18th Brazilian Congress of Thermal Sciences and Engineering
November 16-20, 2020, (Online)

ENC-2020-0566
NONLINEAR TURBULENT REYNOLDS AVERAGE MODEL BASED THE NON-PERSISTENCE TENSOR

Bruno Jorge Macedo dos Santos
Angela Ourivio Nieckele
Dept. Mechanical Eng., Pontifical Catholic University of Rio de Janeiro, PUC-Rio, Rio de Janeiro, RJ, Brazil
brunojorge@aluno.puc-rio.br
nieckele@puc-rio.br

Abstract. *Reynolds average turbulence models need closure equation to determine the Reynolds stress tensor with the mean kinematic tensors. Traditionally, linear models based on the Boussinesq hypothesis are employed. However, it is well known that linear models fail in several applications and different non-linear models have been proposed. In the present work, an evaluation of a nonlinear model that is based on the rate-of-strain tensor and non-persistence tensor to predict a fully developed channel flow is examined. The time average velocity filed and Reynolds Stress Tensor components are compared with DNS data. Predictions of the mean velocity and shear components of the Reynolds stress were equivalent as the one obtained with the linear model. However, an improvement is obtained with respect to the normal components. While the linear models are unable to predict them, the present model is capable of presenting a reasonable prediction of two components, but is also fails for the third one.*

Keywords: *Reynolds stress modeling, nonlinear turbulent model, channel flow*

1. INTRODUCTION

Among the different strategies to predict a turbulent flow, Reynolds Average Navier-Stokes (RANS) are still the most popular, since it demands smaller computational effort. However, this methodology needs closure equations to determine the Reynolds stress tensor. The traditional models are based on the Boussinesq hypothesis, which assumes a linear relation between the Reynolds Stress Tensor, and the rate-of-strain tensor, with a proportionality parameter called as turbulent viscosity (Pope, 2000).

Aiming to improve the prediction of the linear models, non-linear models have been developed, employing combinations of the rate-of-strain tensor and vorticity tensor (Lien et al, 1991). However, within this approach, it is important to guarantee that the tensor has Euclidean invariance, objectivity and it is frame-invariant (Weis & Hutte, 2004 and Thompson & Mompean, 2010). To this end, Thompson et al. (2010) proposed to evaluate the Reynolds tensor based on rate-of-strain and non-persistence tensors. Furthermore, Nieckele et al. (2016) conducted *a priori* analysis of six different non-linear models based on these tensors, by employing DNS data, and demonstrated an improvement of the Reynolds stress prediction.

Both linear and non-linear models also need additional models to determine the proportionality parameters between the Reynolds stress and the mean kinematic tensors employed. For example, within the linear group of models, there are several different models to determine the turbulent viscosity, varying from algebraic models to differential models (Pope, 2000). These additional models also need some closure, and several employ the turbulent kinetic energy κ to represent the magnitude of the velocity fluctuations. However, there is no consent with regard to the best form to represent the turbulent scale. Among the most popular ones, it is possible to mention, the models based on the dissipation of the turbulent energy ($\kappa - \varepsilon$ family models, Rodi & Mansur, 1993) or specific rate of dissipation ($\kappa - \omega$ family models, Menter, 1994). The conservation equation to determine the turbulent kinetic energy can be obtained with a limited number of simplifying hypothesis. However, the conservation equation for both dissipation of the turbulent energy and specific rate of dissipation require strong simplifying hypotheses, and do not render good results for several applications. Aiming to provide a more robust model, Alves (2014) proposed to employ the norm of the strain deformation tensor $\dot{\gamma}$ to evaluate the characteristic turbulent length and in this way, to avoid the solution of ε or ω.

Murad *et al.* (2018) and Murad *et al.* (2020) conducted *a posteriori* analysis with some of the models proposed by Nieckele *et al.* (2016). The proportionality parameters were based on $\kappa - \varepsilon$. They showed that the *posteriori* analysis predicted the same behavior of the Reynolds stress as observed with the *a priori* analysis.

