^{-x}, \text{with } x = 6, 7, \ldots, 20,$$

and the selected ones can be found in Tab. 2.

Table 2. Selected values of the regularization parameter α using the discrepancy-based rule of Section 3, for each data misfit function in Eqs. (10)-(12).

Misfit Function	Eq. (10)	Eq. (11)	Eq. (12)
Regularization Parameter (α)	10^{-15}	10^{-14}	10^{-7}

The true and reconstructed source parameters, as well as the corresponding data misfit values, can be found in Tab. 3. Such results are associated with the regularization parameters in Tab. 2. The best solutions, their corresponding data misfit, and regularization parameters values are shown in Tab 4.

Table 3. True and reconstructed source parameters for each data misfit function in Eqs. (10)-(12) and the regularization parameters in Tab. 2

Misfit Function	x [m]	y [m]	z [m]	Q [g/s]	Misfit Value
Eq. (10)	-21.81	43.39	127.00	3.60	0.12
Eq. (11)	279.93	-90.01	147.27	3.60	0.12
Eq. (12)	-197.84	139.49	123.06	3.83	0.14
True Parameters	0	0	115.00	3.20	-

Table 4. True and best reconstructed source parameters for each data misfit function in Eqs. (10)-(12). The corresponding data misfit values and regularization parameters are also shown.

Misfit Function	x [m]	y [m]	z [m]	Q [g/s]	Misfit Value	Reg. Parameter (α)
Eq. (10)	-21.81	43.39	127.00	3.60	0.12	10^{-6}
Eq. (11)	73.53	1.19	120.59	3.35	0.11	10^{-20}
Eq. (12)	-50.89	70.22	113.52	3.47	0.13	10^{-9}
True Parameters	0	0	115.00	3.20	-	-

Figure 2 presents the comparison between true and the estimated parameters in Tabs. 3-4. The left and right columns show the spatial positions in the X-Y and Y-Z planes, respectively. In addition, Misfit 1,2 and 3 stand for the data misfit functions in Eqs. (10), (11) and (12), respectively.

The inverse problem of source estimation is hard to solve since it is subject to a high number of uncertainties. They are caused mainly by diffusion, misplaced sensors, modeling inaccuracies, noise in the data and sensors malfunction. An effective inverse modelling technique, that combines composite misfit functions in Tikhonov-type regularization, a discrepancy-based choice of the regularization parameter, and the minimization with GA is proposed.

Considering the size of the computational domain and the fact that sensors were placed away from the emission source, it is possible to assert that reconstructions are satisfactorily accurate. More precisely, reconstructed spatial positions are close to the true one, and the estimated emission strengths almost match the actual ones.

Notice that, the parameters in Tab. 4, for the misfits in Eqs. (11)-(12), are more accurate than the corresponding ones in Tab. 3, as expected, since the best solutions are presented in the referred table. This also illustrates the accuracy of the proposed method, which is a result of the forward and inverse problem solutions. In other words, an accurate representation of the dispersion model is used as well as an inversion technique that addresses appropriately the uncertainties of the problem under consideration.

