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AN OVERVIEW OF NONLINEAR PIEZOELECTRIC ENERGY HARVESTING USING A NONLINEAR TWO-DEGREES-OF-FREEDOM PORTAL FRAME PLATFORM

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Abstract: The objective of this work is to present an overview of energy harvesting using a nonlinear two-degrees-of-freedom portal frame platform with a nonlinear piezoelectric material coupled to one of its columns and externally base-excited. The nonlinear platform possesses two-to-one internal resonance between its two vibration modes and presenting the saturation phenomenon. The nonlinearities of the piezoelectric material are considered by a nonlinear mathematical relation. Here, it is considered an electro-dynamical shaker with harmonic output. The employed methodology to carry out the analysis of this work was: the application of the method of multiple scales to find the best configuration of the parameters, and to find some kind of phenomena due to the two-to-one internal resonance; several numerical simulations were carried out to optimize the energy harvesting through parametrical variations, bifurcation diagrams, stability diagrams. It will be analyzed: the influence of the nonlinearity of the piezoelectric material and of the electro-dynamical shaker on the energy harvesting. Results showed great influence of the nonlinearity of the material and using the electro-dynamical device. It was possible to gain considerably in energy harvesting and stability of the system.

Keywords: Energy harvesting, electro-dynamical shaker, saturation phenomenon, nonlinear piezoelectric material, nonlinear portal frame platform

INTRODUCTION

In the past few years, there is a great demand for electrical energy due to the high level of population growth. With the great world demand for electrical energy, many researchers, in Brazil and in the World, have concentrated their efforts to seek for new energy sources. One of the newest and most promising means of energy transduction is through the kinetic energy, which is one of the most abundantly available energy source found in the environment (Rocha, 2016).

Among the possible energy harvesting devices that are capable to extract/scavenge ambient vibration energy induced in some kind of structures, the piezoelectric materials have been of great interest in the technical-scientific community. These materials are commonly called intelligent materials or multi-functional due to their significantly response for stimulus of different physical natures (Preumont, 2006; Priya and Inman, 2009). These materials have been used as a low-power energy transducer from ambient vibration that enables harvest energy from wasted vibration energy. The technique of energy harvesting using these kind of piezoelectric devices have been studied by Preumont (2006); Priya and Inman (2009); Erturk *et al.* (2009); Erturk and Inman (2011); Litak *et al.* (2012); Syta *et al.* (2015), Stanton *et al.* (2010); Jalili (2009); Stephen (2006), among others.

However, the piezoelectric materials possess a certain nonlinearity between its strain constant and the electric field (DuToit and Wardle, 2007; Twiefel *et al.*, 2008). The researchers Crawley and Anderson (1990) demonstrated experimentally this nonlinearity exhibiting a significantly dependence in the induced deformation of the material. More recently, Tripplet and Quinn (2009) purposed an approximation between the results of a nonlinear model to the experiment results separated in two coefficients that are the linear piezoelectric coefficient and the nonlinear piezoelectric coefficient, which is the adjust to the experimental curve. With that, many works showed the importance of the nonlinear coefficient to the results of the energy harvesting based on piezoelectric materials, showing the gain or loss of energy depending on the value of the nonlinear coefficient (Iliuk *et al.*, 2013a,b, 2014; Balthazar *et al.*, 2014; Rocha, 2016). An overview of the nonlinearities presented by the piezoelectric material was carried out by Daqaq *et al.* (2014).

Generally, the energy harvesting is introduced when the piezoelectric material is coupled to a vibrating structure. However, some kind of structures may possess certain nonlinearity presenting different kind of behaviours. A structure of two-degrees-of-freedom (2DOF) having quadratic nonlinearities under two-to-one internal resonance involving its two modes of vibration, i.e., $\omega_2 = 2\omega_1 + \sigma$, where ω_2 is the natural frequency of the second mode, ω_1 is the natural frequency of the first mode, and σ is a detuning factor, and an external resonance between an external excitation and the second mode, i.e., $\Omega = \omega_2 + \sigma$, there is a phenomenon called *saturation*. When the amplitude of excitation is small, only the second mode is excited. As the amplitude reaches a critical value, which depends on the damping, the excited mode becomes saturated and the vibration energy "spills over" into the other mode so that the first mode begins to vibrate. This is the *saturation phenomenon* described by many authors, for example, Nayfeh *et al.* (1973); Nayfeh (2000); Nayfeh and Mook (2008); Nayfeh and Balachandran (2008); Haddow *et al.* (1984); Mook *et al.* (1985); Mankala and Quinn (2004); Quinn (2007); Rocha (2016). A big gamma of works considering saturation were studied as in Golnaraghi (1991); Oueini *et al.* (1997); Oueini (1999); Pai *et al.* (1998); Pai and Schulz (2000); Balthazar *et al.* (2003); Shoeybi and Ghorashi (2005); Warminski *et al.* (2013); Felix *et al.* (2014); Tusset *et al.* (2015); Felix *et al.* (2005); Pratt *et al.* (1999); Hall *et al.* (2001); Ashour and Nayfeh (2002), among others.