At the present work, the performance of a non-linear Reynolds stress model, based on the non-persistence straining tensor, to predict a channel flow is evaluated. The non-persistence straining tensor $\boldsymbol{P}$ is an objective tensor that can measure the ability of the fluid to avoid been stretched by the flow, playing an important hole in turbulent flow. It is defined based on $\boldsymbol{W}^*$, that is the relative vorticity, i.e., the vorticity computed with respect to the rate of rotation of the eigenvectors of $\boldsymbol{D}$ ($\boldsymbol{W}^* = \boldsymbol{W} - \boldsymbol{\Omega}^{\boldsymbol{D}}$), where $\boldsymbol{\Omega}^{\boldsymbol{D}} = \dot{\boldsymbol{e}}_{\boldsymbol{k}}^{\boldsymbol{D}}\, \boldsymbol{e}_{\boldsymbol{k}}^{\boldsymbol{D}}$, where $\boldsymbol{e}_{\boldsymbol{k}}^{\boldsymbol{D}}$ is the unit eigenvector of D_{ij} and $\dot{\boldsymbol{e}}_{\boldsymbol{k}}^{\boldsymbol{D}}$ is the material time derivative of $\boldsymbol{e}_{\boldsymbol{k}}^{\boldsymbol{D}}$. Here, the proportionality parameters are based on the turbulent kinetic energy κ and norm of the of the strain deformation tensor $\dot{\gamma}$.

2. MODELING

The channel plates separation is $2\,H$. The flow is considered as incompressible. The dimensionless velocity U_i and coordinates x_i are

$$U_i = \frac{U_i^*}{u_\tau} \quad ; \quad x_i = \frac{x_i^*}{\rho\, u_\tau / \mu} \tag{1}$$

where $u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity, τ_w is wall shear stress, ρ density and μ is the molecular viscosity. The dimensionless conservation of mass is

$$\frac{\partial\, U_i}{\partial\, x_i} = 0 \tag{2}$$

The flow is fully developed, thus a periodic boundary condition is applied in the main flow direction (x −direction), with pressure p decomposed in an area-average pressure $\bar{p}$ and a perturbation $\tilde{p}$, $p = \bar{p}(x) + \tilde{p}\,(x,y)$, with the gradient of the average pressure in the axial direction balanced by the wall shear. The dimensionless Reynolds average Navier Stokes equations can be written as

$$U_j \frac{\partial U_i}{\partial x_j} = \frac{1}{Re_\tau} e_i e_x - \frac{\partial \hat{p}}{\partial x_i} + \frac{\partial\,(2\,D_{ij})}{\partial x_j} + \frac{\partial a_{ij}}{\partial x_j} \tag{3}$$

where $e_i e_x = 1$ when $i = x$ (the main flow direction) and 0 when $i = y$ (in the normal direction), with $Re_\tau = \rho\, u_\tau\, H\,/\mu$, as the friction Reynolds number. This term is due to the gradient of the average pressure in the axial direction. Here, a modified pressure is defined as $\hat{p} = \tilde{p} + 2/\,3\,\kappa$, where κ is the turbulent kinetic energy. D_{ij} is the dimensionless symmetric part of the mean velocity gradient (rate of strain)

$$D_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{4}$$

and a_{ij} is the dimensionless traceless Reynolds stress.

$$a_{ij} = -\frac{\overline{u_i' u_j'}}{u_\tau^2} + \frac{2}{3}\,\kappa\;\delta_{ij} \quad ; \qquad \kappa = \frac{1}{2}\,\frac{\overline{u_k' u_k'}}{u_\tau^2} \tag{5}$$

with u_i' as the velocity fluctuation, and δ_{ij} is the delta de Kronecker function.