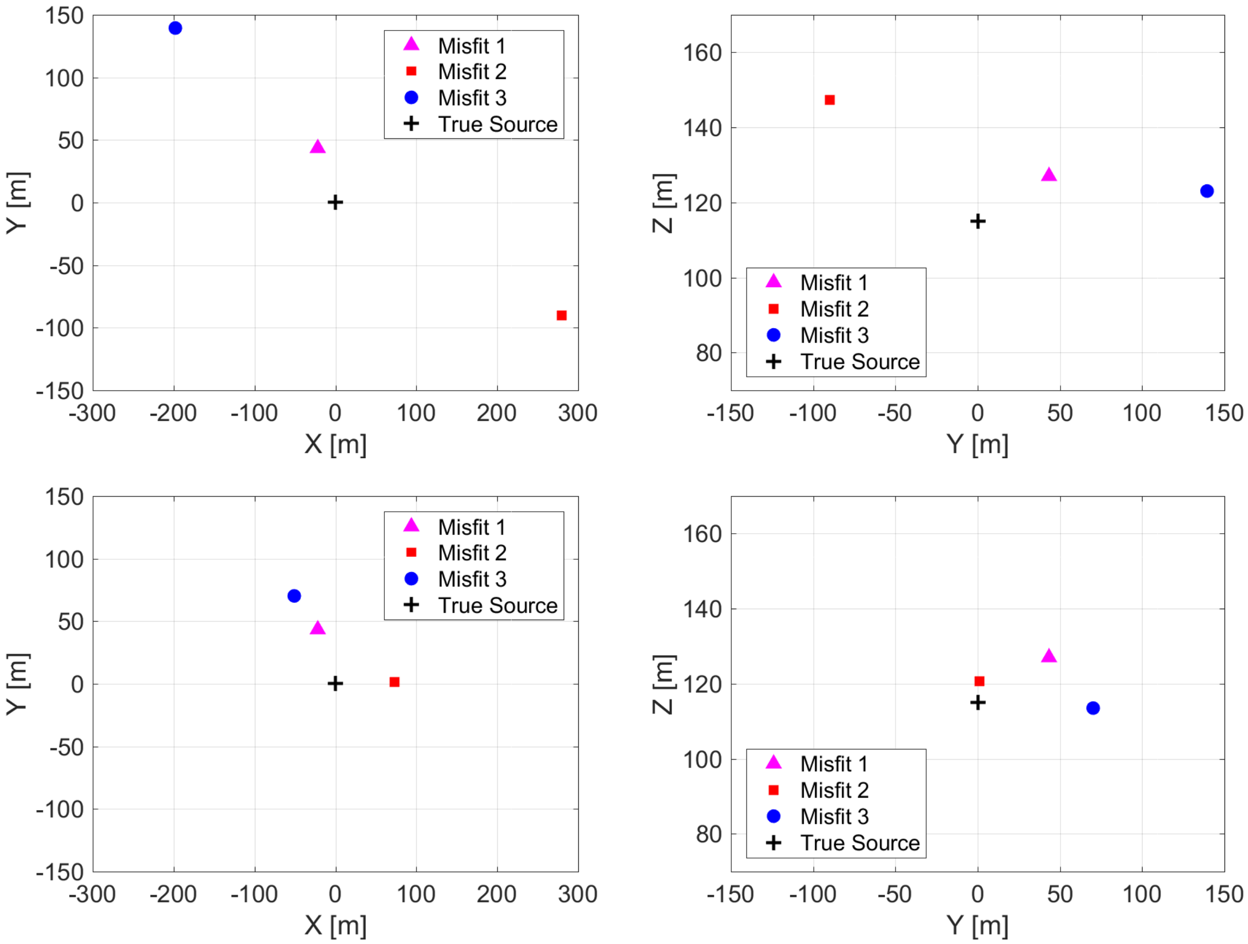

Figure 2. Comparison between true and the estimated parameters in Tab. 3 (top line) and Tab. 4 (bottom line).

5. CONCLUDING REMARKS

An accurate inversion methodology to estimate source parameters of atmospheric emissions is proposed. Such accuracy relies mainly on appropriate solutions for the forward and inverse problems. A more precise description of the forward problem improves considerably the inverse problem solution. Thus, to take it into account, the GLS formulation with parametric profiles for the wind and diffusion terms in the PDE in Eq. (5) is applied. Using data misfit functions that handle more appropriately the uncertainties and noise underlying the problem under consideration also improves the estimation. Hence, we addressed this task by using composite data misfits. The regularization parameter is chosen according to the concentration data through a Morozov-like discrepancy-based rule. This helps to avoid the reproduction of noise by solutions and the introduction of bias. Also to reduce the introduction of bias, the employed penalty term is the negative form of the Boltzmann-Shannon entropy, which has been frequently used as a penalty term alternatively to the square of the Euclidean norm. The resulting Tikhonov-type functional is then nonlinear and not necessarily convex, i.e., it may have different local minimum points. Therefore, the global minimization strategy is addressed by GA.

The proposed methodology is evaluated with experimental data from the Copenhagen Tracer Experiments and both, the forward and inverse problems solutions are satisfactorily accurate. Thus, the reconstructed source positions and emission strengths are close to the true one. This illustrates that the proposed inversion tool is sufficiently versatile to solve realistic emission source identification problems.

As an extension of this methodology, it is possible to add a potential correction or refinement of the result obtained by GA through a local search algorithm. Such a local search can be undertaken by some gradient descent method or some other metaheuristic technique, such as *Simulated Annealing* (Camps Echevarría *et al.*, 2019). Alternatively to GA, it is also possible to consider, for example, *Particle Swarm Optimization* (Camps Echevarría *et al.*, 2019), according to Albani *et al.* (2020).

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7. REFERENCES

Addepalli, B., Sikorski, K., Pardyjak, E. and Zhdanov, M., 2011. "Source characterization of atmospheric releases using stochastic search and regularized gradient optimization". *Inverse Probl. Sci. Eng.*, Vol. 19, No. 8, pp. 1097–1124. doi:10.1080/17415977.2011.589901.