Some kind of electromechanical devices are capable to be used as an external exciter. One of them is the electro-dynamical shaker whose excitation performed by the shaker has been used by several authors considering a harmonic kind input voltage (Xu *et al.*, 2005, 2007; Lee *et al.*, 2008; Lenci *et al.*, 2012; Litak *et al.*, 2010; Alevras *et al.*, 2014; Avanço *et al.*, 2015).

Therefore, this work presents the analysis of energy harvesting from a two-degrees-of-freedom structure considering two-to-one internal resonance. The saturation phenomenon will be evaluated through the method of multiple scales, and in the following, numerical simulations and discussions about parametrical analysis related to the behaviour and energy harvesting of the system will be commented.

This work was organized as follows: In the first section was discussed a completely introduction about the themes of the research. The second section showed the mathematical and physical model to be studied and its governing equations of motion. The third section showed the numerical simulations and some discussions about the parametrical variations. The fourth and last section presented the conclusions about this work.

Hence, the next section shows the mathematical and physical model to be studied and its governing equations of motion.

ENERGY HARVESTING PORTAL FRAME MODEL

The schematic system illustrated in Fig. 1 represents a two-degrees-of-freedom portal frame with a piezoelectric material coupled to one of its columns under excitation induced to this base via an electro-dynamical shaker.

The portal frame structure consists of two columns clamped in their bases with length *h* and a horizontal beam pinned to the columns at both ends with length *L* so that both columns and beam have flexural stiffness *EI*. The mass of the mid-span of the beam is *M* and the masses of the columns are *m*. The structure is modelled as a lumped mass system with two-degrees-of-freedom whose generalized coordinates are q_1, which is related to the horizontal displacement (first mode) of mass *M* with natural frequency ω_1, and q_2, which is related to the vertical displacement (second mode) of mass *M* with natural frequency ω_2. The linear stiffness of the columns and beam can be evaluated by a Rayleigh-Ritz procedure using cubic trial functions. Geometric nonlinearity is introduced by considering the shortening due to bending of the columns and beam.

The nonlinear piezoelectric material distributed along the column is considered as an electric circuit consisting of a resistor R_p, a produced charge Q_p, and a capacitance C_p of the capacitor. Moreover, the dimensionless relations of its strain constant is given by $d(q_1) = \theta(1+\Theta|q_1|)$ defined by Triplett and Quinn (2009), where θ is the linear piezoelectric strain coefficient and Θ is the nonlinear piezoelectric strain coefficient.

The electro-dynamical shaker is considered as an electromechanical system. The mechanical part of the shaker consists of its base with mass m_0, stiffness k_0 and damping c_0 with displacement represented by S_0. The electrical part of the shaker consists of an electrical circuit RL with resistance R_0, inductance L_0 and an electric charge Q_{s0}, where $I_0 = dQ_{s0}/dt$. The input voltage of the shaker represents the induced external force in the base, consequently, exciting the main portal frame structure, that is $F_{ext} = e_0 \cos\omega_n t$, where e_0 is de amplitude of the external excitation and ω_n is the frequency of the external excitation. The external frequency is set near resonance with the second mode, which is the twice of the frequency of the first mode, i.e., $\omega_n \approx \omega_2 + \sigma_2$ and $\omega_2 \approx 2\omega_1 + \sigma_1$. These are the conditions to saturation phenomenon occurs.