Following the proposition of Thompson et al. (2010) and Nieckele et al. (2016), to better capture the Reynolds stress characteristics, the traceless Reynolds stress is modeled based on the rate of strain tensor D_{ij} (as traditionally done) and the non-persistence tensor P_{ij} as

$$a_{\boldsymbol{ij}} = \alpha_D\; 2\; D_{ij} + \beta_P\; P_{ij} \quad , \tag{6}$$

where

$$P_{ij} = D_{ik} W_{kj}^* - W_{ik}^* D_{kj} \quad ; \quad W_{\boldsymbol{ij}}^* = W_{\boldsymbol{ij}} - \Omega_{ij}^D \quad ; \quad W_{\boldsymbol{ij}} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \tag{7}$$

The parameters $\alpha_D$and β_P must be written as a function of the flow variables. Traditionally α_D is modeled based on the turbulent kinetic energy κ and its dissipation ε, and additional conservation equation for both variables must be solved. However, it is well known, that the equation for ε has several uncertainties, and it fails in several applications. Thus, following the recommendation of Alves (2014), these two parameters were determined here based on the turbulent kinetic energy κ and the norm of strain deformation tensor $\dot{\gamma}$. By defining the deformation tensor as

$$\gamma_{ij} = 2\; D_{ij} \quad , \tag{8}$$

its norm can be obtained by

$$\dot{\gamma} = \sqrt{2\, D_{ij} D_{ij}} \tag{9}$$

2.1 Model $\kappa - \dot{\gamma}$

The dimensionless parameters of the non-linear Reynolds stress models are

$$\alpha_D = f_\mu C_\mu \frac{\kappa}{\dot{\gamma}} \quad ; \quad \beta_P = f_\beta C_\beta \frac{\kappa}{\dot{\gamma}^2} \tag{10}$$

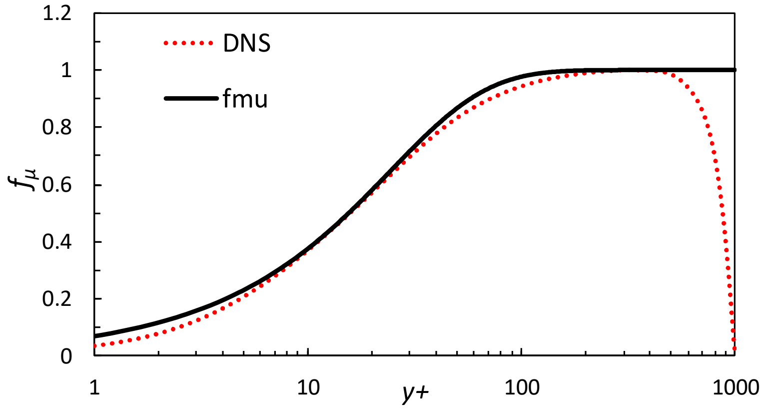
where the constants $C_\mu = 0.261$ and $C_\beta = 1.67$ where adjusted based on the DNS data (Thais *et al.*, 2012), as well as the damping functions f_μ and f_β to be applied only at the wall region, $y^+ \leq 100$.

At the present work, the following damping functions were defined based on the wall distance $y^+ = \rho\, u_\tau y\, /\mu$ as

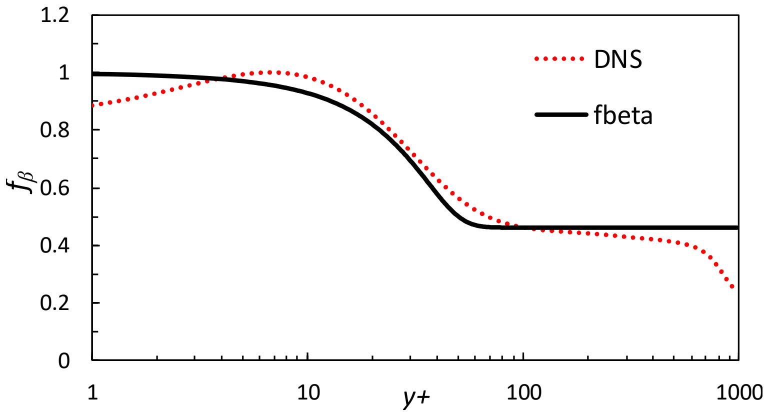
$$f_\mu = \tanh\left(0.07 {y^+}^{0.75}\right) \tag{11}$$