Albani, R. and Albani, V., 2020. "An Accurate Strategy to Retrieve Multiple Source Emissions in the Atmosphere". *Atmos. Environ.*, Vol. 233, p. 117579. doi:10.1016/j.atmosenv.2020.117579.

Albani, R., Albani, V. and Silva Neto, A., 2020. "Source Characterization of Airborne Pollutant Emissions by Hybrid Metaheuristic/Gradient-based Optimization Techniques". To appear on *Environ. Pollut.*

Albani, R. and Albani, V., 2019. "Tikhonov-type regularization and the finite element method applied to point source estimation in the atmosphere". *Atmos. Environ.*, Vol. 211, pp. 69–78. doi:10.1016/j.atmosenv.2019.04.063.

Albani, V. and De Cezaro, A., 2019. "A Connection Between Uniqueness of Minimizers and Morozov-like Discrepancy Principles in Tikhonov-type Regularization". *Inverse Probl. Imaging*, Vol. 13, No. 1, pp. 211–229. doi: 10.3934/ipi.2019012.

Albani, V., De Cezaro, A. and Zubelli, J., 2017. "Convex Regularization of Local Volatility Estimation". *Int. J. Theor. Appl. Finan.*, Vol. 20,, No. 1, p. 1750006. doi:10.1142/S0219024917500066.

Arya, S.P., 2001. *Introduction to Micrometeorology*. Academic Press.

Camps Echevarría, L., Llanes Santiago, O., Campos Velho, H. and Silva Neto, A., 2019. *Metaheuristics for Optimization Problems*, Springer International Publishing, chapter 3 in Fault Diagnosis Inverse Problems: Solution with Metaheuristics, pp. 43–83.

Cover, T. and Thomas, J., 2006. *Elements of Information Theory*. John Wiley & Sons, 2nd edition.

Gryning, S.E., Holtslag, A.M., Irwin, J.S. and Sivertsen, B., 1987. "Applied modelling based on meteorological scaling parameters". *Atmos. Environ.*, Vol. 21, pp. 79–89. doi:10.1016/0004-6981(87)90273-3.

Gryning, S.E. and Lyck, E., 2002. "The copenhagen tracer experiments: Reporting of measurements". Technical Report 1054, Technical University from Denmark, Denmark. Forskningscenter Risoe.

Hanna, S.R., 1989. "Confidence limit for air quality models as estimated by bootstrap and jacknife resampling methods". *Atmos. Environ.*, Vol. 23, pp. 1385–1395. doi:10.1016/0004-6981(89)90161-3.

Holland, J., 1975. *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor, Michigan; re-issued by MIT Press (1992).

Hughes, T., Franca, L. and Hulbert, G., 1989. "A new finite element formulation for computational fluid dynamics: VIII. the Galerkin/Least-Square method for advective-diffusive equations". *Comput. Method. in Appl. M.*, Vol. 73, pp. 173–189. doi:10.1016/0045-7825(89)90111-4.

Ma, D., Deng, J. and Zhang, Z., 2013. "Comparison and improvements of optimization methods for gas emission source identification". *Atmos. Environ.*, Vol. 81, pp. 188–198.

Ma, D., Wang, S. and Zhang, Z., 2014. "Hybrid algorithm of minimum relative entropy-particle swarm optimization with adjustment parameters for gas source term identification in atmosphere". *Atmos. Environ.*, Vol. 94, pp. 637–646. doi:10.1016/j.atmosenv.2014.05.034.

Morozov, V., 1966. "On the solution of functional equations by the method of regularization". *Dokl. Math.*, Vol. 7, pp. 414–417.

Ulke, A.G., 2000. "New turbulent parameterization for a dispersion model in the atmospheric boundary layer". *Atmos. Environ.*, Vol. 34, pp. 1029–1042.

Wade, D. and Senocak, I., 2013. "Stochastic reconstruction of multiple source atmospheric contaminant dispersion events". *Atmos. Environ.*, Vol. 74, pp. 45–51. doi:10.1016/j.atmosenv.2013.02.051.

Yee, E., Lien, F.S., Keats, A. and D'Amours, R., 2008. "Bayesian inversion of concentration data: Source reconstruction in the adjoint representation of atmospheric diffusion". *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 96, No. 10-11, pp. 1805–1816. doi:10.1016/j.jweia.2008.02.024.

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