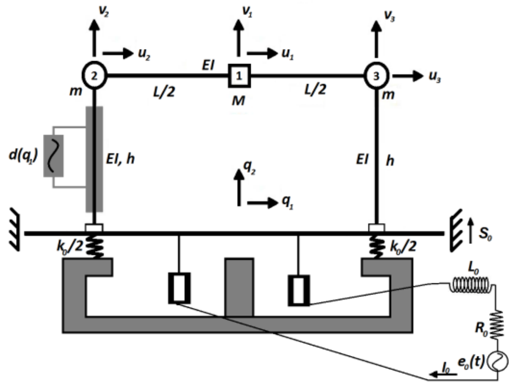


Figure 1 - Schematic model of a two-degrees-of-freedom portal frame with a piezoelectric coupled to a column under excitation induced by the electro-dynamical shaker coupled to its base

Knowing the nodal displacements of the main structure given that Eq. (1).

$$\begin{aligned} u_1 &= q_1 & u_2 &= u_1 + \frac{B}{4}v_1^2 & u_3 &= u_1 - \frac{B}{4}v_1^2 \\ v_1 &= S_0 + q_2 & v_2 &= S_0 - \frac{A}{2}u_1^2 & v_3 &= S_0 - \frac{A}{2}u_1^2 \end{aligned} \tag{1}$$

The dimensionless equations of motion of the full system is given by the Eqs. (2)-(6) (Rocha, 2016).

$$x_1'' + \mu_1 x_1' + x_1 + \alpha_1 x_1 x_2 = \beta_1 x_1' Y' + \theta\left(1+\Theta\left|x_1\right|\right)\delta_1 V_p \tag{2}$$

$$x_2'' + \varepsilon Y'' + \mu_2 x_2' + \omega_2^2 x_2 + \alpha_2 x_1^2 + G_0 = \theta\left(1+\Theta\left|x_1\right|\right)\delta_2 V_p x_2 \tag{3}$$

$$Y'' + \delta_m \varepsilon x_2'' + \mu_0 Y' + \omega_0^2 Y + \gamma_1 U + G_1 = \beta_2 x_1'^2 + \theta\left(1+\Theta\left|x_1\right|\right)\delta_1 V_p \tag{4}$$

$$U' + \gamma_2 U - \gamma_3 Y' = E_0 \cos\Omega\tau \tag{5}$$

$$V_p' - \theta\left(1+\Theta\left|x_1\right|\right)\left(\delta_3\left(x_1+Y\right)+\delta_4 x_2^2\right)+\delta_3 V_p = 0 \tag{6}$$

where

$$x_1 = \frac{q_1}{h} \quad x_2 = \frac{q_2}{L} \quad Y = \frac{S_0}{h} \quad V_p = \frac{Q_p}{q_0} \quad \tau = \omega_1 t \quad U = \frac{I_0}{i_0} \quad \omega_1 = \sqrt{\frac{2\left(k_c - mgA\right)}{2m+M}} \quad \hat{d}\left(x_1\right) = \frac{h}{q_0}d\left(q_1\right)$$

$$\mu_1 = \frac{c_1}{\left(2m+M\right)\omega_1} \quad \mu_2 = \frac{c_2}{M\omega_1} \quad \beta_1 = \frac{2mAh}{\left(2m+M\right)} \quad \varepsilon = \frac{h}{h} \quad G_0 = \frac{g}{\omega_1^2 L} \quad E_0 = \frac{e_0}{L_0\omega_1 I_0} \quad \delta_m = \frac{M}{M_T}$$

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{k_b}{M}} \quad \omega_0 = \frac{1}{\omega_1}\sqrt{\frac{k_0}{M_T}} \quad \mu_0 = \frac{c_0}{M_T\omega_1} \quad \alpha_1 = \frac{Ak_bL}{\left(2m+M\right)\omega_1^2} \quad \alpha_2 = \frac{Ak_bh^2}{2M\omega_1^2L} \quad G_1 = \frac{\left(2m+M\right)g}{M_T\omega_1^2 h}$$

$$\gamma_1 = \frac{KI_0}{M_T\omega_1 h} \quad \gamma_2 = \frac{R_0}{L_0\omega_1} \quad \gamma_3 \frac{Kh}{L_0 I_0} \quad M_T = 2m + M + m_0 \quad \beta_2 = \frac{2MAh}{M_T}$$

$$\delta_1 = \frac{q_0^2}{\omega_1^2 h^2\left(2m+M\right)C_p}, \quad \delta_2 = \frac{Bq_0^2}{2M\omega_1^2 hC_p}, \quad \delta_3 = \frac{1}{RC_p\omega_1}, \quad \delta_4 = \frac{BL^2}{4RC_p\omega_1 h}, \quad \Omega = \frac{\omega_n}{\omega_1}$$

To calculate the harvested power through the piezoelectric material, Eqs. (7) and (8) are given as dimensionless instantaneous and average harvested power, respectively.

$$P = R_0 V'^2 \tag{7}$$

$$P_{avg} = \frac{1}{T}\int_0^T P\left(\tau\right)d\tau \tag{8}$$

where $R_0 = R_p(\omega_1 q_0)^2$.