$$f_\beta = \left\{0.025 + \exp\left[-0.0305 - 0.0236\, y^+ - 0.0013\, {y^+}^2\right]\right\}^{0.21} \tag{12}$$

Fig. 1 shows that the proposed dimensionless damping functions present a good agreement with the DNS data at the wall region, $y^+ \leq 100$.



(a) f_μ damping function



(b) f_β damping function

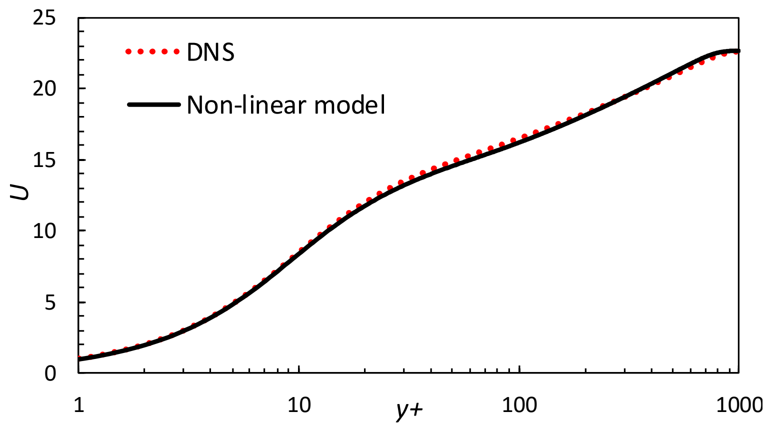
Figure 1. Comparison between the dimensionless damping functions with the DNS data for f_μ and f_β.

Once the model coefficients depend on the turbulent kinetic energy, this variable needs to be obtained. To this end, formulation of Rodi & Mansur (1993) was selected.

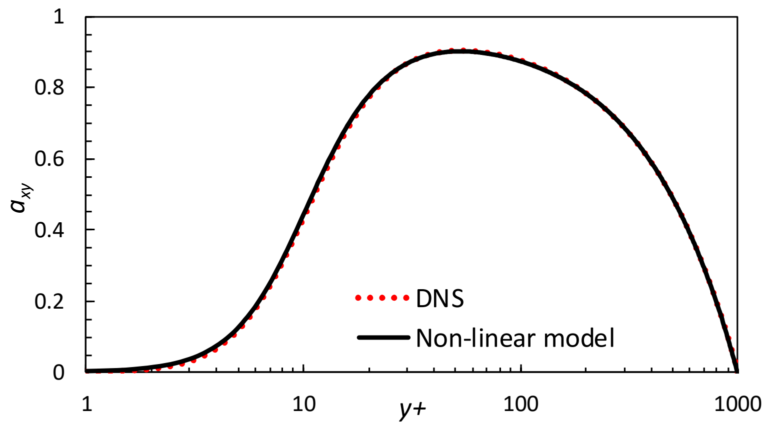
3. RESULTS

To evaluate the model, the fully developed flow between two parallel plates is solved for friction Reynolds number $Re_\tau = 1000$, and the time-average velocity and the components of the Reynolds stress tensor are compared with the DNS data of Thais *et al.* (2012).

Figure 2(a) compares the time average velocity field obtained with the present non-linear model, employing $\kappa - \dot{\gamma}$, with the DNS time average velocity data. A very good agreement can be seen. The agreement of the shear Reynolds stress obtained with the present model and the DNS data (Fig. 2b) is also excellent. These results show that the non-linear model with the proposed near wall damping factors can predict a good time average axial velocity as well as the shear Reynolds stress.



(a) time average axial velocity



(b) a_{xy} shear Reynolds stress

Figure 2. Comparison between the dimensionless velocity field and dimensionless Reynolds stress with DNS reference.

Here it must be said, that exactly the same result was obtained by both time average axial velocity and shear Reynolds stress, by the following linear model.

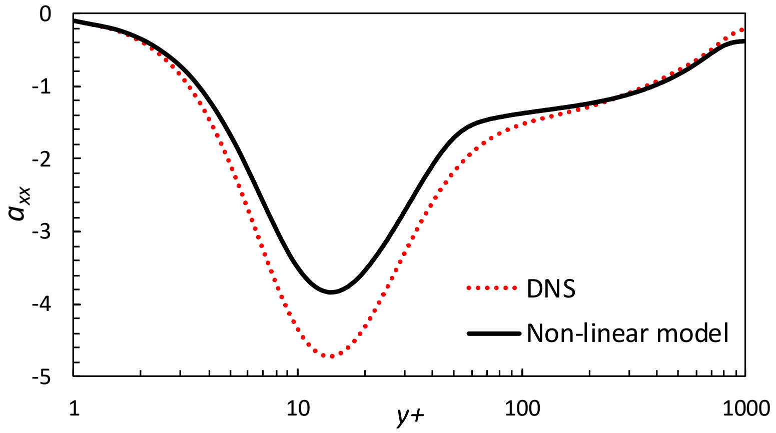
$$a_{ij} = \alpha_D \, 2 \, D_{ij} \quad ; \quad \alpha_D = f_\mu C_\mu \frac{\kappa}{\dot{\gamma}} \tag{13}$$

This means that the contribution of the non-persistence tensor P_{ij} for the present case is negligible. However, the linear model is unable to predict any normal Reynolds stress component.

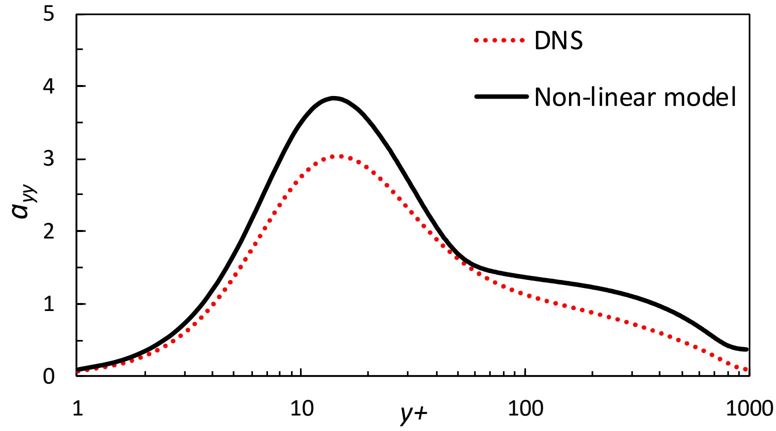
The present model is also unable to predict the a_{zz} component, however, reasonable predictions of a_{xx} and a_{yy} were obtained, as can be observed in Fig. 3. Component a_{xx} (Fig. 3a) is underestimated, while component a_{yy} (Fig. 3b) is over predicted. By the present analysis, it is shown that the inclusion of the non- non-persistence tensor P_{ij} to evaluate the Reynolds stress is positive, capturing flow behavior that the linear models is incapable. However, it also shows that important phenomena are still not capture by the present model.

4. CONCLUSIONS

A non-linear model based on the rate of strain tensor D_{ij} and the non-persistence tensor P_{ij}, both objective tensors was applied to the fully developed channel flow. The model parameters depend on the turbulent kinetic energy and the norm of the deformation tensor. Damping function were developed and showed a very good agreement with the DNS values.