Therefore, in the next section will be shown data about the numerical simulations and discussions.

ANALYTICAL AND NUMERICAL ANALISYS AND DISCUSSIONS

The numerical simulations carried out in this work are given using the parameters of Tab. 1 through the software MATLAB.

Table 1 - Dimensional parameters of the portal frame system

Parameters	Values	Means
$g[m/s^2]$	9.81	Gravity acceleration
$M[kg]$	2.00	Beam mass
$m[kg]$	0.50	Column mass
$m_0[kg]$	15.88	Base mass
$c_1[Ns/m]$	1.55	Column damping
$c_2[Ns/m]$	3.14	Beam damping
$c_0[Ns/m]$	534	Base damping
$EI[Nm^2]$	128	Linear flexural stiffness
$k_0[kg/m]$	86176	Base stiffness
$L[m]$	0.52	Beam length
$h[m]$	0.36	Column length
$e_0[V]$	40	Amplitude of the shaker
$R_0[\Omega]$	0.3	Electric resistance of the shaker
$L_0[mH]$	2.626	Inductance of the shaker
$K[N/A]$	130	Electromagnetic force of the shaker
$\omega_n[rad/s]$	148	Frequency of the shaker
$R_p[k\Omega]$	100	Electric resistance of the piezoelectric
$C_p[\mu F]$	1	Capacitance of the piezoelectric
θ	Vary	Linear piezoelectric coefficient
Θ	0 or 1	Nonlinear piezoelectric coefficient

All the parameters of Tab. 1 will be considered as default except when it is exposed. The nonlinear piezoelectric material will be considered linear as default so that the nonlinear contribution will be presented before its usage.

In the next section, the method of multiple scales will be performed in order to confirm the existence of saturation phenomenon in the portal frame.

Method of Multiple Scales (MMS)

The method of multiple scales is a method to find an approximated analytical solution of a nonlinear system. This method will be applied only to the main portal frame foundation excited by a representative harmonic force to find the interactions of the modal coupling of its degrees-of-freedom. Considering the method to a first-order approximation solution and neglecting Eqs. (4)-(6) and their terms, the possible solutions of Eq. (2) and (3) are given by Eq. (9).

$$\begin{aligned} x_1 &= \varepsilon x_{11}(T_0,T_1)+\varepsilon^2 x_{12}(T_0,T_1) \\ x_2 &= \varepsilon x_{21}(T_0,T_1)+\varepsilon^2 x_{22}(T_0,T_1) \end{aligned} \tag{9}$$

where ε is a small dimensionless parameters of the order of the amplitude of oscillation and $T_n = \varepsilon^n t$ are the time scales. In order to have the damping and nonlinear terms in the same perturbation equations, it was scaled the damping μ_i by letting to $\varepsilon\mu_i$.

Substituting Eqs. (9) into Eqs. (2) and (3) and equating coefficients of order ε, it is obtained

Order ε

$$\begin{aligned} D_0^2 x_{11} + x_{11} &= 0 \\ D_0^2 x_{21} + \omega_2^2 x_{21} &= 0 \end{aligned} \tag{10}$$

Order ε^2

$$\begin{aligned} D_0^2 x_{12} + x_{12} &= -2D_0 D_1 x_{11} - \mu_1 D_0 x_{11} - \alpha_1 x_{11} x_{21} \\ D_0^2 x_{22} + \omega_2^2 x_{22} &= -2D_0 D_1 x_{21} - \mu_2 D_0 x_{21} - \alpha_2 x_{11}^2 + E_0 \cos\Omega\tau \end{aligned} \tag{11}$$

where $D_n = \partial/\partial T_n$.

The solutions of Eqs. (10) can be written in the form

$$\begin{aligned} x_{11} &= A_1(T_1)\exp(iT_0)+cc \\ x_{22} &= A_2(T_1)\exp(i\omega_2 T_0)+cc \end{aligned} \tag{12}$$

where cc are complex conjugate terms.

Substituting Eqs. (12) into Eqs. (11), leads to Eqs. (13).

$$\begin{aligned} D_0^2 x_{12} + x_{12} &= -i\left(2A_1' + \mu_1 A_1\right)\exp\left(iT_0\right) - \alpha_1 \begin{bmatrix} A_1 A_2 \exp\left(i\left(1+\omega_2\right)T_0\right) + A_1 \bar{A}_2 \exp\left(i\left(1-\omega_2\right)T_0\right) + \ldots \\ \bar{A}_1 A_2 \exp\left(i\left(-1+\omega_2\right)T_0\right) + \bar{A}_1 \bar{A}_2 \exp\left(-i\left(1+\omega_2\right)T_0\right) \end{bmatrix} + cc \\ D_0^2 x_{22} + x_{22} &= -i\omega_2\left(2A_2' + \mu_2 A_2\right)\exp\left(i\omega_2 T_0\right) - \alpha_2\left[A_1^2 \exp\left(2iT_0\right) - 2A_1\bar{A}_1 + \bar{A}_1^2 \exp\left(-2iT_0\right)\right] + \ldots \\ & \qquad \frac{E_0}{2}\exp\left(i\Omega T_0\right) + cc \end{aligned} \tag{13}$$

Depending on the value of the natural frequencies, there are terms that connect themselves in equations related to x_1 and x_2. Because of the quadratic nonlinearity present in the system, when the system has 2:1 internal resonance and external resonance, has a different particular solution. Therefore, introducing detuning parameters σ_1 and σ_2 according to

$$\begin{aligned} \omega_2 &= 2\omega_1 - \varepsilon\sigma_1 = 2 - \varepsilon\sigma_1 \\ \Omega &= \omega_2 + \varepsilon\sigma_2 \end{aligned} \tag{14}$$

the solvability conditions (secular terms) are given as follows in Eqs. (15).

$$\begin{aligned} & -i\left(2A_1' + \mu_1 A_1\right) - \alpha_1 \bar{A}_1 A_2 e^{-i\sigma_1 T_1} = 0 \\ & -i\omega_2\left(2A_2' + \mu_2 A_2\right) - \alpha_2 A_1^2 e^{i\sigma_2 T_1} + \frac{E_0}{2} e^{i\sigma_2 T_2} = 0 \end{aligned} \tag{15}$$

Introducing polar notation in Eqs. (15), and eliminating θ_1 and θ_2, the approximated modal equations of the system with 2:1 internal resonance and external resonance, separated into real and imaginary parts, is given by Eqs (16).

$$\begin{aligned} a_1' &= -\frac{\mu_1}{2} a_1 - \frac{\alpha_1}{4} a_1 a_2 \sin\gamma_1 \\ a_2' &= -\frac{\mu_2}{2} a_2 + \frac{\alpha_2}{4\omega_2} a_1^2 \sin\gamma + \frac{E_0}{2\omega_2 a_2} \sin\gamma_2 \\ \gamma_1' &= \left(\frac{\alpha_2 a_1^2}{4\omega_2} - \frac{\alpha_1 a_2^2}{2}\right)\frac{\cos\gamma_1}{a_2} - \frac{E_0}{2\omega_2 a_2}\cos\gamma_2 + \sigma_1 \\ \gamma_2' &= -\frac{\alpha_2 a_1^2}{4\omega_2 a_2}\cos\gamma_1 + \frac{E_0}{2\omega_2 a_2}\cos\gamma_2 + \sigma_2 \end{aligned} \tag{16}$$

where $\gamma_1 = \theta_2 - 2\theta_1 + \sigma_1 T_1$ and $\gamma_1 = \sigma_2 T_1 - \theta_2$.

Using the values of the parameters in Tab. 1, which shows the 2:1 internal resonance and the external resonance, Eqs. (16) are numerically integrated.

Figure 2 shows the numerical integration of Eqs. (16), which shows the modal displacements of the vertical (a_2 in black) and horizontal (a_1 in red) coordinates. It is possible to observe the vibration energy exchange between the two modes continuously dissipating until a steady state where the amplitude of the horizontal keeps to be higher than the vertical, i.e., the excited second mode partially transferred vibration energy to the first one after the saturation of its motion.

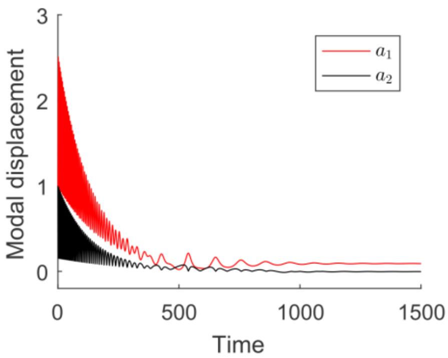


Figure 2 - Modal oscillation of the amplitudes of the system considering 2:1 internal resonance and external resonance: $a_1(0) = a_2(0) = 1$

With the method of multiple scales was possible to find the existence of saturation in the portal frame structure. Knowing that, the next step is to implement the presence of the electro-dynamical shaker and the piezoelectric material in order to harvest energy from the wasted vibration energy transferred to the columns.