(a) normal a_{xx}Reynolds stress component



(b) normal a_{yy}Reynolds stress component

Figure 3. Comparison between dimensionless Reynolds stresses with DNS data.

The results of the simulations showed an improvement of the normal Reynolds stress prediction, although the model was not able to determine the a_{zz} component. This result indicates that there are important contributions to the Reynolds stress which are not being represented by the tensors selected, and other combinations of the mean kinematic tensors must be considered.

Murad et al (2018) analyzed of a quadratic rate of strain Reynolds stress tensor model, and observed that while a_{xx} component was underestimated and a_{yy} presented an inverted sign. Further, that model was able to predict the a_{zz} component. By these two analysis, a new non-linear model that incorporates both tensor (quadratic rate of strain $\boldsymbol{D}^2$ and non-persistence tensor $\boldsymbol{P}$), seems to be a good candidate and it will be investigated in a future work.

5. ACKNOWLEDGEMENTS

The authors thank the Brazilian Government agencies CAPES and CNPq for the continuous support during the development of this research.

6. REFERENCES

Alves, F., 2014. *O uso da base de dados DNS para construção de modelos RANS utilizando decomposições tensoriais e coeficiente de bases normalizadas.* Ms.C. thesis, Univerisdade Federal Fluminense, Niterói, Brasil.

Lien, F.; Chen, W.; Leschziner, M. 1991, Low-Reynolds-Number Eddy-Viscosity Modelling Based on Non-Linear Stress-Strain/Vorticity Relations. Engineering Turbulence Modelling and Experiments, p.91-100.

Menter, F., 1994, Two-equation eddy-viscosity turbulence models for engineering applications. AIAA Journal, 1994. P.1598-1605.

Murad, F. W., Sampaio, L. E. B., Thompson, R. L., Nieckele, A. O., 2018. “Evaluation of a quadratic rate of strain Reynolds stress tensor model for a chanel flow employing DNS data”. Turbulence, Heat and Mass Transfer.

Murad, F. W., Nieckele, A. O., Sampaio, L. E. B., Thompson, R. L., 2020. "Evaluation of different levels of approximation of the Reynolds Stress Tensor for channel flow employing DNS data". In 5th Thermal Fluids Engineering Conference. New Orleans, LA, USA.

Nieckele, A. O., Thompson, R. L., Mompean, G., 2016. "Anisotropic Reynolds stress tensor representation in shear flows using DNS and experimental data" J. of Fluids, Vol. 17, pp: 602-632.

Pope, S. B., 2000. Turbulent Flows. Cambridge University Press.

Rodi, W. Mansour, N.N. 1993. Low Reynolds number $\kappa - \varepsilon$ modelling with the aid of direct simulation data. J. Fluid Mech, 250: 509-529, 1993.

Thais, L., Gatski, T. B., Mompean, G., 2012. "Some dynamical features of the turbulent flow of a viscoelastic fluid for reduceddrag". J. Turbulence. Vol. 13, pp: 1-26.

Thompson,R.L, Mompean, G. "An alternative assessment of weak-equilibrium conditions in turbulent closure modeling", .Int. J. Eng. Sci., 48, pp: 1633–1640 (2010).

Thompson, R. L., Mompean, G., Thais, L., 2010. "A methodology to quantify the nonlinearity of the Reynolds Stress Tensor". J. Turbulence, Vol. 33, pp:1-27.

Thompson, R. L., Sampaio, L. E. B., Alves, F. A. V. B.,Thais, L., Mompean, Gilmar., 2016. "A methodology to evaluate statistical errors in DNS data of plane channel flows". Computers and Fluids. Vol. 130, pp: 1-7.

Weis, J., Hutter, K. 2004. On Euclidean invariance of algebraic Reynolds stress models in turbulence, J. Fluid Mech., 518, pp: 63–68 (2004).

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.