Full Portal Frame System Numerical Analysis

From this part of this work, the portal frame with a piezoelectric coupled to its column under excitation of an electro-dynamical shaker in its base, i.e., all Eqs. (2)-(6) will be considered. The interesting is study the influence of the nonlinear piezoelectric material on various numerical analysis, as much the linear piezoelectric materials as the amplitude of external of excitation.

First of all, the analysis of the linear piezoelectric coefficient and amplitude of excitation will be investigated without the nonlinear piezoelectric contribution ($\Theta = 0$). After that, the same analysis will be evaluated with the presence of the nonlinearity of the piezoelectric material ($\Theta = 1$), and then, comparing both cases.

The next subsection will analyze the energy harvesting considering just the linear piezoelectric case.

Case of Sole Linear Piezoelectric Material

In this first analysis, the case of sole linear piezoelectric material is considered ($\Theta = 0$). The harvested power strongly depends on the linear piezoelectric coefficient, hence, the variation of the linear coefficient versus the harvested power is carried out by Fig. 3. It is possible to observe the increase of the harvested power until the value of $\theta = 0.3038$ when the maximum average power reaches $P_{max} = 79.79$. With $\theta > 0.3038$, the amount of power decreases until zero abruptly.

The bifurcation diagrams constructed with the same parametrical analysis, illustrated by Figs. 4a and 4b, which presents the behaviour of horizontal and vertical motions, respectively, shows that in the interval $0 \leq \theta \leq 0.3830$ the system is periodic-2. When $\theta > 0.3038$, the system tends to be periodic-1 in a first moment, and after that, its amplitude becomes higher enough that can damage the structure.

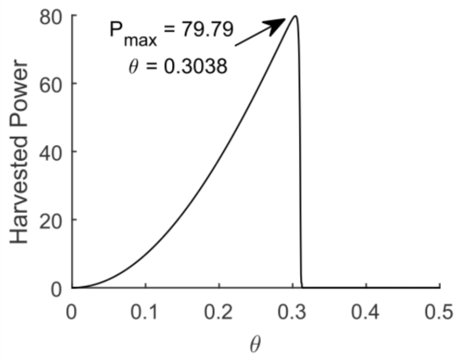


Figure 3 - Parametrical analysis of linear piezoelectric coefficient versus dimensionless average harvested power of the sole linear piezoelectric case (Θ = 0)

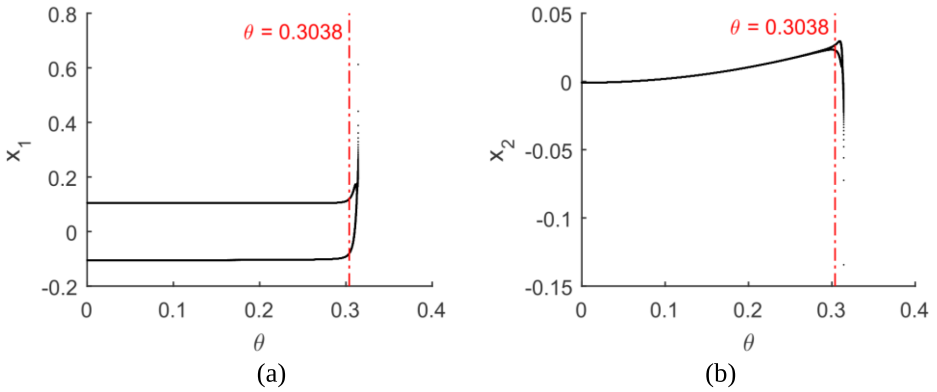


Figure 4 – Bifurcation diagram of linear piezoelectric coefficient versus dimensionless average harvested power of the sole linear piezoelectric case (Θ = 0)

A general analysis of the amplitude and energy harvesting of the structure is given by Figs. 5a, 5b and 5c. These figures showed the parametrical variation of the linear piezoelectric coefficient versus the dimensionless amplitude of excitation related to the steady state amplitude of motion of the structure, Fig. 5a, and related to the harvested power, Figs. 5b and 5c.

In Fig. 5a, the coloured area shows when the system presents amplitudes high enough to damage the system, that region was called unsafe because its intensity. The safe area that the vibrations work with small amplitudes represents that physically correspond to a structural vibration. The maximum values that the system will not be damage and will be possible to harvest energy is when $\theta = 0.2915$ and $E_0 = 456.76$.

The same safe area related to the harvested power, in Figs. 5b and 5c, shows that the amount of power increases from the maximum obtained before, which is $P = 79.79(\theta = 0.3038, E_0 = 205.84\ (e_0 = 40V))$, to $P_{lin} = 174.1$.

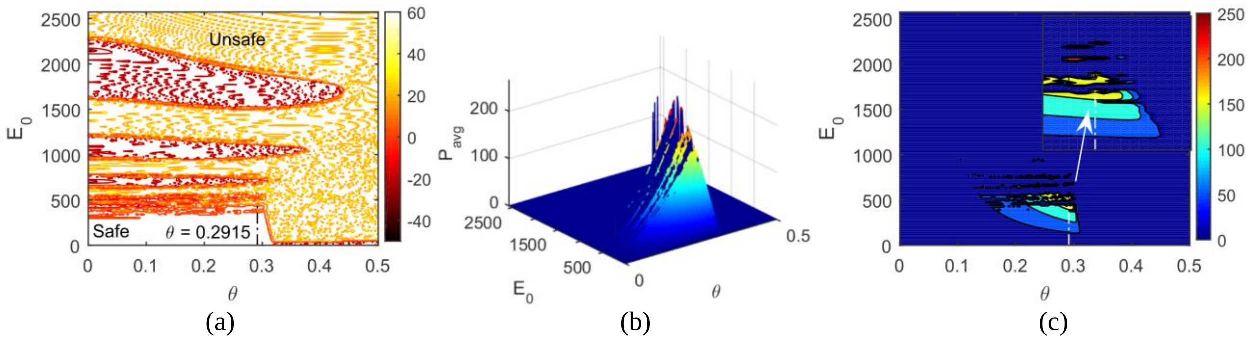


Figure 5 – Sole linear piezoelectric case (Θ = 0), (a) contour of the maximum displacement of the structure related to the parametrical variation of E_0 and θ; (b) surface and (c) colored contour of the average harvested power related to the parametrical variation of E_0 and θ

In the next subsection, the case of nonlinear piezoelectric material with the same analysis of this subsection will be exposed in order to be compared and concluded the important influence of the linear piezoelectric coefficient to this kind of system.

Case of Nonlinear Piezoelectric Material

From here, the nonlinear piezoelectric coefficient will be considered as Θ = 1, i.e., totally nonlinear. As the harvested power was analyzed in the linear case, the same parametrical variation was carried out in Fig. 6. With the nonlinear coefficient, the maximum average amount of power was obtained with a smaller value of the linear piezoelectric coefficient, which is $\theta = 0.2763$, however with a higher value of power, which is $P_{max} = 83.85$. After $\theta >$ 0.2763, the amount of power tends to be zero.

The bifurcation diagrams of the horizontal and vertical motion illustrated in Figs. 7a and 7b, respectively, shows that the system continues to be periodic-2 in the linear coefficient ratio where the amount of power increases, after that the system tends its amplitude of motion to infinite faster than in the linear case.

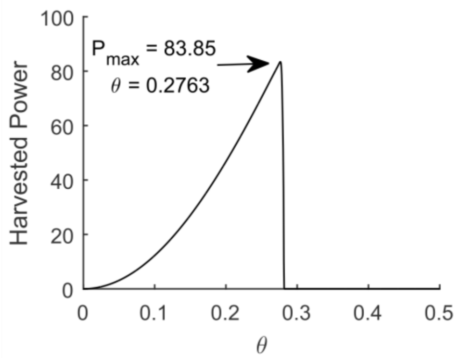


Figure 6 - Parametrical analysis of linear piezoelectric coefficient versus dimensionless average harvested power of the nonlinear piezoelectric case (Θ = 1)

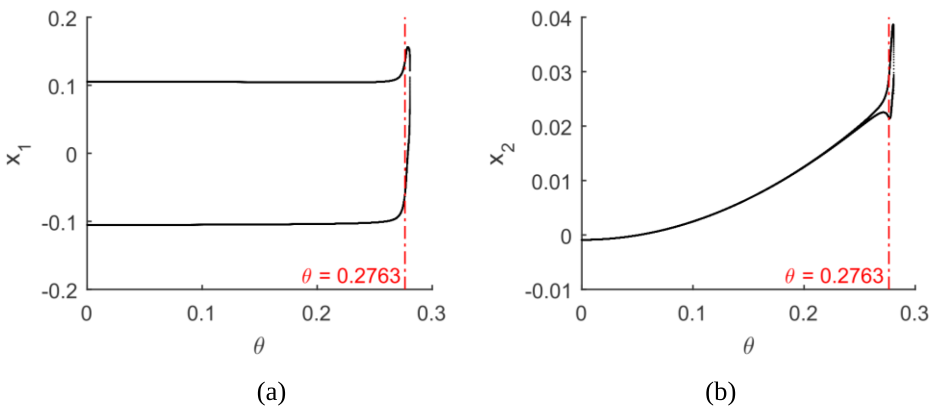
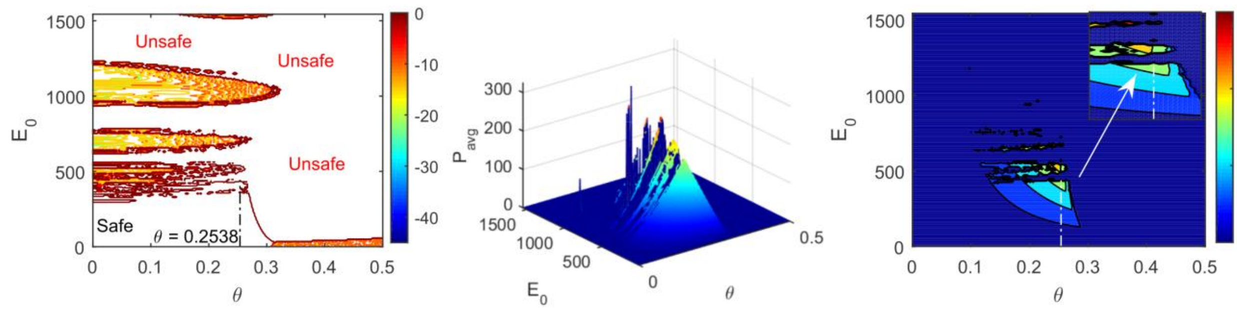


Figure 7 - Bifurcation diagram of linear piezoelectric coefficient versus dimensionless average harvested power of the nonlinear piezoelectric case (Θ = 1)

A general analysis of the amplitude and energy harvesting of the structure is given by Figs. 8a, 8b and 8c, however considering the nonlinear case. The figures showed the parametrical variation of the linear piezoelectric coefficient versus the dimensionless amplitude of excitation related to the steady state amplitude of motion of the structure, in Fig. 8a, and related to the harvested power, in Figs. 8b and 8c.

In Fig. 8a, the coloured area shows the unsafe region where the amplitudes of motion are very high. Considering the nonlinear coefficient, the safe region where the amplitudes are small enough to harvest the vibration energy, which is given by $\theta = 0.2538$ and $E_0 = 438.29$, is smaller than the linear case.

In Figs. 8b and 8c, even with a small safe region the harvested power is higher than in the sole linear case, which was $P_{lin} = 174.1$ ($\theta = 0.2915$, $E_0 = 456.76$) and becomes $P_{nlin} = 177.4$.



(a) (b) (c)

Figure 8 - Nonlinear piezoelectric case (Θ = 1), (a) contour of the maximum displacement of the structure related to the parametrical variation of E_0 and θ; (b) surface and (c) colored contour of the average harvested power related to the parametrical variation of E_0 and θ

With the whole discussion of the results of numerical simulations considering and not considering the nonlinear piezoelectric coefficient, some conclusions are taken out.

CONCLUSIONS

This work presented the analysis of the nonlinear piezoelectric coefficient related to the energy harvesting of a two-degrees-of-freedom portal frame structure excited by an electro-dynamical shaker. Moreover, the method of multiple scales was carried out to find out the saturation phenomenon between the two modes of vibration of the main structure.

With the method of multiple scales was possible to develop an approximated analytical solution to the modal amplitudes of the horizontal and vertical coordinates of the portal frame structure. This approximation showed that, with the 2:1 internal resonance and the external resonance, saturation phenomenon is presented in the main structure, making possible to harvest energy from the column`s vibration.

The nonlinear piezoelectric coefficient has a foremost importance to the energy harvesting because depending on its value the amount of power can increase or decrease. As it was observed in the last two subsections, there was some changes in the safe values of the amplitude of excitation and linear piezoelectric coefficient. Considering the nonlinear coefficient, the safe values decreases, however the energy harvesting increases. A summary of the safe regions comparing the linear and nonlinear cases is shown in Tab. 2.

Table 2 - Summary of the safe regions of the paper

Case Θ	θ	E_0	Average Power
0	0.2915	456.76	174.1
1	0.2538	438.29	177.4

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